Rotating Bubble Benchmark

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This case represents the flow inside a spherical bubble rising in a rotating fluid with a gravity perpendicular to the rotation axis. This is a pure Navier-Stokes case, there is no magnetic field or buoyancy.

The code solves the Navier-stokes equation:

$$\partial_t u + u\nabla u + 2\Omega \times u = -\nabla p + \nu \nabla^2 u \tag{1}$$

with particular boundary conditions that correspond to the flow surrounding the bubble.

The resulting flow is important at the center of the bubble, making it suitable to benchmark *full-sphere* codes.

1 the setup

The rotation axis Ω is parallel to the z-axis, while gravity is along the x-axis. The velocity **imposed** at the surface of the sphere is:

$$u_\theta = -u_0 \cos\theta \cos\phi$$
$$u_\phi = u_0 \sin\phi$$

which is the gradient of a pure $\ell = 1, m = 1$ spherical harmonic $Y_1^1(\theta, \phi)$.

2 parameters

- $0 \leq r \leq 1$
- $u_0 = \sqrt{\frac{3}{2\pi}} \simeq 0.69099$ (because of spherical harmonic normalizations).
- $\nu = 0.01$
- $\Omega = 10$

3 results

The flow converges quickly towards a stationary solution, in which $m = 1$ is dominant. There is also a non-zero axisymmetric (around the rotation axis) flow produced, but all spherical harmonic modes are populated.

3.1 Should be reported:

- Total kinetic energy: $E_c = 0.0618062$.
- Angular momentum (along the z-axis): $L_z = 0.02777102$.
- Kinetic energy for $m = 0$: $E_c^0 = 4.3445 \times 10^{-4}$.
- Kinetic energy for $m = 1$: $E_c^1 = 0.0612593$.
- Kinetic energy for $m = 2$: $E_c^2 = 1.17436 \times 10^{-4}$.
- $U_x(r = 0)$: -0.00825753.
- $U_y(r = 0)$: 0.0382824.
- $U_z(r = 0)$: 0.

Optionally, a measure of performance can be reported: $Perf = N/(p.t.f)$, where N is the number of independent variables ($N = R^3$), p the number of cpu cores used, t the time in seconds, and f the cpu frequency in GHz. For my reference case, 500 radial grid points, $m_{max} = 31$, and variable ℓ_{max} going from 1 at the center to 63 at the outer shell ($N = 842214$), running on 8 cores at 2.5GHz, with 79ms per iteration, I report $Perf = 5.33 \times 10^{-4}$.

3.2 Additional information (not requested):

The figures are a snapshot at $t = 400$, but at $t = 50$ the flow is already well converged. $\ell_{max} = m_{max} = 63$ were used for the spherical harmonic truncation with 200 radial grid points (finite differences). The time-step was set to 10^{-3}.

The following figures show the flow and the spectra, they have been draw using the data in the associated .tar.gz file, (which also contains matlab routines for plotting them). A full 3D data file is also available in *matlab* and *hdf5* format.

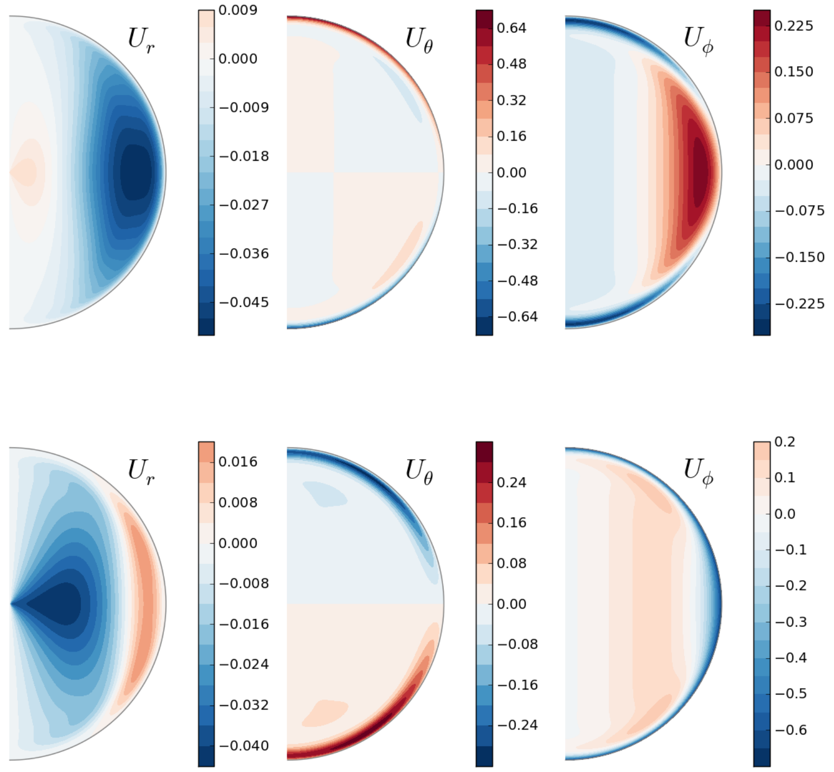

Figure 1: Velocity field in the xz plane (top, $\phi = 0$) and in the yz plane (bottom, $\phi = \pi/2$).

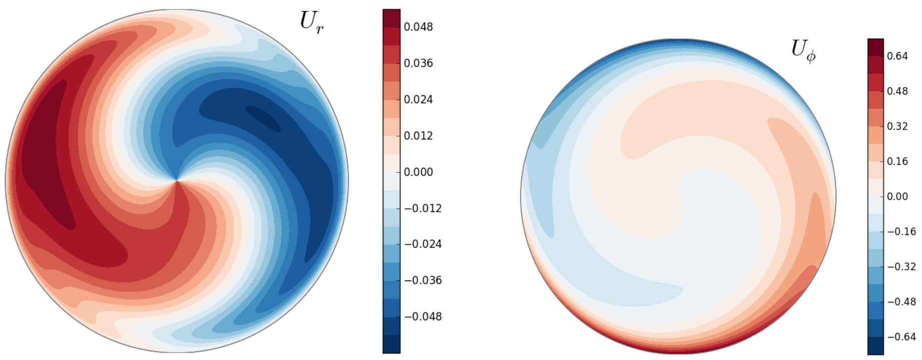

Figure 2: Velocity field in the xy plane (the equatorial plane, perpendicular to the rotation axis).

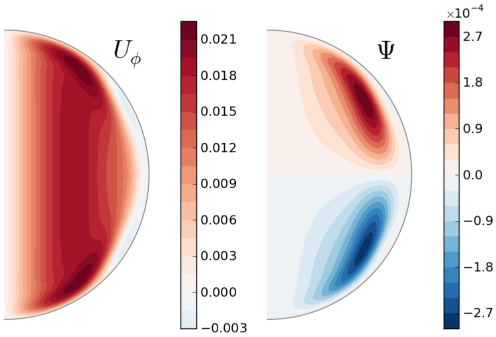

Figure 3: Zonal velocity field ($m = 0$) in the meridional plane (including the rotation axis z).

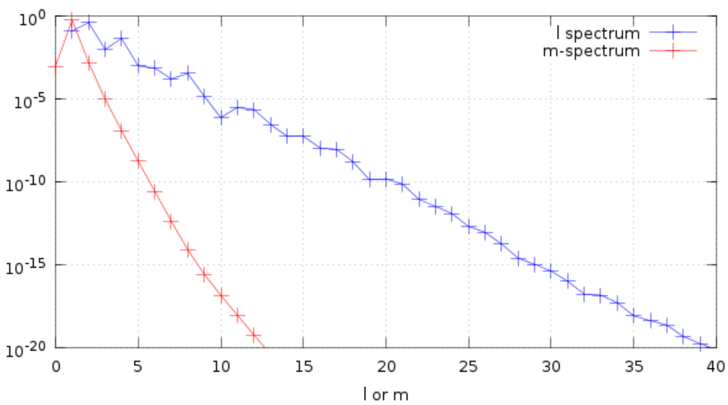

Figure 4: Spectra of kinetic energy at $r = 0.95$. Blue for ℓ-spectrum E_ℓ, red for m-spectrum E_m. The spectra are defined as: $E_m(r) = \sum_\ell |\mathbf{u}_\ell^m(r)|^2$ and $E_\ell(r) = \sum_m |\mathbf{u}_\ell^m(r)|^2$.