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On the ability of phase transitions and viscosity layering to induce long wavelength heterogeneity in the mantle

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Abstract. The ability of phase transitions and a viscosity jump at 660 km depth to induce long-wavelength flow in the mantle is systematically investigated using three-dimensional numerical simulations of internally-heated convection at convective vigors (as indicated by the Rayleigh number *Ra*) ranging from mildly supercritical to greater than the Earth's current regime. Cases with neither a viscosity jump nor phase changes display a steadily decreasing wavelength with increasing *Ra*, as expected. Increasing the lower mantle viscosity by a factor of 30 induces significantly longer wavelengths, but the "reddening" effect decreases with increasing *Ra*; this is explained in large part by the decrease in effective Rayleigh number due to increasing the lower mantle viscosity. In contrast, the effect of phase transitions is minor at low *Ra* but increases sharply at geodynamically relevant *Ra*, potentially increasing the mean wavelength of thermal heterogeneity by factors of greater than 10. Conclusions regarding the relative importance of the two effects are thus highly dependent on *Ra*.

Introduction

The lateral spectrum of heterogeneity in the Earth's mantle is observed by seismic tomography to be "red", i.e., dominated by long wavelengths [*Su and Dziewonski*, 1992], whereas laboratory and numerical experiments indicate that thermal convection with simple material properties tends to adopt an aspect ratio similar to the depth of the layer [*Schubert*, 1979, 1992], giving a much broader lateral spectrum. This discrepancy between observation and simple experiments is particularly pronounced in cases driven by complete or mostly internal heating [*Houseman*, 1988; *Travis et al.*, 1990; *Parmentier et al.*, 1994], which, to first order, may be most representative of the heating mode in Earth's mantle [*Schubert*, 1979; *Davies and Richards*, 1992]. Many mechanisms have been proposed for causing "reddening" (i.e., increasing the dominant wavelengths of thermal heterogeneity), in particular (i) plate lengths imposing a long wavelength to the flow [*Davies*, 1988; *Zhong and Gurnis*, 1994] (although such an explanation seems to propel the question one step further: what causes the plates to be long?) (ii) the thermal insulation effect of mobile continents [*Gurnis and Zhong*, 1991], (iii) temperature-dependent viscosity [*Daly*, 1980; *Tackley*, 1993], (iv) depth-dependent properties such as viscosity and thermal expansivity [*Balachandar et al.*, 1992; *Hansen et al.*, 1993; *Zhang and Yuen*, 1995], and (v) an endothermic phase transition [*Tackley et al.*, 1993]. While these may all be important, this paper focuses on mechanisms (iv) (in particular, viscosity stratification) and (v).

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The idea that an endothermic phase transition at 660 km depth is capable of inducing long wavelength flow in the mantle was first put forward by *Tackley et al.* [1993], based on a comparison of two simulations in three-dimensional (3-D), spherical-shell geometry. Recently, however, a more complete study of 100% internally-heated 3-D spherical convection [*Bunge et al.*, 1996a, 1996b] indicated that in the parameter range studied, the spectral effect of the endothermic phase transition was quite minor in comparison to the effect of a 30-fold viscosity increase at that depth. The purpose of this paper is thus to examine these claims, comparing and contrasting the spectral "reddening" effects of a phase change and viscosity jump, and focusing in particular on how these scale with convective vigor, as indicated by the Rayleigh number (*Ra*). This study is performed by means of numerical simulations of 100% internally-heated convection in a Cartesian box, at *Ra* ranging from mildly supercritical to values higher than that characterizing the current Earth's mantle.

Model

For computational convenience, 3-D Cartesian geometry is assumed. This allows a much wider exploration of parameter space than would be possible in 3-D spherical geometry with currently available resources. Geometry is not expected to affect the observed trends, but may have an influence on the exact planform when the dominant wavelength becomes comparable to the size of the box [*Tackley*, 1996]. An aspect ratio 4 box with reflecting side boundaries is assumed. While this aspect ratio is perhaps a factor of 2 smaller than the effective aspect ratio of the Earth, it allows some first-order results to be obtained at a parameter range (that is, in terms of obtaining realistic heat flow and velocities) which is realistic for the Earth. Furthermore, the maximum possible wavelength is $4\text{x}2\sqrt{2}=11.3$ (corresponding to a downwelling in one corner of the box and an upwelling in the opposite corner), comparable to the (nondimensional) distance around the equator. As shown in several studies [*Travis et al.*, 1990; *Parmentier et al.*, 1994], the preferred aspect ratio for internally-heated convection in the absence of either of the effects studied here is very much smaller than 4, so that any reddening can be clearly discerned.

Depth-dependent properties associated with compressibility [*Balachandar et al.*, 1992; *Hansen et al.*, 1993; *Zhang and Yuen*, 1995] and temperature-dependent viscosity [*Daly*, 1980; *Tackley*, 1993] both have a reddening effect on mantle convection. In order to isolate the effect of phase changes and a viscosity jump on the dominant wavelengths, these two effects are thus left out of the model, so that the Boussinesq approximation with constant material properties is assumed. The lack of compressibility, depth-dependent properties and temperature-dependent viscosity may limit the applicability of this study to theoretical guidance rather than being a realistic model of the Earth; however, this is probably true of all current

3-D models since they lack plate tectonics, the first-order surface expression of mantle convection. The resulting equations are well known and reported, for example, in *Tackley* [1993, 1996].

The model includes both major phase transitions: the exothermic olivine to spinel transition at around 410 km depth and the endothermic spinel to perovskite + magnesiowüstite transition at around 660 km depth. The phase buoyancy parameters are +0.0778 and -0.1778 respectively (with reasonable material properties these correspond to Clapeyron slopes of roughly +2.5 and -4.0 MPa K^{-1}).

Solutions are obtained using the finite-volume multigrid code STAG3D, the details and benchmarking of which have been reported elsewhere [*Tackley*, 1993, 1996]. The buoyancy associated with vertical deflection of a phase-change boundary is parameterized as a sheet mass anomaly at the nominal phase-change depth, an approximation which is similar to the "effective α" approach [*Christensen and Yuen*, 1985] and appears to give a first-order validity sufficient for the present study [*Tackley*, 1994]. Numerical resolution is constant in the horizontal directions but with vertical grid refinement near 660 km and the surface. The number of grid cells used depends on the Rayleigh number with the maximum number used being 256x256x64. Tests at Ra_H=4x10^8 indicate no change in the character of the solution as resolution is increased from 128x128x32 cells to 256x256x64 cells, indicating that further resolution increases are unnecessary, although the solutions are Ra_H=8x10^9 are probably only marginally resolved. Cases were run until they reached a statistically steady-state, which typically took between 0.01 and 0.1 diffusion times, roughly equivalent to between 3 and 30 billion years.

Three sets of runs were performed, (i) reference cases, isoviscous with no phase changes, (ii) isoviscous cases with phase changes, and (iii) cases with a 30-fold viscosity jump at 660 km but no phase changes. A viscosity jump of around a factor of 30 is indicated by several geoid/topography studies [for reviews see *King*, 1995a, 1995b] as well as by the thickening of seismically observed slab thermal heterogeneities as they enter the lower mantle [*Grand*, 1994]; in addition, it provides compatibility between this study and the study of [*Bunge et al.*, 1996a, 1996b]. Internally-heated Rayleigh numbers Ra_H ranging from 2x10^3 to 8x10^9 (based on the upper mantle viscosity) are considered. These are roughly equivalent to temperature-based Rayleigh numbers Ra_T of 10^3 to 10^8 respectively.

Results

Figure 1 shows cold downwellings for two sets of cases, one at a low Ra_H of 10^6, (corresponding to $Ra_T \approx 10^5$) and one at high Ra_H of 4x10^8 ($Ra_T \approx 10^7$). At the lower *Ra*, the phase transitions cause a slight planform change, but the dominant wavelength, as indicated by the spacing of downwellings, is not much affected. The case with a viscosity jump, however, has a much longer wavelength flow pattern. In contrast, at the higher *Ra*, the reverse situation appears to be true. i.e., phase transitions cause a very strong reddening, whereas a viscosity jump has only a minor effect. From this visual comparison, it thus appears that the effect of the phase transition on wavelength increases with *Ra*, whereas the effect of a viscosity jump decreases with *Ra*.

These trends are now quantified by a spectral analysis of the thermal heterogeneity. This is performed by taking a horizontal, two-dimensional Fourier transform of the 3-D temperature field, followed by binning and summation of the power in spectral modes according to their spatial wavenumber k (given by $\sqrt{k_x^2+k_y^2}$, where k_x and k_y are the spatial wavenumbers in the x and y directions respectively) to produce the one-dimensional horizontal spectrum as a function of depth. The mean wavelength (defined as $\lambda_{mean} = \sum\left(\frac{2\pi}{k}a_k\right)/\sum a_k$, where a_k is the rms. amplitude at wavenumber k) is then calculated from the vertically-averaged version of this spectrum.

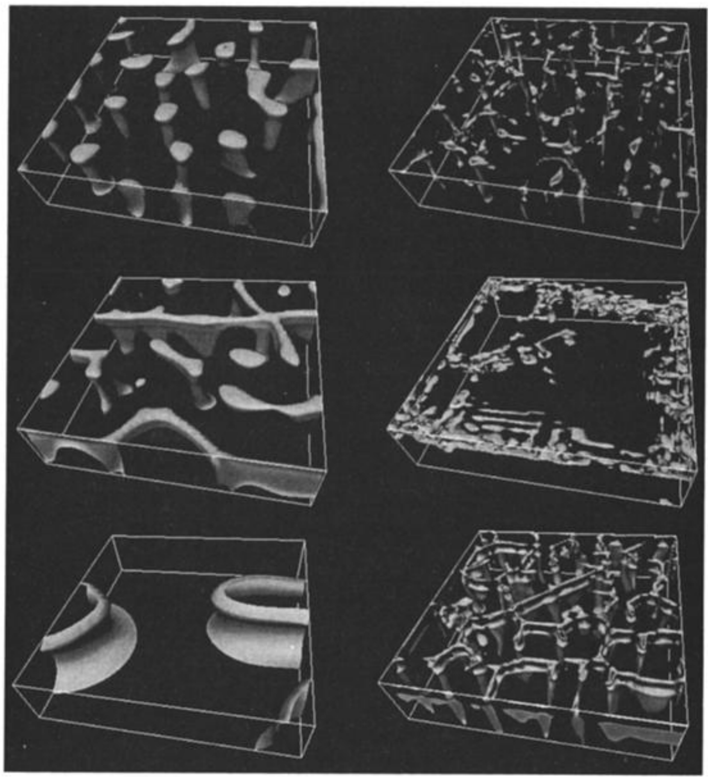

Figure 1. Cold downwellings for various cases at Ra_H=10^6 (left column) and Ra_H=4x10^8 (right column). Reference cases (top row), cases with phase transitions (middle row) and cases with a 30-fold viscosity increase at 660 km depth (bottom row). Plotted are isocontours of temperature anomaly showing where nondimensional temperature is 0.1 lower than the horizontally-averaged geotherm (the nondimensionalization used results in similar non-dimensional temperature variations at all *Ra*).

Figure 2a shows this mean wavelength as a function of *Ra*, for the three sets of calculations. The reference cases display a steady decrease in mean wavelength with increasing Rayleigh number, from ~3 at Ra_H=2x10^3 ($Ra_T \approx 10^3$) to <0.2 at Ra_H=8x10^9. ($Ra_T \approx 10^8$). This trend is expected from previous internally-heated results [*Parmentier et al.*, 1994], and is also displayed, though perhaps to a lesser degree, in basally-heated convection [*Malevsky and Yuen*, 1993]. With complexities included, the trends apparent in the visual comparison of Figure 1 are clearly apparent, i.e., the effect of a viscosity jump is large at low *Ra* but decreases as *Ra* is increased, whereas the effect of phase transitions is small at low *Ra* but increases sharply once the appropriate *Ra* range is reached. At very low *Ra*, the three curves approximately converge at a wavelength which is similar to the most unstable mode as predicted by linear stability analysis.

The plotting of viscosity-jump cases against a *Ra* defined using the upper mantle viscosity may be misleading, since the

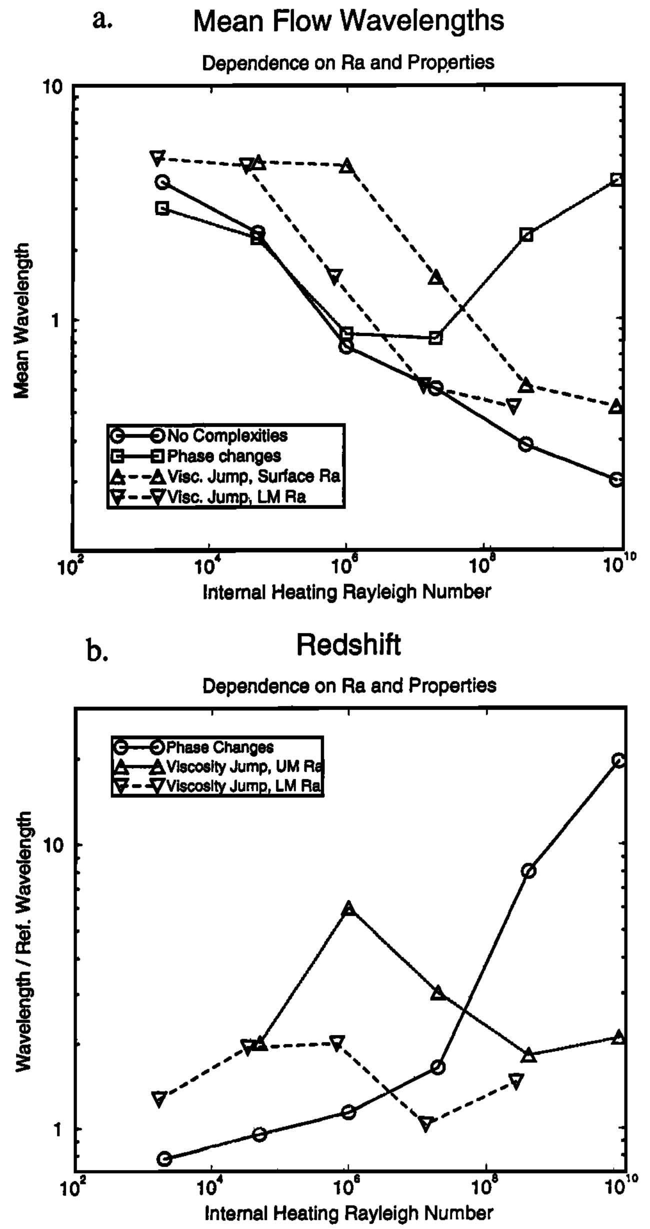

Figure 2. a. The mean wavelength of thermal heterogeneity plotted against Ra_H, for the three sets of calculations. Viscosity jump cases are also plotted against the lower mantle Ra_H. b. Redshift (see text for definition) as a function of Ra_H for phase transitions and a viscosity jump.

much higher lower mantle viscosity results in greatly reduced convective vigor. Thus, these results are also plotted against a *Ra* defined using the lower mantle viscosity; when so plotted, they lie much closer to the reference cases. This indicates that most, though not all, of the reddening is simply caused by a decrease in effective *Ra*: lower *Ra* results in longer-wavelength flow, as shown by the reference cases discussed above.

The *Ra* scaling of the reddening caused by the two physical complexities can be quantified by defining a "redshift" which normalizes results by the background trend displayed by the reference cases:

$$\text{Redshift}(Ra) = \frac{\lambda_{mean}(Ra)}{\lambda_{mean\_ref}(Ra)} \qquad (1)$$

where λ_{mean} is the mean wavelength for a particular case and "ref" indicates the reference cases. The *Ra* dependence of this redshift is plotted in Figure 2b, and very clearly illustrates the trends. A viscosity jump appears to give a large redshift (up to ~6) at low *Ra*; however, when *Ra* is defined by the lower mantle viscosity, the redshift is less than 2 at all *Ra*, and is lower at high *Ra*. In contrast, the redshift caused by phase transitions is small (or even less than unity) at low *Ra*, but increases sharply beyond Ra_H~10^7 (Ra_T~10^6)

Discussion and Conclusions

It is clear from these results that conclusions regarding the relative influence of phase transitions and viscosity jumps on increasing the dominant wavelengths of mantle convection are highly regime dependent, such that at low *Ra*, the effect of a viscosity jump is dominant, whereas at high *Ra*, the effect of phase changes are dominant. The latter is not surprising since the effect of an endothermic phase transition on all aspects of mantle flow is well-known to increase with *Ra* [*Christensen and Yuen*, 1985; *Yuen et al.*, 1994]. The *Ra* range in which the redshift increases sharply corresponds to the range in which "avalanches" (i.e., the intermittent pooling of downwellings above the 660 km discontinuity and the subsequent vigorous flushing of pooled material into the lower mantle) [*Christensen and Yuen*, 1985; *Tackley et al.*, 1993; *Solheim and Peltier*, 1994] become increasingly important. At low *Ra*, avalanches do not occur. At intermediate *Ra*, avalanches are weak, downwelling material pools only briefly above the 660 km discontinuity, and large-scale structure is not affected. However, when *Ra* is high enough for strong layering to take place, (i) avalanches become less frequent, and (ii) due to temperature differences that build up between the upper and lower mantles during periods of local layering, avalanches become much more vigorous, with each one associated with considerable mass exchange between upper and lower mantles. In this regime, an individual avalanche may be sufficiently large and vigorous to organize flow over a wide area, as large as the entire computational domain of these calculations, thereby inducing long-wavelength flow and thermal heterogeneity. At even higher *Ra*, previous simulations [*Christensen and Yuen*, 1985; *Yuen et al.*, 1994] indicate that avalanches would shut off altogether, completely layered convection would result, and the dominant wavelengths would probably decrease, as indicated in basally heated phase-change calculations [*Christensen and Yuen*, 1985; *Yuen et al.*, 1994] and enforced two-layered calculations [*Glatzmaier and Schubert*, 1993].

To date, 3-D spherical studies of phase-transition modulated mantle convection [*Tackley et al.*, 1993, 1994; *Bunge et al.*, 1996a, 1996b] have only reached the intermediate regime, where phase changes have a relatively small effect on the dominant wavelengths. In particular, the long-wavelength flow pattern observed by [*Tackley et al.*, 1993] was probably due largely to the combination of depth-dependent parameters (including viscosity) and relatively large (40%) fraction of basal heating.

The present convective regime of the Earth is more uncertain since effects such as plates, the rigidity of subducted slabs, phase diagram complexity, and uncertainties in parameter values make the true situation much more complicated than that modeled here. Although it is impossible to define a unique *Ra* for Earth, a comparison of model boundary layer thickness with the thickness of oceanic lithosphere (essentially comparing the heat flux), would place the Earth at slightly above $Ra_H=10^8$ on Fig. 2. Seismic and other evidence, however [*Jordan et al.*, 1993; *Tackley*, 1995; *Vanderhilst*, 1995], suggest that the Earth is currently only a weak avalanche regime, and thus other mechanisms, such as plates, are probably dominant in determining the observed long-wavelength heterogeneity. Additionally, the total viscosity increase over the mantle may be much larger than the factor of 30 modeled here [*van Keken et al.*, 1994]. Of course, the *Ra* scaling demonstrated here and elsewhere [*Christensen and Yuen*, 1985; *Yuen et al.*, 1994] indicates that strong avalanches and perhaps complete layering should have been important earlier in Earth's evolution, when the *Ra* was much higher [*Christensen and Yuen*, 1985; *Steinbach et al.*, 1993].

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