The role of viscous heating in Barrovian metamorphism of collisional orogens: thermomechanical models and application to the Lepontine Dome in the Central Alps

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ABSTRACT

Thermal models for Barrovian metamorphism driven by doubling the thickness of the radiogenic crust typically meet difficulty in accounting for the observed peak metamorphic temperature conditions. This difficulty suggests that there is an additional component in the thermal budget of many collisional orogens. Theoretical and geological considerations suggest that viscous heating is a cumulative process that may explain the heat deficit in collision orogens. The results of 2D numerical modelling of continental collision involving subduction of the lithospheric mantle demonstrate that geologically plausible stresses and strain rates may result in orogen-scale viscous heat production of 0.1 to >1 μW m⁻², which is comparable to or even exceeds bulk radiogenic heat production within the crust. Thermally induced buoyancy is responsible for crustal upwelling in large domes with metamorphic temperatures up to 200 °C higher than regional background temperatures. Heat is mostly generated within the uppermost mantle, because of large stresses in the highly viscous rocks deforming there. This thermal energy may be transferred to the overlying crust either in the form of enhanced heat flow, or through magmatism that brings heat into the crust advectively. The amplitude of orogenic heating varies with time, with both the amplitude and time-span depending strongly on the coupling between heat production, viscosity and collision strain rate. It is argued that geologically relevant figures are applicable to metamorphic domes such as the Lepontine Dome in the Central Alps. We conclude that deformation-generated viscous dissipation is an important heat source during collisional orogeny and that high metamorphic temperatures as in Barrovian type metamorphism are inherent to deforming crustal regions.

Keywords: Alps; Barrovian metamorphism; Lepontine Dome; numerical modelling; viscous heating.

INTRODUCTION

Intermediate-pressure (Barrovian) metamorphism is a long-known, first-order feature that typifies many ancient and nearly all modern orogenic regions in collision settings (e.g. Miyashiro, 1973; Winter, 2001). However, the pressure–temperature–time evolution of Barrovian rocks remains difficult to explain (e.g. Thompson & Ridley, 1987). ‘Barrovian metamorphism: where’s the heat?’ asked Jamieson et al. (1998). Whereas viscous heating is undeniable for high strain, preservation of tectonic overpressures in mineral assemblages is not established (even though overpressures may well occur in a rock’s history). A number of thermo-mechanical models demonstrated that nappé stacking of heat-producing, radioactive continental crust usually fails to provide the amount of heat needed to attain the peak metamorphic temperatures commonly obtained from thermobarometric calculations (e.g. Jamieson et al., 1998; Engi et al., 2001). An answer is to assume that orogens have formed where materials with especially high radioactive heat production existed before orogeny (e.g. Chamberlain & Sonder, 1990; Huerta et al., 1998; Goffé et al., 2003).

However, the thermal equilibration time remains typically too slow with respect to independent geochronological constraints on the timing of thrusting and the subsequent thermal peak of metamorphism. Such considerations point towards the significant bulk effects of viscous heating during orogeny. A common weakness of most thermo-mechanical modelling of convergent orogenic wedges (e.g. Allemand & Lardeaux, 1997; Beaumont et al., 2001; Burov et al., 2001; Doin & Henry, 2001; Jamieson et al., 2002; Pysklywec et al., 2002) is that the heat derived from mechanical work is neglected: a recent exception is Babeyko et al. (2002). It is argued that first, in a deforming orogen, viscous heating actually is an in situ physical mechanism potentially capable of accounting for the thermal budget shortfall on the appropriate timescale. Two-dimensional (2D) numerical experiments are presented that incorporate viscous heating in modelling collision orogens. We argue that these results are applicable to classical examples of $P-T$–$t$ paths such as in the Central Alps, without the need for anomalously high heat production in the crust.
VISCOUS HEATING: PHYSICAL PRINCIPLES AND GEOLOGICAL CONSIDERATIONS

Several authors (e.g. Graham & England, 1976; England & Thompson, 1984; Molnar & England, 1990; Genser et al., 1996; Kincaid & Silver, 1996) have advocated frictional heating, also termed viscous dissipation, as a significant contributor to the heat budget of metamorphism. The magnitude of viscous heating $H_S$ in 3D deformation (see Table 1 for symbols and units) depends on the magnitude of the deviatoric stress tensor the rock can sustain ($\sigma$) and the strain rate tensor ($\dot{e}$), which is expressed in the equation:

$$H_S = \sigma_{xx}\dot{e}_{xx} + \sigma_{yy}\dot{e}_{yy} + \sigma_{zz}\dot{e}_{zz} + 2(\sigma_{xy}\dot{e}_{xy} + \sigma_{xz}\dot{e}_{xz} + \sigma_{yz}\dot{e}_{yz})$$

where $\dot{e}_{ij} = 1/2[(\partial v_i/\partial x_j) + (\partial v_j/\partial x_i)]$, and $i$ and $j$ are coordinates ($x$, $y$, $z$). Simplifying the mode of deformation permits a reduction of Eq. (1a) as follows (e.g. Turcotte & Schubert, 2002):

(i) simple shear:

$$H_S = \dot{\gamma}_{xz} \tau_{xz}$$

where $\dot{\gamma}_{xz} = \partial v_x/\partial z$ is the shear strain rate and $\tau_{xz}$ is the shear stress;

(ii) homogeneous flattening/thickening in the $z$ direction (3D pure shear):

$$H_S = \frac{3}{2} \dot{\gamma}^2_{zz} \sigma_{zz}$$

where $\dot{\gamma}_{zz} = \partial v_z/\partial z$ is the flattening/thickening strain rate and $\sigma_{zz}$ is the deviatoric stress in the flattening/thickening direction;

(iii) shortening/extension in the $x$ direction (2D pure shear):

$$H_S = 2\dot{\gamma}_{xx} \sigma_{xx},$$

where $\dot{\gamma}_{xx} = \partial v_x/\partial x$ is the shortening/extension strain rate and $\sigma_{xx}$ is the deviatoric stress in the shortening/extension direction.

This source of heat has been argued for various geodynamic processes such as mantle convection (e.g. Hansen & Yuen, 1996; Yuen et al., 2000; Gerya & Yuen, 2003a), lithospheric delamination (e.g. Schott et al., 2000), plateau uplift (Yuen & Schubert, 1981), subduction (e.g. Peacock, 1992) and related ‘cold plumes’ (Gerya & Yuen, 2003b), slab breakoff (Gerya et al., 2004) continental collision (e.g. Stüwe, 1998), shear zones (e.g. Brun & Cobbold, 1980; Leloup et al., 1999) and even for microstructural processes (Tenczer et al., 2001) and reaction texture formation (Stüwe & Sandiford, 1994). Nevertheless, viscous heating as a significant source of energy within the lithosphere remains rather unpopular among geologists. Below, we consider and refute the most frequent arguments used to question the concept:

First, the geological evidence for viscous heating is no more than fragmentary. But absence of evidence is not evidence of absence, and regarding viscous heating as a speculative source of energy on this basis contradicts a fundamental, intrinsic and non-removable part of thermodynamics, which is present in the law of conservation of energy (e.g. Landau & Lifshitz, 1963). Viscous heating is a mechanically ubiquitous, quantitatively well-established phenomenon that does not require geological verification.

Second, heat conduction of rocks is fast and thus allows the dissipation of the heat before it can contribute to the thermal budget of the deforming zone (e.g. Poirier et al., 1979). However, heat generated within any source region is spread by conduction with a characteristic timescale ($t_d$) that depends on the width $L$ of the region (e.g. Stüwe, 2002, symbols in Table 1)

$$t_d = \frac{L^2}{\kappa}$$

with $\kappa = k/\rho c_P$.

Thus the duration of heat dissipation via conduction grows as the square of the width ($L$) of the deforming/heat-producing zone (Fig. 1). For instance, if the viscous heat produced within a 100-m wide shear zone dissipates in $c. 1000$ yr, then heat generated within a 1 km wide shear zone requires $c. 10^7$ yr for a similar degree of conductive cooling. Therefore, the viscous heat produced within regionally significant zones of deformation (up to several km in thickness) will not
dissipate over geologically relevant time of millions of years (Fig. 1).

Third, viscous heating is a self-destructive mechanism. As viscosity $\eta$ links strain rate $\dot{\gamma}$ to shear stress $\tau$:

$$\tau = \eta \dot{\gamma},$$

(e.g. Ranalli, 1995) the responsible shear stress diminishes while generating heat and thermal softening tends to impede the system (Graham & England, 1976). In other words, as viscosity decreases as temperature increases, viscous heating decreases under constant strain rate $\dot{\gamma}$, but increases under constant shear stress. A mathematical formulation is required to define rates and temperature changes for which viscous dissipation may choke itself. If shear strain rate $\dot{\gamma}$ is kept constant, replacing shear stress $\tau$ in Eq. (1b) by its expression in Eq. (3) demonstrates that viscous heating decreases according to:

$$\frac{\partial H_S}{\partial t} = \dot{\gamma}^2 \frac{\partial \eta}{\partial t},$$

provided $(\partial \eta / \partial t) < 0$.

With heat diffusion $\partial \eta / \partial t \to 0$, hence $H_S \to$ constant, which means that viscous heating becomes stable, producing a positive thermal anomaly within the deforming zone. Moreover, such $H_S$ is non-null and so shear heating is not suppressed (e.g. Stüwe, 1998; Turcotte & Schubert, 2002). Conversely, if shear stress $\tau$ is maintained constant, then the shear strain rate $\dot{\gamma}$ increases with decreasing viscosity and, as $(\partial \eta / \partial t) < 0$, $H_S$ actually increases with time, according to:

$$\frac{\partial H_S}{\partial t} = -\dot{\gamma} \frac{\partial \eta}{\partial t}$$

Fourth, viscous heating is a local perturbation around shear zones and faults that require fast movement and high shear stress over long periods (Scholz, 1980), but is negligible on a regional scale. Also, assuming weakness of deeply buried rocks (e.g. Ranalli, 1995) the magnitude of temperature increase because of viscous heating is small compared with other temperature perturbations. Separating mechanical from thermal effects is a weakness of most models. The conversion of mechanical work into heat, which is a direct feedback of deformation on metamorphism, is usually neglected for the sake of simplicity but can be a significant inaccuracy. Critically, sequential or neighbouring shear-heating events are cumulative and the conductive dissipation of the heat generated does not equate to disappearance: the quantity of heat becomes distributed into a widening zone. Deformation lasting million(s) of years consists of many smaller events in time and space, ranging from grain- to kilometre-scale, distributed flow. The heat effect is additive, the dissipation time depending on the overall width of the deformation zone. This cumulative effect provides the thermal contribution because of viscous heating. As a first approximation, this thermal contribution may be estimated by integrating Eq. (1) over time $t$:

$$\Delta T_{\text{max}} = \int_{t_i}^{t_f} \frac{H_S(t)}{\rho C_p} \, dt$$

where $\Delta T_{\text{max}}$ is the maximum temperature increase (assuming no heat diffusion). Under relatively constant effective regional stresses ($\tau_{xz}, \sigma_{zz}$ or $\sigma_{xx}$), the effect of cumulative viscous heating for the different deformation modes may be estimated by integrating Eqs (1b), (1c) and (1d) with respect to time:

(i) regional simple shear:

$$\Delta T_{\text{max}} = \frac{\tau_{xz} \dot{\gamma}_{xz}}{\rho C_p},$$

where $\dot{\gamma}_{xz} = \int_{t_i}^{t_f} \dot{\gamma}_{xz}(t) \, dt$ is the bulk shear strain;

(ii) regional flattening/thickening:

$$\Delta T_{\text{max}} = \left( \frac{3}{2} \right) \frac{\sigma_{xz}}{\rho C_p} \ln \left( \frac{h_1}{h_0} \right)$$

where $h_0$ and $h_1$ are initial and final thicknesses, respectively;

(iii) regional shortening/extension:

$$\Delta T_{\text{max}} = 2 \frac{\sigma_{xx}}{\rho C_p} \ln \left( \frac{l_1}{l_0} \right),$$

where $l_0$ and $l_1$ are initial and final lengths, respectively.

The magnitudes of effective regional stresses $(\tau_{xz}, \sigma_{zz}, \sigma_{xx})$ in Eqs (6b)–(6d) can be estimated from the force required to maintain the average topographic elevation $h$ (Turcotte & Schubert, 2002, p. 223) as:

$$\left( \tau_{xz}, \sigma_{zz}, \sigma_{xx} \right) = \rho g h.$$
Taking $\rho = 3000 \text{ kg m}^{-3}$ and typical variations in topography (0.5–5 km) yields orogenic stresses of 15–150 MPa. These are minimum values as stress amplification because of deformation localization may occur in heterogeneous tectonic settings (Molnar & Lyon-Caen, 1988; Kusznir, 1991):

$$ (\tau_{xz}, \sigma_{zz}, \sigma_{xx}) = \rho gh \frac{L_x}{L_d} $$

where $L_x$ is the length of the deformation system and $L_d$ is the length of the localised, actively deforming zone. These rough stress estimates are consistent with in situ measurements yielding values of the order of tens of MPa (e.g. Turcotte & Schubert, 2002). In the case of regional simple shear, plotting the dependence of $\Delta T_{\text{max}}$ on effective shear stress ($\tau_{xz}$) and bulk shear strain ($\gamma_{xz}$) shows that, for conservative values of effective orogenic stress and bulk shear strain ($>1$), the maximum increase in temperature reaches several tens or even hundreds of degrees (Fig. 2). Obviously, under relatively slow deformation, the actual values of temperature increase may be considerably smaller, because of heat diffusion (Fig. 1). It is worth noting that viscous heating has a strain-rate dependent timescale adjusted to progressing deformation [Eq. (1)], thus providing immediate feedback to metamorphic temperatures (Stüwe, 1998). In contrast, radiogenic heating has a timescale >1 Myr for $\Delta T_{\text{max}} = 100 ^\circ \text{C}$, independent of deformation; as discussed by Kincaid & Silver (1996), heat production because of radiogenic decay lasts five to 10 times longer than regional metamorphism dated in orogenic zones.

A final argument is that fluids infiltrating along fault zones should rapidly carry heat away.

Because fluid and rock have comparable heat capacities, the mass of circulating fluids passed through rocks must be comparable to the mass of the rocks themselves to have efficient cooling effects (e.g. Connolly & Thompson, 1989). The heat balance is hardly achievable at peak metamorphic conditions. Dehydration of rocks takes place during diagenesis and the early stages of metamorphism. Higher grade rocks do not contain volume fractions of fluids large enough to remove heat produced by mechanical work.

Such qualitative considerations point towards a significant bulk effect of viscous heating during orogeny. Yet, these considerations need to be supported by quantitative estimates on both amount and dynamics of viscous heating in realistic orogenic situations. We therefore performed systematic, high resolution 2D numerical modelling of continental collision to determine both an average intensity, and the spatial and temporal patterns of viscous heat production within the deforming continental lithosphere. In order to do this in a more realistic way, numerical models of collision were developed that incorporate viscoplastic pressure–temperature–strain-rate-dependent rheology, pressure–temperature-dependent conductivity and partial melting of the crust. Details on boundary conditions and technical description of the analytical and modelling techniques are given in the Appendix. The results are applied to examine the effects of viscous heating on the thermal evolution of the central Alps.

**RESULTS FROM NUMERICAL EXPERIMENTS**

Thirty-seven numerical experiments using 17 configurations (Table 3) have been run with a finite-difference grid of $354 \times 88$ irregularly spaced Eulerian points (Fig. 3a), and with over 7 million markers to portray fine details of the temperature, material and viscosity fields. By varying the experimental parameters the primary interest is in quantifying the extent and the pattern of viscous heating as a function of upper- and lower-crustal compositions and rheology, and convergence rate. Our discussion essentially refers to successive stages of Model 10 (Figs 4–6) characterized by the rather slow convergence rate of 2 cm yr$^{-1}$ and the moderate ductile strength of the crust composed of upper felsic and lower mafic layers (Tables 2 & 3). Similar evolutions were identified in other models, with variances that are discussed below.

**First-order features**

A double-verging geometry, which branches at the point where the mantle is subducting, occurs from the beginning (Fig. 4a,b). The strain rate distribution (Fig. 5e,f) and the Y-shaped pattern of viscous heating (Fig. 4e,f) also fit this pattern. The amount of heat
produced by viscous dissipation is on average 0.1–0.3 $\mu$W m$^{-3}$ (Table 3), which corresponds to 10–30% of radiogenic heat generation ($\approx 1 \mu$W m$^{-3}$) in the crust. The degree of viscous heating shows strong lateral variations and often reaches values 1–10 $\mu$W m$^{-3}$ along localized (yet several kilometres thick) deformation zones (Fig. 4e–h). Thus, viscous heating provides an important source of heat in movement zones. The timescale for heat diffusion ($t_d$) within these zones is on the order of several million years (Fig. 1) and is comparable with the timescale of deformation (see timing in Fig. 4a–d), which results in noticeable positive temperature anomalies along these zones (see isotherms in Fig. 4a–d). For example, temperature along the plate interface is sufficiently increased to produce partial melting of subducted crustal rocks (Fig. 4c,d, red coloured zone).

**Tectonic stress and overpressure**

Figure 6 shows the distribution of deviatoric stresses and non-lithostatic pressure component in the same Model 10. Inclinations of the principal stress axes (Fig. 6a,b) fan according to the double-verging pattern. Deviatoric stresses (Fig. 6a–d) vary from 10 to 100 MPa in the crust, in accordance with estimates presented in the introduction. Strong positive (up to 100%) and negative (up to 50%) non-lithostatic pressure regions develop in the upper (brittle) part of the mantle lithosphere (Fig. 6e–h). There are no strong (>20%) overpressures in the thickened orogenic crust; this appears to be because of the relatively low brittle strength of crustal rocks used in our numerical experiments (Table 3). However, relatively high (10–20%) overpressures (Fig. 6g,h) occur in crustal rocks at the lower tip of the wedge-shaped tectonic/subduction channel (Fig. 4d), which is equivalent to overpressures of 200–600 MPa at 70–90 km depth. These overpressures are related to the onset of the forced upward-directed return flow (Fig. 5c,d) of subducted crustal and hydrated mantle rocks from the channel contributing to the exhumation of high-pressure complexes (e.g. Dobretsov & Kirdyashkin, 1992; Mancktelow, 1995; Gerya & Stöckhert, 2002).

**Effect of rheology on viscous heating**

We compared the distribution of viscous heating in models with rheologically different lower crusts and different convergence rates (Fig. 7). An increase in the
convergence rate causes a proportional increase in the amount of viscous heating in the crust for the same amount of shortening (compare Model 10 to Model 14, and Model 15 to Model 16; Table 3). Models with relatively strong lower crusts (dominated by plagioclase creep) develop significant amounts of viscous heating in the crust (0.18–0.46 $\mu$W m$^{-3}$) after 150 km of convergence, Table 3) and produce structural domes.
of 10–15 km amplitude that facilitate exhumation of lower crustal rocks. Models with weak lower crusts (dominated by wet quartzite creep) show insignificant viscous heating in the crust (0.02–0.06 $\mu$W m$^{-3}$ after 150 km of convergence, Table 3), while fundamentally homogeneous crustal thickening does not allow exhumation of deeper crustal rocks. Therefore, the general cross-sectional structure of the orogen (Fig. 7a–d)
as well as the intensity of crustal viscous heating (Fig. 7e–h) strongly reflects the rheological strength of the lower crust. On the other hand, all models show strong viscous heating in the mantle (Fig. 7e–h, Table 3) along the upper surface of the subducting slab, where temperatures trigger partial melting of mantle rocks.
Local perturbations of metamorphic temperatures

As follows from Fig. 1, viscous heating produced in deformation zones of several kilometre thickness (Fig. 5e–h) should have important local effects on the metamorphic temperature when deformation takes place on the timescale of several million years or shorter. Positive temperature anomalies along deformation zones should therefore be characteristic of relatively fast convergence rates. This conclusion is supported by comparing two cases using the same numerical setup as for experiment 14 but with a higher convergence rate of 5 cm yr\(^{-1}\). In one case viscous heating is ignored (Fig. 8a–d), in the other case (Fig. 8e–h), viscous heating contributes to the thermal pattern. From Fig. 8i–l, it can be readily seen that viscous heating (i) provides a significant (25–200 °C) increase in metamorphic crustal temperatures (Fig. 8i–l), (ii) facilitates upwelling of lower crustal rocks (Fig. 8d,h), (iii) triggers melting of the subducted crust at the plate interface (Fig. 8c,g), and (iv) produces decoupling and decreases bending of the plates (Fig. 8d,h).

Temporal variability and overall intensity of viscous heating

Under a constant convergence rate, the amount of viscous heating tends to decrease with time (Fig. 9a, Table 3) while viscosity decreases (Fig. 5a–d) with increasing temperature (Fig. 4a–d). The amount of viscous heating at mantle depths remains more stable (Fig. 9b, Table 3) and generally exceeds 10 \(\mu W \cdot m^{-3}\) along the plate interface (Figs 4e–h & 7e–h). For models with constant horizontal stresses, increase and decrease of convergence rate and viscous heating with time are both obtained (cf. Models 2, 5, 7, 9, 11; Table 3, Fig. 9). The configuration and position of the convergence condition domain (Table 3) have no significant influence on the numerical results. For example, the reference Model 10 (Table 2) with convergence velocity defined within the mantle lithosphere only shows shear heating intensity and dynamics very similar to those of Model 12, for which the convergence velocity was defined within both the crust and the mantle lithosphere (Table 3, Fig. 9).

A pronounced positive correlation is found between the overall intensity of crustal viscous heating and instantaneous convergence rate (Fig. 10). This correlation suggests that a significant contribution (>0.1 \(\mu W \cdot m^{-3}\)) of viscous heating into the crustal heat balance can be expected when the convergence rate exceeds 1 cm yr\(^{-1}\), particularly if the lower crust is strong. Therefore, viscous heating may become a dominant heat source in collision orogens with rapid convergence rates (e.g. Himalayas).

### Table 2. Material properties used in 2D numerical experiments

<table>
<thead>
<tr>
<th>Material</th>
<th>Material properties</th>
<th>(\rho \left( \text{g} \cdot \text{cm}^{-3} \right))</th>
<th>(K \left( \text{W} \cdot \text{m}^{-1} \cdot \text{K}^{-1} \right))</th>
<th>(\eta_{\text{op}} \left( \text{Pa} \cdot \text{s} \right))</th>
<th>(\eta_{\text{opt}} \left( \text{Pa} \cdot \text{s} \right))</th>
<th>(\eta_{\text{mp}} \left( \text{Pa} \cdot \text{s} \right))</th>
<th>(\eta_{\text{mip}} \left( \text{Pa} \cdot \text{s} \right))</th>
<th>(\mu \left( \text{Pa} \cdot \text{s} \right))</th>
<th>(\nu \left( \text{m} \cdot \text{s}^{-1} \right))</th>
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</thead>
<tbody>
<tr>
<td>Felsic and sedimentary crust</td>
<td>2800 (solid)</td>
<td>0.64 (+ 807\left( T - 77 \right) \times 10^{-6})</td>
<td>455 (+ 374\left( T - 77 \right) \times 10^{-6})</td>
<td>-</td>
<td>-</td>
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<td>-</td>
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<tr>
<td>Mafic crust</td>
<td>3000 (solid)</td>
<td>0.73 (+ 120\left( T - 77 \right) \times 10^{-6})</td>
<td>455 (+ 374\left( T - 77 \right) \times 10^{-6})</td>
<td>-</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>Lithosphere–asthenosphere</td>
<td>3000 (solid)</td>
<td>0.73 (+ 120\left( T - 77 \right) \times 10^{-6})</td>
<td>455 (+ 374\left( T - 77 \right) \times 10^{-6})</td>
<td>-</td>
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<tr>
<td>Hydromelt in the subduction zone</td>
<td>3000 (solid)</td>
<td>0.73 (+ 120\left( T - 77 \right) \times 10^{-6})</td>
<td>455 (+ 374\left( T - 77 \right) \times 10^{-6})</td>
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<td>Reference: (\text{a})</td>
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</table>

\(\text{a}\) \(G = 1000 \text{ kg m}^{-2} \cdot \text{s}^{-1} = \frac{1}{3} \times 10^{12} \text{ Pa} \cdot \text{s} \cdot \text{m}^{-1} \cdot \text{Pa} \cdot \text{s} \cdot \text{m}^{-1} \) for all rock types.

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GEOLOGICAL APPLICATION

Stüwe (1998) suggested that shear (dissipative) heat, episodically released during nappe stacking, contributed...
Table 3. Parameters of selected numerical experiments.

<table>
<thead>
<tr>
<th>Model</th>
<th>Material</th>
<th>$\iota_{\text{in}}$</th>
<th>$H_x$</th>
<th>$\iota_{\text{in}}$</th>
<th>$H_x$</th>
<th>Domain</th>
<th>Type</th>
<th>Convergence</th>
<th>Topography</th>
<th>$H_x$ (mW m$^{-2}$)</th>
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<td></td>
<td></td>
<td>(mW m$^{-2}$)</td>
<td>(mm a$^{-1}$)</td>
<td></td>
<td>(mW m$^{-2}$)</td>
<td>(mm a$^{-1}$)</td>
<td>(km)</td>
<td>(Myr)</td>
<td>(m a$^{-1}$)</td>
<td>(m)</td>
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<tr>
<td>1 (buad)</td>
<td>Strong felsic</td>
<td>0.75</td>
<td>1</td>
<td>Strong felsic</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
<td>$x = 1410-1440$; $z = 55-57$</td>
<td>$r_x = 5$ cm a$^{-1}$</td>
<td>0</td>
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<tr>
<td></td>
<td>Felsic</td>
<td>0.75</td>
<td>1</td>
<td>Felsic</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
<td>$x = 1425-1455$; $z = 55-57$</td>
<td>$\sigma_{xx} = -100$ MPa</td>
<td>100</td>
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<tr>
<td>2 (bua)</td>
<td>Strong felsic</td>
<td>0.75</td>
<td>1</td>
<td>Strong felsic</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
<td>$x = 1425-1455$; $z = 55-57$</td>
<td>$r_x = 3$ cm a$^{-1}$</td>
<td>250</td>
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<tr>
<td></td>
<td>Mafic</td>
<td>0.75</td>
<td>1</td>
<td>Mafic</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
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<td>$r_x = 5$ cm a$^{-1}$</td>
<td>500</td>
</tr>
<tr>
<td>3 (buag)</td>
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<td>1</td>
<td>Strong felsic</td>
<td>0.75</td>
<td>1</td>
<td>1</td>
<td>$x = 1410-1440$; $z = 55-57$</td>
<td>$r_x = 3$ cm a$^{-1}$</td>
<td>100</td>
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<tr>
<td></td>
<td>Felsic</td>
<td>0.75</td>
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<td>Felsic</td>
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<tr>
<td>4 (bush)</td>
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<td>Felsic</td>
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<td>1</td>
<td>1</td>
<td>$x = 1410-1440$; $z = 55-57$</td>
<td>$r_x = 3$ cm a$^{-1}$</td>
<td>500</td>
</tr>
<tr>
<td>5 (bui)</td>
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<td>1</td>
<td>Felsic</td>
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significantly to thermal perturbations in the Eastern Alps. In the present paper we draw attention to the Lepontine Dome (Fig. 11), a prominent structural dome of the post-Mid-Eocene Central Alps delineated by the concentric distribution of isograds that cut earlier, Cretaceous to Eocene contacts of the Penninic nappe pile (Wenk, 1956; Trommsdorff, 1966; Niggli, 1970). This relationship suggests a time lag, not discussed here, between high pressure/temperature syn-nappe metamorphism, and the lower pressure/temperature Lepontine thermal event summarized in Fig. 12. Previous authors have attributed this thermal pattern to (i) subduction of crustal fragments rich in heat-generating radioactive elements (e.g. Goff et al., 2003 and references therein) or (ii) extrusion of hot crustal units previously subducted to mantle depth (Engi et al., 2001). Modelling of these processes applied to the Alps had to include a highly radiogenic crust and/or viscous heating to obtain agreement with geological information (Roselle & Engi, 2002). Analogue modelling shows that crustal subduction during the formation of the Lepontine is a mechanically disputable hypothesis (Burg et al., 2002) and high-resolution tomography reveals subduction of the European mantle lithosphere only (Lippitsch et al., 2003). These observations persuaded us to examine the effects of viscous heating produced by Oligocene–Miocene crustal imbrication on the thermal evolution of the Central Alps during subduction of the mantle lithosphere.

Lepontine Dome

The 40 Myr-old high-pressure mineral assemblages suffered rapid decompression in the early stages of the development of the Lepontine Dome (Gebauer, 1999). The thermal overprint is dated at 38–35 Ma near the northern margin of the dome and peak metamorphic temperatures were established over most of the Lepontine at c. 32 Ma (see details and methods in Hunziker et al., 1992; Gebauer, 1999). More generally, metamorphic conditions lasted from 35 to 25 Ma (Grujic & Mancktelow, 1996; Frey & Mählmann, 1999). Thermobarometry documents an increase in peak metamorphic temperatures from 500–550 °C at the northern limit of the dome to 680 °C at the southern border, against the Insubric Line (Engi et al., 1995). Highest temperatures do not match maximum metamorphic pressures of c. 7 kbar in the core. Pressures decrease asymmetrically northward (to 5.5 kbar) and southward (down to 4.5 kbar; Engi et al., 1995; Todd & Engi, 1997). In terms of thermobarometric uncertainties, these differences in $P$–$T$ conditions might not be as large as stated. In any case, they portray, along with the age ranges, the finite state of a diachronous, Barrovian-type metamorphic field (Engi et al., 1995). Rb–Sr and Ar–Ar ages of c. 21–23 Ma in the centre and 28 Ma in the south (Engi et al., 1995) refer to already dominant cooling stages. Fission track ages
of detrital grains suggest that the core of the Lepontine became exposed at c. 14 Ma (Spiegel et al., 2000) and cooling/exhumation continued after then (Grasemann & Mancktelow, 1993). Zones of rapid cooling migrated from the east to the west but geological and geochronological data are insufficient to differentiate erosion-dominated versus tectonic-dominated denudation (Schlunegger & Willett, 1999).
In brief, the Lepontine Dome is a short-lived (<20 Ma) thermal event accompanied by decompression during overall shortening of the area.

Strain

A wealth of structural data within the Lepontine Dome documents polyphase and penetrative deformation that has absorbed an important part of the shortening strain (Merle et al., 1989; Steck & Hunziker, 1994; Grujic & Mancktelow, 1996; Rütti, 2001). Syn-metamorphic, Lepontine-related structures have isoclinally refolded often-transposed older foliations. The regional kinematics are imprecise, yet involved SW-vergent folding that intensifies towards the south where top-to SE shear is identified (Nagel et al., 2002). Intense shear strain is

Fig. 8. Influence of viscous heating on the thermal regime and structure of the collision orogen. Each sketch represents an enlarged 300 × 235 km area of the original 4000 × 670 km Model 14 (Table 3). Vertical scale as in Fig. 4. Left column: model development with no viscous heating. Middle column: model development with viscous heating. Right column: temperature difference between equivalent stages in left and right columns. Rock types and isotherms as in Fig. 4. The negative temperature difference within the subducting mantle slab reflects the steeper subduction angle in the case of viscous heating.
indicated by the ubiquitous, dominant foliation bearing a marked stretching and mineral lineation parallel to fold axes, along with isoclinal folds with inverted limbs several hundreds of metres in length (e.g. Grujic & Mancktelow, 1996; Rütti, 2001). This is consistent with the concept that shear is the likely dominant orogenic deformation process (Burg, 1999), because of the tendency of the lithosphere to undergo shear failure rather than bulk flattening.

Structural information can only yield a minimum estimate of the shear strain, which can be obtained in two ways: (1) using the large-scale, tectonic information and (2) using the internal bulk fabric of the deformed zone. In this first-order approach, we ignore strain related to transcurrent deformation between Europe and Apulia (e.g. Laubscher, 1988) is ignored, which would add to the estimates presented here.

**Large-scale information**

The amount of convergence across the Central Alps since 40 Ma is $150 \pm 50$ km, most of which took place within the Lepontine Dome and the adjacent Southern Alps (Schmid et al., 1996; Escher & Beaumont, 1997). Owing to large uncertainties, it is assumed (as a first approximation) that all of the convergence is accommodated by the Lepontine Dome and that omission of the few 10s kilometres shortening in the Southern Alps part of the section approximately balances, within the uncertainties given above, missing strain information in the gneiss region. The present day width of the Lepontine Gneiss Dome is c. $50$ km, from the amphibolite facies isograd to the Insubric Line (Fig. 11). Shortening (shortening strain $= (l-l_0)/l_0$, with $l$ and $l_0$ the final and initial profile lengths, respectively, e.g. Ramsay, 1967) is thus 50–75%. In the absence of quantitative information, and recognizing the uncertainty in the structural information, half of this shortening is attributed to distributed strain within the rocks. Taking into account the intense, symmetamorphic fabric described by all authors in the region, it is a very crude but fair estimate that is consistent with the 20–30% shortening commonly associated with the presence of strong foliation planes (Ramsay & Huber, 1983). It is assumed that relative movements along shear zones have taken up the remaining half. Conservative estimates thus yield distributed strain ranging from 20 to 35%. This amount of shortening was mostly achieved within 15–30 Myr, depending on the bounding age criteria chosen. These approximations

---

**Fig. 9.** Dynamics of orogenic viscous heating for selected numerical experiments (model numbers in Table 3) (a) averaged by 50 km depth (below the $z = 10$ km sea level) and (b) averaged by 100 km depth. Solid lines – models with prescribed rate of convergence ($v_x$). Dashed lines – models with prescribed horizontal deviatoric stress ($\sigma_{xx}$) whose rate of convergence ($v_x$) changes with time (Table 3).

**Fig. 10.** Correlation between extent of viscous heating in the collision zone (1850–2250 km) expressed as an average by 50 km depth (below the $z = 10$ km sea level) with instantaneous convergence rate (Table 3). Different regimes of crustal viscous heating shown in different greyscales are arbitrarily subdivided by comparison to the average radioactive heating for the entire continental crust used in this work ($=1 \mu W m^{-3}$, Table 2). Different symbols correspond to different rheologies of the ductile lower crust (Table 3): open squares, wet quartzite; open triangles, strong wet quartzite; solid diamonds, plagioclase.
imply a time-averaged convergence rate spanning 1.0–3.5 cm yr\(^{-1}\), which is consistent with plate tectonic reconstitutions for the Alps (e.g. Schmid \textit{et al.}, 1996).

**Fabric information**

The fabric developed synchronous with Lepontine metamorphism has almost completely obliterated and transposed older structures in this region. Visual transposition is reached once superimposed planes make an angle smaller than 5\(^\circ\), and hence is difficult to decipher and measure. The oldest foliation is often reworked by shear zones of the main syn-metamorphic deformation, thus is close to the shear plane of non-coaxial deformation identified from sense-of-shear criteria. Assuming that the dominant foliation results from simple shear, the angular relationship \(\theta\) between shear and foliation planes indicates shear strain \(\gamma = 2/tan \ 2\theta\) (Ramsay & Graham, 1970). An angle of 5\(^\circ\) corresponds to \(\gamma \approx 11\). However, deformation is not simple shear because there are pure shear and/or volume loss components (Marquer \textit{et al.}, 1996). Under general shear, \(\theta\) used for a shear strain calculation is incorrect and the measured angle between shear and foliation planes may be only half the \(\theta\) value (Burg & Laurent, 1978; Platt & Behrmann, 1986). Introducing this logic, a conservative regional shear strain of \(\gamma_{xz} \geq 5\) results. Structural geologists have established that most mylonitic shear zones formed at \(\gamma_{xz} \approx 10\) (e.g. Simpson, 1983; Watts & Williams, 1983; Burks & Mosher, 1996; Dutrue & Burg, 1997). The mylonitic character of the main deformation phase throughout the Lepontine Dome suggests that a regional figure of \(\gamma_{xz} \geq 5\) is a plausible minimum.

**Viscous heating in the Lepontine Dome**

According to Eq. (6b) the upper limit of temperature increase because of a shear strain of 5 distributed over the Lepontine Dome would be \(\Delta T_{\text{max}} = 25–250\ \degree\text{C}\) for
effective regional shear stresses $\tau_{xy}$ of 15–150 MPa, respectively (Fig. 2). The bulk deformation structure of numerical experiments with relatively strong lower crusts (Figs 7a,b & 13b,c) resemble the large-scale structure of the Central Alps (Fig. 13a) and suggest a significant role for strong viscous heating effects (Fig. 7e,f, Table 3). Granulites identified as the lower crust of Western Europe may constitute this strong rheological layer in the Alps (e.g. Schmid et al., 1996). Additional numerical experiments with identical rheological setups but with a convergence rate of 5 cm a$^{-1}$ confirm that 30–50 km wide positive temperature anomalies, as in the Lepontine Dome, are produced above the plate boundary, and maintained within the crust because of viscous heating (e.g. Figs 8i–l & 13c).

In these models, viscous heating promotes upwelling of lower crustal rocks (cf. Fig. 8c,g), which adds to the positive temperature anomaly in the middle crust (Fig. 8k). Thermally induced buoyancy further amplifies this anomaly and crustal doming. This is further emphasized by Model 17 (Fig. 13c, Table 3), characterized by strong viscous heating ($=1.5 \mu W m^{-3}$); its geometry and peak metamorphic temperature distribution after 200 km convergence show notably more similarity with the Lepontine Dome (Fig. 13a) than Model 12 (Fig. 13b, Table 2) characterized by low viscous heating ($=0.2 \mu W m^{-3}$).

Although these numerical experiments are 2D, we speculate that thermally induced buoyancy in 3D will produce elongated dome-like structures in collision zones, such as the Lepontine Dome and the dome-shaped Tauern Window in the Alps (see also Stüwe, 1998). We attribute the fact that seismicity of the Central Alps is no deeper than c. 15 km (Fig. 13a), to ongoing viscous heating and subsequent rheological softening of the crust as a consequence of sustained collision.

CONCLUSION

Barrovian metamorphism is generally taken to have occurred because of increased radiogenic heating of a thickened crust (England & Thompson, 1984). This mechanism has been tested for the Lepontine Dome in the Central Alps but is not sufficiently consistent with geological information (Roselle & Engi, 2002). The numerical modelling developed here, using conservative estimates of parameters and variables, confirms that viscous heating is an alternative, in situ energy source associated with distributed deformation during continental collision. This mode of heat production may explain the closed shape of post-40 Ma isograds and may be sufficient to produce an additional 100–200 °C temperature increase in the Lepontine gneiss region. Many authors attributed the decompressional structural evolution of the Lepontine Dome to forced, upward extrusion assisted by efficient erosion of crustal units. Alternatively, our numerical modelling suggests that viscous heating has the potential to affect both temperature distribution and large-scale structural patterns within the deforming crust, notably accelerating exhumation of lower crustal rocks. Based on the similarities between models with strong lower crust and the geology of the central Alps, we conclude that the combination of viscous heating and moderate tectonic overpressure contribute to the heat/pressure budget of the Barrovian-type metamorphic field gradients inherent to any collisional orogens, without implying any external heat and pressure source.

ACKNOWLEDGEMENTS

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REFERENCES


Fig. 13. Comparison between (a) a geological profile across the Central Alps with deep structures from Burg et al. (2002) and dashed red lines showing the isotherms for peak metamorphic temperature in the Lepontine Dome from the maps of Steck & Hunziker (1994); Engi et al. (1995) and Todd & Engi (1997); dark circles are loci of seismic events after Deichmann et al. (2000), and (b) Model 12 and (c) Model 17 (Table 2). (b) and (c): same legend as Fig. 4. Same horizontal/vertical scales for (a)–(c). Model 17, which displays a cross sectional structure with crustal recumbent folding/imbrication that can be compared with the geological section involves more viscous heating than Model 12. Note the outward-dipping isograd pattern cutting the lithological boundaries in both models, as in the geological case. Blue-green boundary, crust–mantle boundary in models.


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APPENDIX: INITIAL AND BOUNDARY CONDITIONS OF 2D MODELS

General setting

We employed the 2D finite difference code I2VIS with a marker-in-cell technique, which allows for the accurate solution of the governing equations on a rectangular fully staggered Eulerian grid for multiphase flow (Gerya & Yuen, 2003a). The 4000 × 670 km model (Fig. 3) is designed for the study of dynamic processes during continental collision involving subduction of the lithospheric mantle (e.g. Pysklywec et al., 2000). A non-uniform rectangular grid with a resolution varying from 2 to 30 km is used. It provides the highest resolution (2 × 2 km) in the upper central, 400 km wide and 100 km deep ‘orogenic’ area of the model (Fig. 3a). Continuous changes in the grid spacing are prescribed on 25 nodes with c. 12% grid step increment between adjacent nodes. The initial thermal structure of the colliding lithospheres with a 35-km thick crust corresponds to a steady state thermal profile limited by the 1300 °C isotherm at 150 km depth. This implies a moderate thickness of continental mantle lithosphere and lower-crustal temperatures of 400–450 °C (Fig. 3a).

At thicker mantle lithosphere, as in many continental sections, implies lower crustal temperatures and higher effective viscosity of crustal rocks, hence stronger viscous heating for the same convergence rate. Therefore, our results provide robust minimum estimates for the thermal effects of viscous heating during orogeny. The thermal boundary conditions are a 0 °C upper boundary and a zero horizontal heat flow across the vertical boundaries (Fig. 3b). An infinity-like constant external temperature condition along the bottom of the model implies 2045 °C at 1650 km depth, i.e. a constant temperature condition to be satisfied far below the lower boundary of the box, which allows both temperatures and vertical heat fluxes to vary along the permeable box lower boundary.

The velocity boundary conditions are free slip at all boundaries except the lower boundary of the box, which is permeable in both downward and upward directions (Fig. 3b). An infinity-like external free slip condition along the bottom of the model implies free slip at the 1650 km depth, i.e. free slip condition is satisfied far below the lower boundary of the box. In contrast to the model employed by Pysklywec et al. (2000) the source of motion is located within and not at a boundary of the computed region. For different numerical experiments, collision is prescribed either (1) by prescribing a constant convergence rate (v_x) or (2) by applying constant horizontal deviatoric stresses (σ_m) within the convergence condition domain of the lithosphere (red square in Fig. 3b). In some experiments, this domain was implemented within the mantle lithosphere only to account for the actual degree of rheological coupling between mantle and crust. The convergence velocities/stresses within the crust are then calculated by solving governing equations that take into account rheological coupling between the crust and the mantle lithosphere. The dynamic viscosity structure (Fig. 3b) corresponds to the given temperature distribution (Fig. 3a), and characterizes the velocity field associated with the movement of the mantle lithosphere at the initiation of numerical experiments. The nucleation of the subduction/collision area is imposed by a 5–20-km thick and down to 160-km deep (below the bottom of the crust) initial zone of weakness (Fig. 3a) that has a wet olivine rheology in the mantle lithosphere having otherwise a dry olivine rheology (Ranalli, 1995). The inclination angle of this zone changes downwards from 10 to 45°, thus simulating the curvature of the subduction plane (inset, Fig. 3a). Our test numerical experiments have shown that the initial shape of the weak zone provides efficient decoupling between the two converging plates and favours subduction of the incoming mantle lithosphere under the collisional orogen.

Topography

Continental collision affects the topographic evolution of adjacent regions and the models include surface movements in a simplified way. The surface is calculated dynamically at each time-step as a free surface (e.g. Gerya & Yuen, 2003b). To account for the topographic changes, we have implemented a low viscosity (10^{10} Pa s), initially 10-km thick layer above the upper continental crust (Fig. 3a). Its density is either 1 kg m^{-3} (air, above the z = 10 km sea level, with z = 0 = top of the box) or 1000 kg m^{-3} (sea water, below the z = 10 km sea level). The interface between this very weak layer and the top of the continental crust is treated as an erosion/sedimentation surface, which evolves according to the transport equation solved in Eulerian coordinates at each time-step (Gerya & Yuen, 2003b):

$$\frac{\partial \varepsilon_s}{\partial t} = v_x \frac{\partial \varepsilon_m}{\partial x} - v_s + \varepsilon_m,$$

where \varepsilon_s is the vertical position of the surface as a function of the horizontal distance x; v_x and v_s are the vertical and horizontal components of the material velocity vector at the surface; v_{s0} and v_{s0} are, sedimentation and erosion rates, respectively, which correspond to the relation:

\begin{align*}
\varepsilon_s &= 0, v_x = v_{s0} \quad \text{when } z < 10 \text{ km}, \\
\varepsilon_x &= v_x, v_x = 0 \quad \text{when } z \geq 10 \text{ km},
\end{align*}

where v_{s0} and v_{s0} are imposed constant erosion and sedimentation rates, respectively.

Partial melting

Partial melting of the crust is an important metamorphic process during orogeny. Therefore, the numerical models allowed melting of the continental crust in the P-T region between the wet solidus and dry liquidus of crustal rocks (Gerya & Yuen, 2003b). As a first approximation, the volumetric fraction of melt M is assumed to increase linearly with temperature according to the relations:

\begin{align*}
M = 0 & \quad \text{at } T \leq T_{\text{solidus}}, \\
M = \frac{(T - T_{\text{solidus}})}{(T_{\text{liquidus}} - T_{\text{solidus}})} & \quad \text{at } T_{\text{solidus}} < T < T_{\text{liquidus}}, \\
M = 1 & \quad \text{at } T \geq T_{\text{liquidus}},
\end{align*}

where T_{\text{solidus}} and T_{\text{liquidus}} are the wet solidus and dry liquidus temperatures of the crust, respectively (see Table 2 for definitions).

The effective density, \rho_{\text{eff}} of partially molten rocks is calculated from:

$$\rho_{\text{eff}} = \rho_{\text{solid}} - M(\rho_{\text{solid}} - \rho_{\text{molten}})$$
where \( \rho_{\text{solid}} \) and \( \rho_{\text{molten}} \) are the densities of solid and molten rock, respectively, which vary with pressure and temperature according to the relation:

\[
\rho_{PT} = \rho_0[1 - \alpha(T - T_0)][1 + \beta(P - P_0)],
\]

where \( \rho_0 \) is the standard density at \( P_0 = 0.1 \) MPa and \( T_0 = 298 \) K; \( \alpha \) and \( \beta \) are the thermal expansion and compressibility coefficients, respectively (Table 1).

The effect of latent heat (e.g. Stüwe, 1995) is included by increasing the effective heat capacity \( (C_{\text{p,e}}) \) and thermal expansion \( (\alpha_{\text{e}}) \) of the partially molten rocks \((0 < M < 1)\), calculated as

\[
C_{\text{p,e}} = C_p + Q_L \left( \frac{\partial H}{\partial T} \right)_P,
\]

\[
\alpha_{\text{e}} = \alpha + \frac{Q_L}{T} \left( \frac{\partial H}{\partial T} \right)_P,
\]

where \( C_p \) is the heat capacity of the solid crust, and \( Q_L \) is the latent heat of melting of the crust (Table 2).

**Rheological model**

Viscosity dependent on strain rate, pressure and temperature is defined in terms of deformation invariant (Randall, 1995) as:

\[
\eta_{\text{Barr}} = \left( \frac{\dot{\varepsilon}_t}{\dot{\varepsilon}_0} \right)^{2/n} F(A_d)^{1/n} \exp \left( \frac{E + V}{nRT} \right),
\]

where \( \dot{\varepsilon}_t = 1/2(\dot{\varepsilon}_x \dot{\varepsilon}_x + \dot{\varepsilon}_y \dot{\varepsilon}_y) \) is the second invariant of the strain rate tensor and \( A_d, E, V \) and \( n \) are experimentally determined flow law parameters (Tables 1 & 2). \( F \) is a dimensionless coefficient dependent on the type of experiments on which the flow law is based. For example:

\[
F = 2^{(1-\alpha)/n} \quad \text{for triaxial compression and}
\]

\[
F = 2^{(1-2\alpha)/n} \quad \text{for simple shear.}
\]

For rocks containing relatively small melt fractions, \((M < 0.1)\), the ductile rheology is combined with a brittle rheology to yield an effective viscoplastic rheology. For this purpose the Mohr-Coulomb yield criterion (e.g. Randall, 1995) is implemented by limiting creep viscosity, \( \eta_{\text{Barr}} \), as follows:

\[
\eta_{\text{Barr}} \leq \frac{(N_1 P + N_2)(1 - \lambda)}{(4\dot{\varepsilon}_0)^{1/2}},
\]

where \( P \) is dynamic (non-lithostatic) pressure, \( N_1 \) and \( N_2 \) are empirical constants (Brace & Kohlstedt, 1980). The pore fluid pressure \( \lambda \) (Table 1) controls the brittle strength of fluid-containing porous or fractured media. A hydrostatic gradient with a pore pressure coefficient \( \lambda = 0.4 \) is generally accepted for the upper crust (e.g. Sibson, 1990). Hydrocarbon exploration wells have shown that in sedimentary basins the transition from a hydrostatic to a near-lithostatic pore pressure gradient generally occurs at c. 3–5 km depth (e.g. Sibson, 1990). However, the KTB and Kola boreholes have shown that a hydrostatic pore pressure gradient can reach down to more than 9 km depth and a temperature of 265 °C (Kukkonen & Clauzer, 1994; Gräwe & Stockkert, 1997; Huenges et al., 1997).

For simplicity, we assumed a continuous transition from the hydrostatic pore fluid pressure \((\lambda = 0.4)\) at the surface to a characteristic pore fluid pressure \((\lambda_{\text{pot}} = 0.5–0.9)\) at 10 km depth. Intermediate pore fluid pressures are thus assumed at shallower depths <10 km, with an effective pore fluid pressure calculated as follows (Gerya et al., 2002):

\[
\lambda = \begin{cases} 
0.4(10 - \Delta z_{\text{ev}}) + \lambda_{\text{pot}} \Delta z_{\text{ev}} & \text{when } 0 \leq \Delta z_{\text{ev}} \leq 10 \text{ km} \\
\lambda_{\text{pot}} & \text{when } \Delta z_{\text{ev}} > 10 \text{ km}
\end{cases}
\]

where \( \Delta z_{\text{ev}} \) is the depth beneath the calculated dynamic erosion/sedimentation surface in km. The brittle strength of the mantle is assumed to be high because of the absence of free pore fluid \((\lambda_{\text{pot}} = 0 \text{ in Eq. (16)})\).

The effective viscosity \( \eta \) of molten rocks \((M > 0.1)\) was calculated by using the formula (Pinkerton & Stevenson, 1992; Bittner & Schmeling, 1995):

\[
\eta = \eta_0 \exp \left[ 2.5 + (1 - M) \left( \frac{1 - M}{M} \right)^{0.49} \right]
\]

where \( \eta_0 \) is an empirical parameter depending on rock composition, \( \eta_0 = 10^{13} \text{ Pa s} \) is taken for molten mafic rocks (i.e. \( 1 \times 10^{14} \leq \eta \leq 2 \times 10^{15} \text{ Pa s} \) for \( 0.1 \leq M \leq 1 \)) and \( \eta_0 = 5 \times 10^{14} \text{ Pa s} \) (i.e. \( 6 \times 10^{12} \leq \eta \leq 8 \times 10^{16} \text{ Pa s} \) for \( 0.1 \leq M \leq 1 \)) for felsic rocks (Bittner & Schmeling, 1995). 10^{16} and 10^{26} Pa s are the lower and upper cut values for viscosity of all types of rocks in our numerical experiments.

**Conservation equations and numerical implementation**

We have considered 2D creeping flow wherein both thermal and chemical buoyant forces are included along with heating from adiabatic compression and viscous dissipation in the heat conservation equation.

We have adopted (Gerya & Yuen, 2003a) a Lagrangian frame in which the heat conservation equation with a thermal conductivity \( k(T, P, C) \) (Table 2) dependent on rock composition \( (C) \), pressure and temperature (e.g. Hofmeister, 1999) that takes the form:

\[
\rho C_p \frac{D T}{D t} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + H_T + H_S + H_R
\]

in which

\[
q_x = -k(T, P, C) \frac{\partial T}{\partial x}, \quad q_y = -k(T, P, C) \frac{\partial T}{\partial y},
\]

\[
H_T = T(x, y) \left( \frac{\partial P}{\partial x} + \frac{\partial P}{\partial y} \right), \quad H_S = \sigma_{xz}\dot{s}_{xz} + \sigma_{yz}\dot{s}_{yz} + 2\sigma_{xx}\dot{s}_{xx},
\]

where \( D/Dt \) represents the substantive time derivative, \( H_T \) is the radioactive heating which depends on advected rock composition \( (C) \) and other notations are shown in Table 1. We emphasize the presence of the viscous heat production term \( H_S \) in the temperature equation because it plausibly has significant but understudied effect on the collisions process.

The conservation of mass is approximated by the incompressible mass conservation (continuity) equation.

\[
\frac{\partial \rho_x}{\partial x} + \frac{\partial \rho_y}{\partial y} = 0.
\]

The 2D Stokes equations for creeping flow take the form:

\[
\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = \frac{\partial P}{\partial x},
\]

\[
\frac{\partial \sigma_{yy}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} = \frac{\partial P}{\partial y} - g\rho(T, P, C, M),
\]

where \( \rho(T, P, C, M) \) depends explicitly on temperature, pressure, rock composition and melt fraction.

We employ viscous constitutive relationships between stress and strain-rate with \( \eta \) representing the effective viscosity, which depends on composition, temperature, pressure, strain-rate and melt fraction:

\[
\sigma_{ij} = 2\eta \dot{\varepsilon}_{ij}.
\]

It is also worth noting that dynamic (and not depth-dependent lithostatic) pressure was consistently used in all calculations including rheological and melting models, thus taking into account effects of non-lithostatic pressures in compressive regions of thickened crust (Petrini & Podladchikov, 2000).