Magnitude Scales for Marsquakes

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Abstract  In anticipation of the upcoming 2018 InSight (Interior exploration using Seismic Investigations, Geodesy and Heat Transport) Discovery mission to Mars, we calibrate magnitude scales for marsquakes that incorporate state-of-the-art knowledge on Mars interior structure and the expected ambient and instrumental noise. We regress magnitude determinations of 2600 randomly distributed marsquakes, simulated with a spectral element method for 13 published 1D structural models of Mars’ interior. The continuous seismic data from InSight will be returned at 2 samples per second. To account for this limited bandwidth as well as for the expected noise conditions on Mars, we define and calibrate six magnitude scales: (1) local Mars magnitude \( M_{Ma} \) at a period of 3 s for marsquakes at distances of up to 10°; (2) \( P \)-wave magnitude \( m_{Ma}^P \); (3) \( S \)-wave magnitude \( m_{Ma}^S \) each defined at a period of 3 s and calibrated for distances from 5° to 100°; (4) surface-wave magnitude \( M_{Ma}^s \) defined at a period of 20 s, as well as (5) moment magnitudes \( M_{Ma}^{FM} \); and (6) \( M_{Ma}^F \) computed from the low-frequency (10–100 s) plateau of the displacement spectrum for either body waves or body and surface waves, respectively; we calibrate scales (4)–(6) for distances from 5° to 180°. We regress stable calibrations of the six scales with respect to the seismic moment magnitude at \( M_w = 5.5 \) by correcting filtered phase amplitudes for attenuation with distance and source depth. Expected errors in epicentral distance and in source depth (25% and 20 km, respectively) translate into magnitude errors of 0.1–0.3 units. We validate our magnitude relations with an independent test dataset of 2600 synthetic marsquakes \((1.0 \leq M_w \leq 7.0)\), for which seismograms are superimposed on the realistic noise predicted by the InSight noise model. Marsquakes with \( M_w < 3.0 \) and epicentral distances of \( \Delta > 15° \) are expected to be hidden in the Mars background noise and will likely not be detectable.

Electronic Supplement: Figures showing preprocessing steps for amplitude and magnitude regression for Mars and results for Earth using the preliminary reference Earth model (PREM).

Introduction

The National Aeronautics and Space Administration (NASA) InSight (Interior exploration using Seismic Investigations, Geodesy and Heat Transport) Discovery Program mission will deploy in 2018 a lander equipped with geophysical and meteorological sensors on the Martian surface (Banerdt et al., 2013), including a single three-component ultra-sensitive very-broadband (VBB) seismometer (Lognonné et al., 2012, 2015). Objectives of the InSight mission are (1) to determine 1D models of Mars’ mantle and core to within ±5% uncertainty in seismic-wavespeeds, as well as 3D velocity models of the crust, and (2) to measure the activity and distribution of seismic events on Mars, including both tectonic and impact seismicity (Banerdt et al., 2013). InSight launched successfully on 5 May 2018 and will land on Mars on 26 November 2018. A nominal operation for one Martian year is anticipated, corresponding to roughly two Earth years.

Without plate tectonics, we expect secular cooling as the driver of sustainable tectonic stress on Mars (Phillips, 1991). Theoretical models for thermoelastic cooling (Phillips, 1991; Knapmeyer et al., 2006; Plesa et al., 2018) and observed surface faults (Golombek et al., 1992; Golombek, 2002) predict an occurrence of 4–40 globally detectable marsquakes per
Martian year, which are estimated to a (body-wave magnitude) $m_b$ 4 event on the Earth. Another source of seismic events on Mars is meteorite impacts. Daubar et al. (2015) and Teanby (2015) predict 0.1–30 regional, respectively, $\sim 8–16$ InSight-detectable impacts per Martian year. Lognonné and Johnson (2007) predict about 10 impacts per year generating amplitudes exceeding $3 \times 10^{-9}$ m/s$^2$.

The Marsquake Service (MQS), as a part of the InSight SEIS Team, is in charge of identification and characterization of seismicity on Mars, as well as management of the seismicevent catalog. The Mars Structure Service is responsible for the determination of 1D and 3D structural models (Panning et al., 2017). In preparation for the data return, a series of single-station event location methods (Panning et al., 2015; Böse et al., 2017) and iterative inversion techniques (Khan et al., 2016; Panning et al., 2017) have been developed and tested (Clinton et al., 2017). In this article, we calibrate magnitude-scaling relations for marsquakes for the MQS that incorporate the state-of-the-art knowledge on Mars’ interior structure and the expected ambient and instrumental noise.

Magnitudes characterize the relative size of a marsquake based on the measurements of peak wave amplitudes of a particular phase and frequency recorded by a seismometer and corrected for the attenuation with distance and possibly depth (e.g., Båth, 1981; Kanamori, 1983). The most commonly used scales on the Earth are the (1) local magnitude $M_L$ also known as Richter magnitude (Richter, 1935), (2) body-wave magnitude $m_b$, (3) surface-wave magnitude $M_s$ (Gutenberg, 1945), and (4) moment magnitude $M_w$ (Hanks and Kanamori, 1979). The first three scales have limited range of applicability and do not satisfactorily measure the size of the largest marsquake source because of amplitude saturation in the narrow frequency ranges they are determined in. The $M_w$ scale, which is based on the concept of the seismic moment $M_0$ and as such is physically meaningful, applies to all earthquake sizes, but is usually more difficult to compute, especially in a single-station inversion.

Existing Earth-magnitude scaling relations cannot be easily adopted to Mars because the two planets have different interior properties and planet sizes and therefore different amplitude–distance relations. Furthermore, InSight is going to deploy a single seismometer on Mars, so there is a need to define magnitude scales for various seismic phases that may be identified and which are applicable to marsquakes from local to teleseismic distances. In this article, we calibrate magnitude scales for Mars by the simulation of seismic-wave propagation through a set of realistic 1D Mars models. We anticipate these relations to be applied to the initial seismic-event catalog that will be produced by InSight. Depending on the observed attenuation, scattering, and 3D effects, these relations may need to be updated once observational data and new models become available.

**Data and Methods**

Despite the theoretical studies of Goins and Lazarevicz (1979) on magnitude-dependent detection thresholds, no marsquake signal could be identified in the seismic recordings of the Viking missions in the mid-1970s (Anderson et al., 1977). In the absence of seismic data, we calibrate our magnitude scales with synthetic waveforms that incorporate the current knowledge on Mars interior structure and the expected ambient and instrumental noise. Our procedure is outlined in Figure 1 and will be described in the following.

**Mars Structural Models and Travel Times**

We use a series of 13 published 1D structural models of Mars’ interior (Fig. 2) as described by Clinton et al. (2017) and Panning et al. (2017) that combine structural models from Rivoldini et al. (2011) and Khan et al. (2016). These

![Figure 1. Calibration of magnitude scales for Mars from synthetic seismograms using a suite of structural 1D models. See the Data and Methods section for details.](image-url)
Figure 2. (a) 13 1D Mars models for $P$- and $S$-wave velocities $V_P$ and $V_S$, (b) density $\rho$, and attenuation $Q_\mu$, used for waveform simulations in this study. Preliminary reference Earth model (PREM) (Dziewonski and Anderson, 1981) is shown for comparison. The color version of this figure is available only in the electronic edition.
models were constructed to meet the currently available Martian geophysical data, including the mean mass and moment of inertia, as well as tidal response in the form of the second degree tidal Love number and global tidal dissipation. Seismic properties ($P$ and $S$ wavespeeds and density) in the silicate portion of Mars are computed using phase equilibrium computations that self-consistently predict radial profiles of above-mentioned properties as a function of composition, temperature, and pressure (Connolly et al., 2009). To compute radial shear attenuation profiles, Khan et al. (2016) combined the phase equilibrium computations with a laboratory-based viscoelastic dissipation model (Jackson and Faul, 2010). This rheological model, which relies on measurements of anhydrous and melt-free olivine, is both temperature and grain-size sensitive and imposes strong constraints on interior structure (Nimmo and Faul, 2013; Khan et al., 2018). In contrast, the shear attenuation models of Rivoldini et al. (2011) were scaled from the preliminary reference Earth model (PREM; Dziewonski and Anderson, 1981). Core properties were calculated using equation-of-state modeling as described in detail by Rivoldini et al. (2011). Finally, the Martian mantle compositions used derive from geochemical and isotopic analyses of Martian rocks and primitive solar system material (e.g., Taylor, 2013; Khan et al., 2018). The emphasis here is on producing physically realistic models that incorporate physical constraints derived from thermodynamic considerations and laboratory measurement. We expect that the predicted set of models is representative of our current a priori knowledge of Mars’ internal structure.

TABLE 1
Parameters and Regression Results for the Six Magnitude Scales

<table>
<thead>
<tr>
<th>Scale $M^\text{Ma}$</th>
<th>Amplitude $A_i$, Band-Pass Filter (BP)</th>
<th>Phase Time Window</th>
<th>Distance $\Delta$ and Depth $z$ Range</th>
<th>Coefficients in Equations (2a) or (2b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M^\text{Ma}_{\text{Sh}}$</td>
<td>$A = \max(\bar{Z}[])$ (m)</td>
<td>$P1 : (P1 + 150 \text{s})$</td>
<td>$\Delta \leq 10^5$</td>
<td>$a_i$, $b_i$, $c_i$, $\sigma$</td>
</tr>
<tr>
<td>$M^\text{Ma}_{\text{Deep}}$</td>
<td>BP: 0.2–0.5 Hz</td>
<td></td>
<td>$0 \leq z &lt; 20 \text{ km}$</td>
<td>0.9 [1.7], 0.01 [0.01], 7.7 [7.8], 0.38 [0.44]</td>
</tr>
<tr>
<td>$M^\text{Ma}_{\text{Sh}}$</td>
<td>$A = \max(\bar{Z}, [N], [E])$ (m)</td>
<td>$P1 : P.P$ or $P1 : S1$ (if PP not existent)</td>
<td>$5^\circ \leq \Delta \leq 10^5$ $0 \leq z \leq 100 \text{ km}$</td>
<td>1.2 [1.0], —, 9.0 [9.9], 0.48 [0.38]</td>
</tr>
<tr>
<td>$M^\text{Ma}_{\text{Sh}}$</td>
<td>BP: 0.2–0.5 Hz</td>
<td></td>
<td>$5^\circ \leq \Delta \leq 10^5$ $0 \leq z \leq 100 \text{ km}$</td>
<td>2.5 [2.1], —, 6.9 [8.4], 0.43 [0.24]</td>
</tr>
<tr>
<td>$M^\text{Ma}_{\text{Sh}}$</td>
<td>$A = \max(\sqrt{(E^2 + N^2)})$ (m)</td>
<td>$R1 : \text{Rend}$</td>
<td>$5^\circ \leq \Delta \leq 10^5$ $0 \leq z \leq 100 \text{ km}$</td>
<td>0.7 [1.1], 0.01 [0.01], 7.9 [7.9], 0.24 [0.16]</td>
</tr>
<tr>
<td>$M^\text{Ma}_{\text{Sh}}$</td>
<td>BP: 0.033–0.066 Hz</td>
<td></td>
<td>$5^\circ \leq \Delta \leq 10^5$ $0 \leq z \leq 100 \text{ km}$</td>
<td>1.2 [1.1], —, 8.7 [9.3], 0.35 [0.33]</td>
</tr>
<tr>
<td>$M^\text{Ma}_{\text{Sh}}$</td>
<td>$A$ from $Z(m/\text{Hz})$</td>
<td>$P1 : R1$</td>
<td>$5^\circ \leq \Delta \leq 10^5$ $0 \leq z \leq 100 \text{ km}$</td>
<td>1.1 [1.2], 0.01 [0.01], 7.5 [7.9], 0.29 [0.25]</td>
</tr>
</tbody>
</table>

Amplitude $A_i$ is either the peak amplitude determined from the selected phase time window (for $M^\text{Ma}_{\text{Sh}}$, $m^\text{Ma}_{\text{Sh}}$, $m^\text{Ma}_{\text{St}}$, $M^\text{Ma}_{\text{St}}$) or from the low-frequency plateau of the displacement spectrum (for $M^\text{Ma}_{\text{Sh}}$ and $M^\text{Ma}_{\text{St}}$). Selected sensor components, time windows, filters, distance, and depth ranges are given. The fit quality is measured by the standard deviation $\sigma$ of the error distributions $M^\text{Ma} \cdot M^\text{Ma}$. Numbers in square brackets are calculated from waveform synthetics computed for the preliminary reference Earth model (PREM). Z, N, E sensor components, up–down, north–south, east–west; P1, first-arriving $P$ wave; S1, first-arriving $S$ wave; R1, first surface-wave arrival assuming speed of 4.0 km/s; Rend, last surface-wave arrival assuming speed of 2.5 km/s; PP, free surface reflection of $P$ wave leaving the source downward; SS, free surface reflection of $S$ wave leaving the source downward.

Additional modeling uncertainties associated with the attenuation models are examined in the Discussion section.

We apply the TauP-2.4.1 toolbox by Crotwell et al. (1999) to predict arrival times of magnitude-relevant phases in each of these models, which we use to cut appropriate time windows for amplitude computations (Fig. S1, available in the electronic supplement to this article). Details on the selected time windows are given in Table 1.

Green’s Functions and Waveform Simulations

We compute Green’s function (GF) databases for all 13 structural models for source depths down to 100 km with the spectral element code AxiSEM (Nissen-Meyer et al., 2014) at a minimum reliable period of 1 s and for a duration of 30 min, as well as at a minimum of 5 s and for a duration of 60 min. As discussed later, we use the 1 s databases for the calibration of body-wave magnitudes and the longer 5 s databases for the calibration of magnitudes based on surface waves.

We use Instaseis (van Driel et al., 2015) to generate synthetic waveforms from the GFs, assuming a magnitude-dependent source time function (Fig. 3a) that produces an amplitude spectrum of shape $M(f) = 1/(1 + (f/f_c)^2)$. Following Brune (1970), we scale the corner frequency $f_c$ of this spectrum with the seismic moment $M_0$ as

$$f_c = 0.49 \times V_S \times (\Delta \sigma / M_0)^{1/3} \text{Hz},$$

assuming a stress drop of $\Delta \sigma = 1$ MPa and shear-wave velocity of $V_S = 3$ km/s, which gives $f_c = 1.36$ Hz for
Noise Models

We determine magnitude-scaling relations in this study from noise-free waveform synthetics. However, we validate these relationships in the presence of realistic noise. The Martian noise model (Fig. 4; Murdoch et al., 2015a,b; Moun et al., 2017) considers various noise sources, including the sensors and systems, as well as the environment (fluctuating-pressure-induced ground deformation, magnetic field, and temperature-related noise) and the nearby lander (wind-induced solar panel vibrations). Strongest noise is expected at frequencies below 0.035 Hz (∼30 s); at higher frequencies, the predicted noise is about 2 orders of magnitude smaller than on the Earth because of the absence of microseisms, which on the Earth are dominant between 0.025 and 0.5 Hz (2–40 s) (Fig. 4). Because of the absence of oceans, we do not expect a microseismic peak on Mars.

Calibration of Magnitude Scales

Traditional magnitude scales developed for the Earth, such as $M_L$, $m_b$, or $M_s$ (as defined, e.g., in the International Association of Seismology and Physics of the Earth’s Interior [IASPEI] standards; see Bormann, 2012, chapter 3), cannot be easily adopted to Mars because interior structure of the two planets is very different. Furthermore, we want our magnitude equations to be applicable to the continuous seismic data streams returned from InSight; even though the vertical component will be sampled by default with 10 samples per second, the horizontals will be sampled with only 2 samples per second, which means that magnitude scales (at least those for $S$ waves) need to be defined at frequencies below 1.0 Hz.

The InSight mission has the goal to characterize as many marsquakes as possible. Therefore, we define in this study six magnitude scales for various seismic phases. We anticipate that for some of the InSight-detected marsquakes, it will be possible to identify multiple phases and thus to compute multiple magnitude types. We define the following six scales, where we use superscript Ma for Mars: correspondent to the local and body-wave magnitude scales on the Earth, we define for Mars (1) $M_{Ma}^L$, (2) $m_{Ma}^b$ (for first-arriving $P$ wave) and (3) $m_{Ma}^b S$ (for first-arriving $S$ wave); all three magnitude scales are computed at a period of 3 s using a fourth-order Butterworth band-pass filter with corner frequencies at 0.2 and 0.5 Hz. We also define (4) a surface-wave magnitude for Mars, $M_{Ma}^S$, which we compute at a period of 20 s (15–30 s band-pass). Finally, we compute the moment magnitudes, (5) $M_{Ma}^{FB}$ and (6) $M_{Ma}^F$, from the low-frequency (10–100 s) amplitude determined from the displacement spectrum using either body waves or the full record (Fig. 4).

As an example, Figure S2 shows the selected time windows and peak values for each magnitude scale for a marsquake simulated at 10° distance.

For magnitude scales without a clear dependence on source depth ($m_{Ma}^b$, $m_{Ma}^b S$, and $M_{Ma}^S$), we assume the following log-linear relationship between magnitude $M_{Ma}^S$, amplitude $A_i$, and epicentral distance $\Delta$ (°):
For magnitude scales with a strong dependence on source depth \( z \) (km) \( (M_{L}^{Ma}, M_{N}^{Ma}, \text{and } M_{F}^{Ma}) \), we use

\[
M_{i}^{Ma} = \log_{10}(A_{i}) + a_{i} \times \log_{10}(\Delta) + c_{i}. \tag{2a}
\]

in which \( A_{i} \) is either the filtered peak displacement amplitude (m) determined within a certain time window from the waveform time series (for \( M_{L}^{Ma}, m_{L}^{Ma}, m_{N}^{Ma}, \text{and } M_{F}^{Ma} \)) or the spectral amplitude (m/\( \sqrt{\text{Hz}} \)) determined from the long-period plateau of the amplitude displacement spectrum (for \( M_{L}^{Ma} \) and \( M_{F}^{Ma} \)). Similar to the Earth, we determine \( A_{i} \) for \( m_{L}^{Ma} \) and \( m_{N}^{Ma} \) from the peak amplitude using all three sensor components \( Z \) (up–down), \( N \) (north–south), and \( E \) (east–west); for \( M_{F}^{Ma} \), we use only horizontal, and for \( M_{L}^{Ma}, M_{N}^{Ma}, \text{and } M_{F}^{Ma} \) only vertical components. When fully installed, the orientation of the InSight VBB components will be known. Details on the selected filters, waveform components, and time windows for each magnitude scale are given in Table 1.

To determine the coefficients \( a_{i}, b_{i}, \text{and } c_{i} \) in equations \( (2a) \) and \( (2b) \), we minimize the magnitude residuals \( M_{Ma} - M_{w} \) for a dataset of 2600 randomly distributed marsquakes with random source mechanisms. Relative to the expected limited magnitude range for marsquakes (e.g., Golombek, 2002; Knapmeyer et al., 2006), we chose a fairly large event size for calibration and simulate all events as

\[
M_{0} = 2.24 \times 10^{17} \text{ N} \cdot \text{m } (M_{w} = 5.5; \text{Kanamori, 1977}).
\]

We assume an impulsive source because equation \( (1) \) predicts a corner frequency above the simulation frequency. Epicenters are chosen to follow a logarithmic distribution in distance to reflect the functional dependencies in equations \( (2a) \) and \( (2b) \); the event depth varies from 0 to 100 km (Fig. 5). We model our marsquakes as double-couple events with random—uniformly distributed—strike, rake, and dip. For each simulated event, a single model is randomly selected. Because the length of the GFs in the 1 s databases is limited to 30 min, we use these databases only for the calibration of local and body-wave magnitudes \( (M_{L}^{Ma}, m_{L}^{Ma}, \text{and } m_{N}^{Ma}) \), and we use the 5 s databases of 60 min duration for the calibration of \( M_{F}^{Ma} \) and \( M_{FB}^{Ma} \).

As specified in Table 1, we filter the simulated waveforms and determine \( A_{i} \) from the respective phase time intervals for each magnitude scale (Fig. 1). To avoid problems caused by our epicentral distance metric for deep events at close distances (distance-depth trade-off), we remove prior to the regression all events at epicentral distances of \( \Delta < 1^\circ \).

Results

Figure 6 shows the decay of amplitude \( A_{i} \) with epicentral distance \( \Delta \) for all six magnitude scales. Although the decay is mostly log–linear, there are for some scales subtle changes in the slope at \( \Delta \sim 10^\circ \) and \( 100^\circ \) caused by peculiarities of crustal and core phases such as shadow zones. Up to a distance of \( 5^\circ \), the seismic-wave propagation occurs mostly in the crust, which makes the amplitudes only weakly dependent on distance. At larger distances, amplitudes are controlled mainly by the attenuation in the mantle, which manifests in a stronger amplitude decay. Based on this observation, we decide to calibrate our magnitude scales for the following distance ranges: (1) \( M_{L}^{Ma} \) for events at \( \Delta \leq 10^\circ \), (2) \( m_{L}^{Ma} \) and \( m_{N}^{Ma} \) for \( 5^\circ \leq \Delta \leq 100^\circ \), and (3) \( M_{F}^{Ma}, M_{FB}^{Ma}, \text{and } M_{F}^{Ma} \) for \( 5^\circ \leq \Delta \leq 180^\circ \). As shown later, each magnitude scale actually extends to much larger distance ranges with acceptable errors.

The amplitudes of some phases and frequency bands show a strong dependence on the source depth, with shallow marsquakes generating larger amplitudes at the planet surface than those at greater depths (Fig. 6). To investigate this dependence in more detail, we plot in Figure 7 the magnitude residuals \( M_{i}^{Ma} - M_{w} \) after a pure distance regression using equation \( (2a) \): \( M_{L}^{Ma} \) (in particular for shallow events at \( z < 20 \) km), \( M_{F}^{Ma} \), and \( M_{F}^{Ma} \) show a clear depth
dependence, but the depth dependence is weak for $m_{Ma}$, $m_{MaS}$, and $M_{MaFB}$. This observation is consistent with observations on the Earth (e.g., Eissler and Kanamori, 1986).

After this preliminary assessment, we run a complete amplitude regression in the scale-dependent distance ranges as specified earlier, including both distance and depth dependencies for $M_{MaL}$, $M_{Mas}$, and $M_{MaF}$ using equation (2b), while keeping equation (2a) for $m_{MaB}$, $m_{MaB}$, and $M_{MaFB}$. For $M_{MaL}$, we use equation (2b) only for shallow events at $z < 20$ km and set $z = 20$ km for all events at greater depth, so that the $b_i$ term effectively adds to the constant $c_i$.

Table 1 summarizes the regression results for all six magnitude scales. We obtain stable magnitude calibrations with respect to $M_w$ 5.5 for all scales with standard deviations $\sigma$ of the residual distributions $M_{Ma} - M_w$ ranging from $\sigma = 0.24$ for $M_{Ma}$ to $\sigma = 0.48$ for $m_{MaS}$. With the exception of $M_{MaL}$, which is valid only for local marsquakes at epicentral distances of up to $10^\circ$, the $m_{Ma}$ and $m_{MaS}$ scales can be extended with acceptable errors (usually < 0.5 magnitude units) to distances from $0^\circ$ to $100^\circ$, scales for $M_{Ma}$ and $M_{MaF}$ from $0^\circ$ to $180^\circ$, and for $M_{MaFB}$ from $5^\circ$ to $180^\circ$ (Fig. 8).

To calibrate our magnitude scales with the seismic moment magnitude $M_w$ over a larger range of magnitudes, we create a second random marsquake dataset of 2600 events for $1.0 \le M_w \le 7.0$ and compare our magnitude estimates with the known moment magnitude of these events (Fig. 9). Because our magnitude scales were computed for different phases in different frequency bands, there is no a priori reason for magnitude scales to agree. Magnitudes scales for the Earth also tend to disagree if computed over large magnitude ranges (e.g., Shearer, 1999). The seismic moment represents a long-period end of the source spectrum, so a high correlation of the six magnitudes scales with the seismic moment implies that the entire spectrum is uniquely determined from the long-period end (Kanamori, 1983), which is inconsistent with the observed magnitude-dependent source spectra in equation (1). Our Mars magnitudes $M_{MaL}$, $m_{Ma}$, and $m_{MaS}$ agree well with $M_w$ for $4.0 < M_w < 6.0$; for smaller and larger events, the seismic moment magnitude tends to be underestimated by up to one full magnitude unit or even more (Fig. 9). For $M_{MaL}$, $M_{MaFB}$, and $M_{MaF}$, a good agreement with $M_w$ is expected for $5.0 < M_w < 7.0$; smaller events

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Figure 5. Distribution of synthetic marsquakes for magnitude calibration. Whereas all event magnitudes are simulated as $M_w$ 5.5 ($M_0 = 2.24 \times 10^{17}$ N·m), source depths and focal mechanisms are kept random. The color version of this figure is available only in the electronic edition.
we generate a second test data set of event seismograms for 2600 randomly distributed marsquakes (1.0 ≤ Mw ≤ 7.0), but this time we superimpose realistic noise. We use the conservative InSight noise model for day (Fig. 4) and add random phase noise time series to the synthetics. As shown in Figure 10, the added noise causes the estimated magnitudes to saturate for small marsquakes (Mw < 2.0 for M_{w}^{Ma} and M_{FB}^{Ma}; Mw < 2.5 for M_{FB}^{Ma}; Mw < 3.0 for M_{FB}^{Ma}; Mw < 3.5 for M_{FB}^{Ma}) for events distances Δ > 15° (∼900 km on Mars), which means that these events are hidden in the Mars background noise and are therefore undetectable. For small marsquakes (Mw < 3.5), M_{L}^{Ma} tends to underestimate Mw and thus M_{0}.

Discussion

Magnitude scales for earthquakes and marsquakes depend on the decay of seismic-wave amplitudes A with increasing distance, which is controlled by elastic (geometrical spreading, multipathing, and scattering attenuation) and anelastic (intrinsic attenuation) processes: whereas elastic processes conserve the energy in the propagating wavefield, anelasticity converts seismic energy to heat. The 1D structural Mars models and AxiSEM waveform simulations used in this study account for both geometrical spreading and anelastic attenuation but neglect scattering and multipathing through 3D structures.

The strongest decay of amplitudes in our synthetics is caused by the geometrical spreading. By conservation of energy, the energy in a unit area of the growing wavefront decreases as r², in which r is the distance from the source.
Body-wave amplitudes hence decay with $1/r$, so we expect $\log_{10}(A)$ to decrease by about one unit per unit change in $\log_{10}(A)$ (Fig. 6), which would correspond to $\Delta M \approx 1$ in equations (2a) and (2b) (Table 1). The decay of surface-wave amplitudes is more complex, also because of their dispersion.

Our marsquake synthetics were computed for radial 1D models. In reality, body waves travel through inhomogeneous 3D structures, which cause amplitudes to increase and decrease because of the focusing and defocusing of seismic rays, respectively. It is expected that lateral structural variations in the Martian crust are strong (Larmat et al., 2008; Bozdag et al., 2017), which manifests as a crustal dichotomy between the southern and northern hemispheres (e.g., Zhong and Zuber, 2001): the crust in the northern lowlands with an estimated average thickness of $\sim 30$ km is only half as thick as in the southern highlands. We expect 3D effects to increase the variability of the waveform amplitudes and thus to lead to a greater spread in the estimated magnitudes compared with the results from our 1D models.

Intrinsic attenuation on Mars is not known well. Khan et al. (2018) estimate that $Q_0$ is between 120 and 200 at seismic periods in the upper mantle, which is consistent with the model suite that we have been using for our magnitude calibration in this article. The magnitude estimate change $\Delta M$ depends on $\Delta Q$ as

$$\Delta M = \frac{\partial M}{\partial Q} \Delta Q = \frac{\pi f r}{c \ln 10} Q_0^{-2} \Delta Q. \quad (4)$$

Assuming that $Q$ in the upper mantle varies around $Q_0 = 150$, the magnitude estimated from an $S$ wave at $r = 2000$ km distance depends on $Q$ by $(\partial M/\partial Q) = 8 \times 10^{-3}$. Therefore, even a change in attenuation by 1/3, that is, $\Delta Q = \pm 50$, results only in a $\Delta M = \pm 0.3$. Local waves that travel mainly through the crust, in which $Q$ is expected being much larger (assume $Q_0 = 600$ and $r = 500$ km), are even less sensitive with $\partial M/\partial Q = 1.4 \times 10^{-4}$, so that even $\Delta Q = 100$ results in small magnitude uncertainties compared with those introduced by the focal mechanism, the depth, and uncertainties in the velocity model. The effect is even smaller for other magnitude scales because they are measured at lower frequencies or for $P$ waves, which are less affected by attenuation.

Goins et al. (1981) developed magnitude and energy equations for moonquakes that account for the effects of intense scattering on the Moon. On Mars, we expect diffusive scattering to be less pronounced because strong scattering requires extreme low-volatile conditions (for very high $Q$), which are not expected on Mars (Banerdt et al., 2013).

To compare our magnitude-scaling results with calibrations for the Earth, we repeat the amplitude regression for terrestrial waveforms for a similar seismic-event catalog. The waveforms are simulated with AxiSEM for the (anisotropic) PREM of Dziewonski and Anderson (1981). The corresponding GF database prem_a_2s is hosted at the Incorporated Research Institutions for Seismology (IRIS, Krischer et al., 2017; see Data and Resources). These GFs were
Figure 8. (a–f) Magnitude residuals as a function of epicentral distance for all six scales. With the exception of $M_{MaL}$ in (a), which is valid only for local marsquakes at epicentral distances of $\Delta < 10^\circ$, all other scales are usable for events up to 100$^\circ$ or even up to 180$^\circ$. Extending the magnitude relations to larger distances increases the magnitude errors with tolerable errors. Thick black lines show median values, and thin black lines show the 5th and 95th percentiles. Only white-shaded distance ranges were considered for magnitude calibration.

Figure 9. Comparison of the six magnitude scales (a–f) with moment magnitude $M_w$ without noise. $M_{MaL}$, $m_{MaB}$, and $m_{MaS}$ agree well with $M_w$ for $4.0 < M_w < 6.0$; for smaller and larger events $M_w$ tends to be underestimated by up to one full magnitude unit or even more. For $M_{MaS}$, $M_{MaB}$, and $M_{FB}$, a good agreement with $M_w$ is expected for $5.0 < M_w < 7.0$; smaller events tend to be underestimated by up to one full magnitude unit or more. The color version of this figure is available only in the electronic edition.
computed at 2 s; we were using 1 s GFs for Mars. The results from magnitude calibration for the Earth are shown in Table 1 and Figures S3–S5. Most of our magnitude scales are defined differently (in terms of frequency) than in the standard Earth magnitude relations (e.g., Bormann, 2012). The surface-wave magnitude $M_s$, however, is defined at 20 s both in our relationship and in the IASPEI reference (e.g., Bormann, 2012, chapter 3). We find that our scaling coefficient $a_i$ determined from the PREM waveform simulations is with $a_i = 1.1$ (Table 1) significantly smaller than in the IASPEI reference as well as in the original publication by Gutenberg (1945), in which $a_i = 1.66$. As pointed out by several authors (e.g., von Seggern, 1977; Herak and Herak, 1993; Rezapour and Pearce, 1998; Bormann, 2012), the IASPEI reference relation, however, suffers from a systematic distance-dependent bias because it had been developed for calibrating $(A/T)_{max}$ over a wide range of periods and distances rather than for calibrating displacement amplitudes $A$ at periods of $T \approx 20$ s. To overcome this bias, Herak and Herak (1993) proposed an alternative $M_s$ relation at 20 s, which holds for $0^\circ < \Delta < 180^\circ$. In this relation, the scaling coefficient is $a_i = 1.1$, which is in excellent agreement with our findings (Table 1) and which is also consistent with relations by von Seggern (1977). The relation by Herak and Herak (1993) has no depth-dependent coefficient but is averaged over earthquakes of different source depths. Our $M_s$ relation for the Earth agrees well with Herak and Herak (1993) for $z = 60$ km (Fig. S6).

Both the distance- and depth-dependent coefficients $a_i$ and $b_i$ in our magnitude relations (Table 1) are quite similar for Mars and Earth even though the distant-dependent decay of amplitudes, in particular for $M_L$, is generally smaller on Mars (Fig. S6). This effect is likely due to the smaller velocities and densities in the Martian crust and upper mantle (Fig. 2). An exemption is $m_b$, for which the amplitudes undergo a stronger attenuation than on the Earth. The main difference in our magnitude relations for Mars and Earth is in the constant $c_i$, which is up to 1.5 units larger on Mars (Table 1). This means that the amplitudes for the same marsquake size are generally greater on Mars (Fig. S6). This discrepancy can be largely attributed to the difference in the planet size: the Earth radius is 1.88 times larger than Mars, so we expect $c_i$ to be $\log_{10}(1.88) \approx 0.3$ times larger on the Earth.

Cross calibration of various magnitude scales on the Earth does not achieve an agreement over a large range of magnitudes (e.g., Shearer, 1999). This is because the Earth magnitude scales are computed for various phases in different frequency bands (on the Earth, $M_L$ is typically calculated at $0.1–3$ s, $m_b$ at $\sim 1$ s, $M_s$ at $\sim 20$ s, and $M_w$ typically at $10–\infty$ s), so there is no a priori reason for magnitude scales to agree. Grouping magnitude scales according to the frequency bands they use actually reaches a much better interscale agreement (Kanamori, 1983). Our marsquake magnitude scales also reach a good agreement with $M_{L0}$ and $M_w$ for limited magnitude ranges only. For $M_{L0}^{Ma}$, $m_b^{Ma}$, and $M_s^{Ma}$, which we defined at 3 s, we observe $M_{L0}^{Ma} \propto 3/2 \times M_w$ for $M_w < 4.5$, and $M_{L0}^{Ma} \propto M_w$ for $M_w \geq 4.5$. For $M_s^{Ma}$, $M_{FB}^{Ma}$, and $M_{F}^{Ma}$, which we defined at 10–100 s, we observe $M_{F}^{Ma} \propto 3/2 \times M_w$ for...
1.0 ≤ M_w ≤ 6.0 (see equations 3a and 3b and Fig. 9). A similar break in 1:1 scaling for small earthquakes is observed on the Earth and is explained by Deichmann (2017) by the observation that the pulse width and equivalently corner frequencies remain practically constant for small earthquakes. Thus, the signals of small earthquakes in an attenuating medium essentially form the impulse response of the medium, which is scaled by the seismic moment. Because we do not expect large marsquakes (M_w > 5.5) to occur on Mars (Knapmeyer et al., 2006), magnitude saturation will likely not be an issue, at least not for frequencies below 1 Hz.

The L1 requirements of the InSight Mars mission demand marsquakes to be located with an accuracy of ±25% (L1-SCI-51; Banerdt et al., 2013). As can be seen from Table 1, location uncertainties on this order translate into errors of ±0.1 to ±0.3 in magnitude on all six scales, calculated as max(|a_i × log_{10}(0.75)|, a_i × log_{10}(1.25)), respectively. With the probabilistic framework developed by Böse et al. (2017), location errors for well-locatable events are expected to be smaller than ±25%, implying that magnitude uncertainties due to errors in Δ are negligible. More critical are errors in the estimated source depth z, which we expect to be large in the single-station-based locations of the InSight mission. An error of 20 km in z leads to a magnitude increase or decrease by 0.2 units (calculated as 20 × b_i) for M_{LB} (shallow events with z < 20 km), M_{LB'}, and M_{FB}. Assuming a wrong depth, particularly for very close events at Δ < 1° (~50 km on Mars) for which the hypocentral instead of the epicentral distance metric should be used, can lead to noticeable magnitude errors. However, observing such a close event during the 2-yr Mars InSight mission is quite unlikely, in particular because most observed surface faults are located at larger distance from the InSight landing site (Knapmeyer et al., 2006).

Conclusions

In this study, we calibrated magnitude scales for marsquakes from synthetic waveform data that incorporate the state-of-the-art knowledge on Mars’ interior structure and the expected ambient and instrumental noise of the VBB InSight seismometer. We determined six magnitude scales for marsquakes: M_{LB}, m_{LB}, m_{DB}, M_{LB}, M_{FB}, and M_{FB'}, which allow us to compute magnitudes for various identified phases. For each magnitude scale, we provide an approved distance cutoff, but we demonstrate that the relations can be extended to larger distances with acceptable magnitude errors with respect to M_w (Fig. 8).

With the exception of m_{LB}, we expect on Mars a weaker decay of seismic-wave amplitudes with increasing epicentral distance compared with the Earth, mostly due to the lack of a lower mantle, where attenuation on the Earth is strong and also because of the smaller planet size of Mars.

We anticipate that our magnitude relations will be applied to the initial seismic-event catalog produced by InSight. Depending on the observed scattering and 3D effects, these relations may need to be updated when new models become available.

Data and Resources

The Green’s function (GF) databases we use here to calibrate our magnitude scales for Mars are publicly available via an HTTP webservice at http://instaseis.ethz.ch/marssynthetics (last accessed June 2018). Readers are referred to Ceylan et al. (2017) for more information on extracting waveforms from these databases, as well as 16 pre-existing ones. The prem_a_2s GF database for the (anisotropic) preliminary reference Earth model (PREM) is hosted at http://ds.iris.edu/ds/products/syngine (last accessed November 2017).

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