

## Chapter 3

# Rheology

To be able to determine the deformation of the Earth, it is essential to know the strength of its materials. The strength of materials in the Earth is largely dependent on temperature, pressure, and rock composition. All of these factors change significantly when going down from the surface into the Earth's interior. Since it is not possible to directly measure this strength at large depth, we have to rely on laboratory measurements, deformation theories, and indirect geophysical methods to determine the strength, or more precisely, the rheology of the crust and mantle. Despite all efforts, the rheology is usually the most poorly known parameter in solving the dynamics of the Earth's interior.

### 3.1 Elastic, viscous and plastic behavior

On short timescales (seconds) the 'solid' Earth looks pretty static and any deformation (such as from earthquakes) is described well with elasticity, from which the wave equation to describe the propagation of seismic waves through the Earth can be derived. On geologically large time scales (millions to billions of years), the Earth mantle behaves basically like a (very viscous) fluid. The third basic mechanism is plastic deformation, which describes the effect that rocks have a finite strength; if differential stresses are larger than a given value (the so-called yield-stress), they will fail. In the lithosphere plasticity typically manifests itself in the form of faults (e.g. the San Andreas fault) or shear-zones.

Elasticity is described by the behavior of a spring: the amount of deformation is linearly related to the applied force, or  $x \propto F$  (see Fig. 3.1). Viscous deformation behaves like a dashpot: the *rate* of deformation is related to the force, or  $\frac{dx}{dt} \propto F$ . Plastic deformation states that the force should never be larger than a given value ( $F \leq F_0$ ). Purely plastic deformation does not allow any deformation to occur if the force is below  $F_0$  (i.e. the slider in fig. 3.1C does not move).

The *Maxwell time*

$$t_M = \frac{\eta}{\mu} \quad (3.1)$$

describes the relative importance of elastic and viscous behavior. For a typical time larger than the Maxwell time, the material deforms mainly in a viscous way. For typical values for the mantle ( $\eta \approx 10^{21}$  Pa s and  $\mu \approx 10^{11}$  Pa), the Maxwell time is  $O(10^3)$  years, so on geological timescales, the mantle behaves as a viscous fluid. The Maxwell time for the lithosphere is much larger, and elasticity cannot always be ignored. Elastic and viscous deformation can also occur simultaneously. This type of deformation is called viscoelastic. The total strain is then the sum of both elastic and viscous strain.

*Question: Is it possible to combine the left and middle panels in Figure 3.1 to describe the principle behavior of viscoelastic deformation? If so, how does each of the three panels look like?*

### 3.2 Constitutive equations

In rheology, an equation that relates kinematic quantities (such as displacement, velocity, deformation, fluxes) to dynamic quantities (forces, stresses) is called a constitutive equation. We will look at both elastic and viscous constitutive equations.

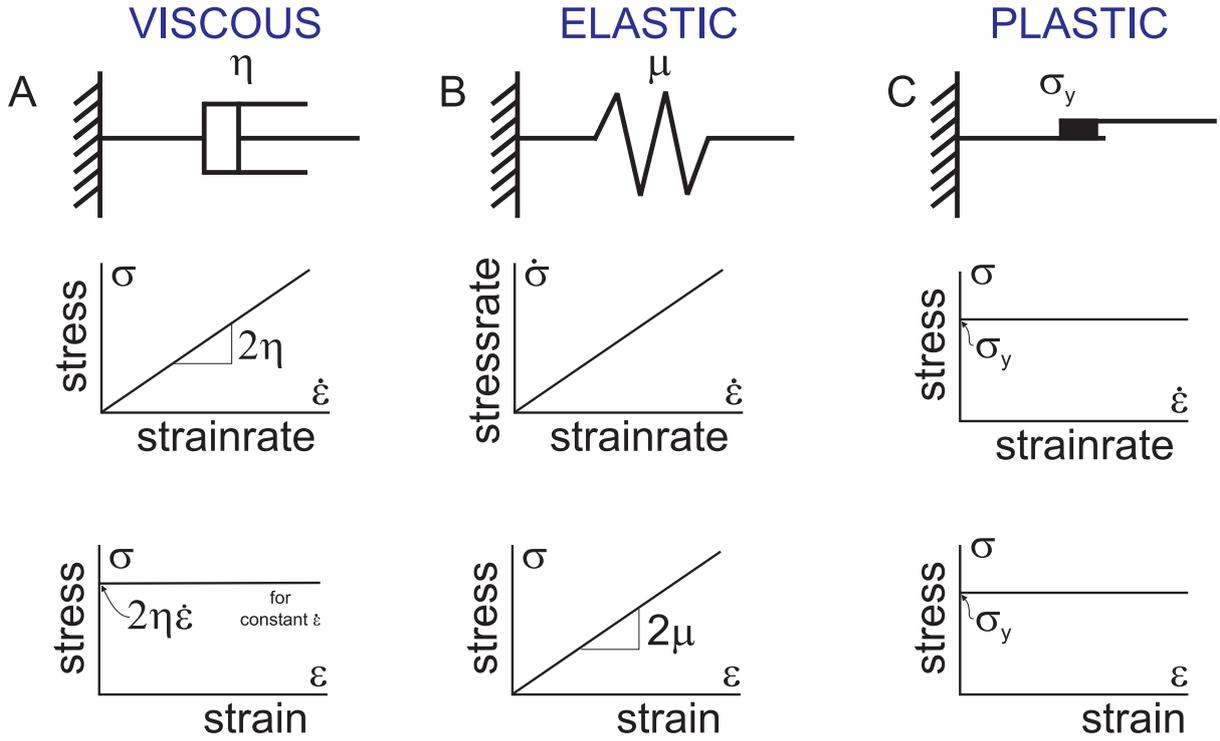


Figure 3.1: Basic properties of viscous, elastic and plastic deformation. Mechanical equivalent (top), stress/strainrate or stress-rate/strainrate response (middle) and stress/strain behavior under constant strainrate conditions (bottom). A: viscous behavior acts like a dashpot, in which the (effective) *viscosity*  $\eta$  describes the relation between force and deformation *rate*. B: elastic behavior acts like a spring, in which the *rigidity*  $\mu$ , also named *elastic shear modulus*, describes the relation between force and deformation. C: plastic behavior which only allows one stress level, the *yield stress*  $\sigma_y$ , regardless of amount of strain or strainrate.

### 3.2.1 Elastic deformation

General elasticity is described by

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \quad (3.2)$$

in which  $\sigma_{ij}$  is the total stress tensor, and  $\epsilon_{kl}$  the deformation-, or strain tensor. The elastic properties of a material are described by  $C_{ijkl}$ , which for an isotropic body reduces to two scalar parameters  $\lambda$  and  $\mu$ , called *Lamé's parameters*. The result is Hooke's law:

$$\sigma_{ij} = \lambda\epsilon_{kk}\delta_{ij} + 2\mu\epsilon_{ij} \quad (3.3)$$

Taking a shear component ( $i \neq j$ , e.g.  $i = 1$  and  $j = 2$ ) gives:

$$\sigma_{12} = 2\mu\epsilon_{12} \quad (3.4)$$

gives the resistance against elastic shearing, and  $\mu$  is called the *shear modulus*, already used in the previous section. Taking any normal components ( $i = j = 1, 2, \text{ or } 3$ ), gives:

$$\sigma_{ii} = (3\lambda + 2\mu)\epsilon_{ii} = 3\kappa\epsilon_{ii} \quad (3.5)$$

with  $\kappa = \lambda + \frac{2}{3}\mu$  the *bulk modulus*, or *incompressibility*.

### 3.2.2 Linear, or Newtonian viscous deformation

The total stress field  $\sigma$  can be subdivided into a *static* part (the thermodynamic pressure, which exists even if the fluid is in rest), and a *dynamic* part  $\tau_{ij}$ , which arises due to resistance against deformation:

$$\sigma_{ij} = -p\delta_{ij} + \tau_{ij} \quad (3.6)$$

For a linear rheology (also called Newtonian rheology), the dynamic stress  $\tau_{ij}$  is linearly related to the rate of deformation rate, or strainrate,  $\dot{\epsilon}_{ij}$ :

$$\tau_{ij} = \lambda' \dot{\epsilon}_{kk} \delta_{ij} + 2\eta \dot{\epsilon}_{ij} \quad (3.7)$$

in which  $\lambda'$  is a material parameter, the strainrate is defined as:

$$\dot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (3.8)$$

with  $\mathbf{u}$  the velocity vector of the fluid. The total stress  $\sigma_{ij}$  and strainrate  $\dot{\epsilon}_{ij}$  are usually split in an isotropic and deviatoric part, because it is the deviatoric part of the stress that causes material to deform:

$$\sigma'_{ij} = \sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} = \sigma_{ij} - \bar{p} \delta_{ij} \quad (3.9)$$

$$\dot{\epsilon}'_{ij} = \dot{\epsilon}_{ij} - \frac{1}{3} \dot{\epsilon}_{kk} \delta_{ij} \quad (3.10)$$

where the mean pressure  $\bar{p}$  in general differs from the static pressure  $p$ . Now the stress becomes

$$\begin{aligned} \sigma'_{ij} &= (\bar{p} - p) \delta_{ij} + \left( \lambda' + \frac{2}{3} \eta \right) \dot{\epsilon}_{kk} \delta_{ij} + 2\eta \dot{\epsilon}'_{ij} \\ &= (\bar{p} - p) \delta_{ij} + \zeta \dot{\epsilon}_{kk} \delta_{ij} + 2\eta \dot{\epsilon}'_{ij} \end{aligned} \quad (3.11)$$

$$(\bar{p} - p) + \zeta \dot{\epsilon}_{kk} = 0. \quad (3.12)$$

where  $\zeta$  is defined as the bulk viscosity, in contrast to  $\eta$  which is the shear viscosity. Their definition is somewhat parallel to the definitions of bulk- and shear modulus in elasticity. By definition of deviator,  $\sigma'_{ii} = \dot{\epsilon}'_{ii} = 0$ , so that  $\bar{p} = p$  if  $\zeta = 0$  (zero bulk viscosity) or  $\dot{\epsilon}'_{kk} = \nabla \cdot \mathbf{u} = 0$  (incompressible fluid). In geodynamic applications  $\zeta = 0$ , the so-called Stokes condition, applies, and the mean pressure always equals the static one. In that case, or in the case of an incompressible fluid,

$$\tau_{ij} = 2\eta \dot{\epsilon}_{ij} \quad (3.13)$$

from which the definition of viscosity follows

$$\eta = \frac{\tau_{ij}}{2\dot{\epsilon}_{ij}} \quad (3.14)$$

for any  $i$  and  $j$ . Sometimes, a different definition  $\dot{\epsilon}_{ij}^* = 2\dot{\epsilon}_{ij}$  for strainrate is used, in which case the viscosity is defined as:  $\eta = \frac{\tau}{\dot{\epsilon}_{ij}^*}$ .  $\eta$  is independent of the state of stress, but it usually is strongly dependent on, e.g., temperature, grainsize, confining pressure, and fluid content. This will be discussed later.

### 3.2.3 Non-linear or non-Newtonian viscous deformation: powerlaw rheology

We can rewrite Eqn. 3.14 to

$$\dot{\epsilon}_{ij} = \frac{1}{2\eta} \tau \quad (3.15)$$

Deformation of most Earth materials is described by a non-linear stress-strainrate relationship, generally described with a powerlaw relation

$$\dot{\epsilon} \propto \tau^n. \quad (3.16)$$

Before continuing, we define the second invariant of the stress and strainrate tensor as:

$$\tau_{II} = \sqrt{\frac{1}{2} \tau_{ij} \tau_{ij}} \quad (3.17)$$

(and similarly we can define  $\dot{\epsilon}_{II} = \sqrt{\frac{1}{2} \dot{\epsilon}_{ij} \dot{\epsilon}_{ij}}$ ). Now we can have the indices right, and write

$$\dot{\epsilon}_{ij} = A \tau_{II}^{n-1} \tau_{ij} \quad (3.18)$$

or

$$\dot{\epsilon}_{ij} = A^{1/n} \dot{\epsilon}_{II}^{(n-1)/n} \tau_{ij} \quad (3.19)$$

where  $A$  usually is, just like  $\eta$  for linear rheology, a function of temperature, pressure, and fluid content. The effective viscosity  $\eta_{eff}$  can now be written as

$$\eta_{eff} = A^{-1} \tau_{II}^{1-n} = A^{-1/n} \dot{\epsilon}_{II}^{(1-n)/n} \quad (3.20)$$

Most linear ( $n = 1$ ) and non-linear ( $n > 1$ ) deformation mechanisms of Earth' materials can be written in the form of Equation 3.18. Before we further elaborate the parameter  $A$  in terms of internal properties (material constants) and external properties (temperature, pressure), we take a look at the microscopic processes that control deformation in the Earth.

*Question: The Pacific plate is moving faster with respect to the high-viscous lower mantle than the North American plate (and so the western Atlantic) lithosphere is. Assuming similar physical circumstances, and that powerlaw rheology applies in the upper mantle, is the effective viscosity of the Pacific upper mantle lower than, higher than, or equal to the one below the western Atlantic?*

### 3.2.4 Plastic rheology

If rocks are subjected to large differential stresses, they break. On a crustal scale this manifests itself in fault zones (such as the San Andreas fault). Laboratory experiments on a large range of different rocktypes indicate that the maximum strength of upper-crustal rocks increases with increasing confining pressure (depth), which is called Byerlee's law. In situ measurements in boreholes seem to indicate that the state of stress of the upper crust is close to the critical stress (the yield stress) predicted by Byerlees law.

The depth-dependent strength of crustal rocks is often represented by a Mohr-Coulomb yield function. This can be written as

$$F = \tau^* + \sigma_N \sin(\phi) - C \cos(\phi) \quad (3.21)$$

with  $\tau^* = \left(\frac{\sigma_1 - \sigma_3}{2}\right)$  and  $\sigma_N = \left(\frac{\sigma_1 + \sigma_3}{2}\right)$ .  $C$  indicates the cohesion of rocks (typically 20-50 MPa) and  $\phi$  the internal angle of friction (typically  $30 - 50^\circ$ ). If  $F < 0$ , stresses are too low for plastic yielding to occur. If stresses are sufficiently large, plastic deformation is activated. Plastic strainrate are in this case given by

$$\dot{\epsilon}^{pl} = \dot{\lambda} \frac{\partial Q}{\partial \sigma_{ij}} \quad (3.22)$$

here  $Q$  is the plastic flow potential, which is given by

$$Q = \tau^* + \sigma_N \sin(\psi) \quad (3.23)$$

where  $\psi$  is the dilation angle (which indicates how much a fault zone "expands" during deformation, and typically  $< 10^\circ$ ). Furthermore,  $\dot{\lambda}$  is a to-be-determined constant that ensures that  $F = 0$ . Mathematically, this is described by the Kuhn-Tucker condition

$$\dot{\lambda} F = 0, \dot{\lambda} \geq 0, F \leq 0 \quad (3.24)$$

### 3.2.5 Viscoelastic rheology

Maxwell viscoelastic rheology combined both viscous and elastic deformation, and is the simplest way to combine both the elastic behavior of the lithosphere on relatively short timescales (e.g. postglacial rebound) and viscous creeping deformation at longer timescale, while at the same time allowing for large deformations. It consists of a elastic spring and a viscous dashpot, combined in series (fig. 3.2).

In a 0D case (ignoring spatial derivatives), elastic deformation is written as

$$\epsilon = \frac{1}{2\mu} \tau \quad (3.25)$$

where  $\epsilon$  indicates strain and  $\tau$  deviatoric stress. It is useful to write this rheology in terms of strainrate. Taking the derivative of eq. (3.25) versus time yields

$$\dot{\epsilon}^{el} = \frac{1}{2\mu} \frac{\partial \tau}{\partial t} \quad (3.26)$$

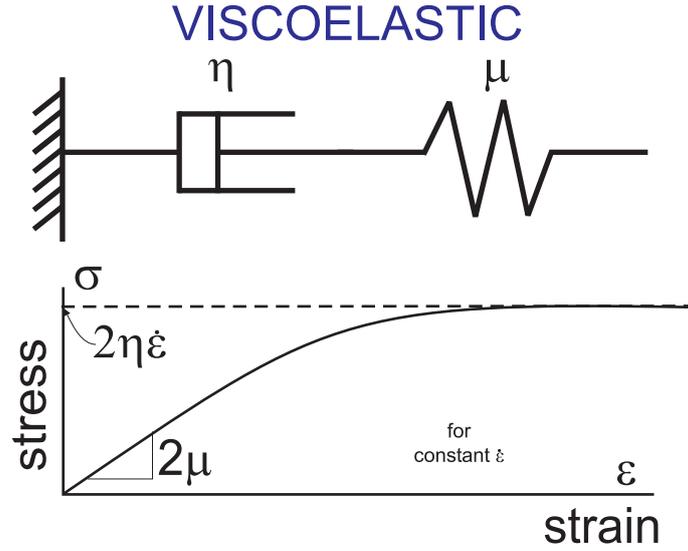


Figure 3.2: Maxwell viscoelastic rheology. Initially the response is elastic; after a certain time the stresses reach the viscous stress and the response is viscous.

A viscous rheology, in 0D, can be written as

$$\dot{\epsilon}^{vis} = \frac{1}{2\eta} \tau \quad (3.27)$$

Maxwell viscoelasticity simply adds the viscous and elastic strainrates:

$$\dot{\epsilon} = \dot{\epsilon}^{vis} + \dot{\epsilon}^{el} = \frac{1}{2\eta} \tau + \frac{1}{2\mu} \frac{\partial \tau}{\partial t} \quad (3.28)$$

Under a constant strainrate boundary condition, with initial condition  $\tau(0) = \tau_0$ , the above equation can be solved analytically for  $\tau(t)$ :

$$\tau(t) = 2\eta\dot{\epsilon} + (\tau_0 - 2\eta\dot{\epsilon}) \exp\left(-\frac{t}{t_M}\right) \quad (3.29)$$

where  $t_M = \frac{\eta}{\mu}$  is the Maxwell relaxation time. If the process you're considering takes place over times  $\gg t_M$ , the rheology can be considered as viscous. If, however, the process happens on smaller timescales, elastic effects may be important. For the Earth,  $\mu$ , is relatively well constrained (from PREM), and varies between  $10^{10} - 10^{11}$  Pa. The effective viscosity,  $\eta$ , however may vary many orders of magnitude. That's why the Maxwell relaxation time may thus vary from several days to several Gyrs.

*Question: The effective viscosity of the mantle is approximately  $10^{21}$  Pas. What is the Maxwell relaxation time for the Mantle?*

Note that 3.28 is only valid for the 0D case. In tensorial format the equation is more complex, which comes from the derivative of a deviatoric stress tensor versus time.

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{vis} + \dot{\epsilon}_{ij}^{el} = \frac{1}{2\eta} \tau_{ij} + \frac{1}{2\mu} \frac{D\tau_{ij}}{Dt} \quad (3.30)$$

where  $D/Dt$  denotes the objective derivative. It is beyond the scope of this chapter to go into details about this (except saying that it can be a bit tricky to implement in a numerical code).

### 3.2.6 Viscoelastoplastic deformation

The present-day state-of-the-art numerical models of lithospheric deformation employ a viscoelastoplastic rheology, which probably gives the most correct representation of the deformation of rocks, which varies from elasto-plastic (brittle) under surface-near conditions to viscous (ductile, creep-like) under mantle conditions. The viscoelastoplastic rheology is written (in tensor form) as

$$\dot{\epsilon}_{ij} = \dot{\epsilon}_{ij}^{vis} + \dot{\epsilon}_{ij}^{el} + \dot{\epsilon}_{ij}^{pl} \quad (3.31)$$

where

$$\dot{\epsilon}_{ij}^{vis} = \frac{1}{2\eta_{eff}} \tau_{ij} \quad (3.32)$$

represents viscous creep with the effective viscosity  $\eta_{eff}$  (which can be a nonlinear function of temperature, pressure, stress, grainsize etc.).

$$\dot{\epsilon}_{ij}^{el} = \frac{1}{2\mu} \frac{D\tau_{ij}}{Dt} \quad (3.33)$$

represents the elastic behavior of materials, and

$$\dot{\epsilon}_{ij}^{pl} = \dot{\lambda} \frac{\partial Q}{\partial \sigma_{ij}} \quad (3.34)$$

the plastic rheology once stresses exceed the yield stress. A viscoelastoplastic rheology behaves like an effective elastic body for small timescales and under low stresses, like a plastic body for large differential stresses and like a viscous material for long timescales and large temperatures.

### 3.3 Microphysical theory and observations

Theoretically, the resisting stress of a material against permanent deformation should be on the order of the rigidity  $\mu$ : on a microscopic scale, elastic deformation is caused by pushing atoms away from their equilibrium positions in the crystal lattice. If the (shear) stress is released, the atoms move back to their equilibrium position, and deformation vanishes. If each atom is pushed away a distance comparable to the atomic distance (which macroscopically corresponds to a strain  $\epsilon_{12} = O(1)$ ), it might end up closer to the equilibrium position of the neighboring atom, and a whole row of atoms might jump one crystal position, which gives permanent deformation (see Figure 3.3). According to Equation 3.4 this requires a stress  $\tau_{12} = O(\mu)$ .

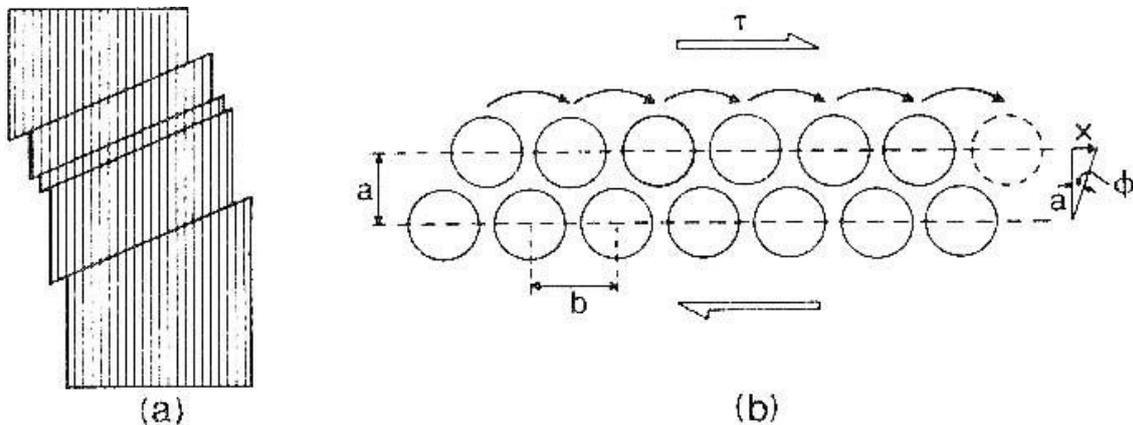


Figure 3.3: a) Ductile deformation of a single crystal by slip along a set of glide plane; b) simultaneous slip of an atomic plane (top) over another (bottom). After [4, ].

However, in the laboratory, the strength of materials turns out to be much less:  $\tau_{12} = O(10^{-3} - 10^{-2}\mu)$  or less. This much lower strength, also called the Frenkel limit of strength of materials, is typically thought to be caused by impurities in the lattice:

1. *vacancies* (missing atoms) or *impurities* (atoms of a different kind) are common in natural materials
2. *dislocations* are lines through the 3-D crystal, where the lattice is imperfect.
3. *subgrain boundaries* are planes through the crystal, where the lattice is misoriented.

More recent work suggested that a self-localizing shear-heating instability might cause materials to fail at approximately the Frenkel limit (elastic energy build up during deformation of crystals is released in thin shear-bands that deform viscously). Experiments on metals indeed indicate that extraordinary temperatures can be reached in such shear bands (several 1000 K). If true, this would be quite remarkable since the theory predicts that it should be operating from a crystal-scale to lithospheric scale.

### 3.3.1 Self-diffusion

Vacancies can diffuse through the material, and macroscopically this diffusion behaves just like the more familiar diffusion of heat. On a microscopic scale, vacancies have to exchange position in the lattice with a neighboring atom. This process is called *self-diffusion* and the diffusion coefficient  $D_{SD}$  is described as:

$$D_{SD} = D_0 \exp\left(-\frac{E + pV}{RT}\right) \quad (3.35)$$

in which  $E$  is the *activation energy*,  $p$  the ambient pressure,  $V$  the *activation energy*,  $T$  the absolute temperature, and  $R = 8.3143 \text{ J/mol, K}$  the universal gas constant. The exponential term (which makes self-diffusion of *Arrhenius* type) indicates that self-diffusion is an activated process, i.e. enough (thermal) energy must be provided to let the process occur with a significant rate.

### 3.3.2 Dislocations

One way to deform a material at much lower stresses than the theoretical strength  $O(\mu)$  of the material is with *consecutive* slip: instead of shifting a whole plane of atoms over one lattice position, it can be done per row of atoms. The location at which a line of atoms is disconnected from their original neighbors, shifted over one lattice spacing (a distance called the Burgers vector  $\vec{b}$ ), and re-connected to their new neighbors is called a dislocation. There is an analogy between a dislocation and a ruck in a carpet: one can move a carpet as a whole over the floor, but much less energy is required if a ruck is made on one end, and then propagated across the carpet, with which the same total slip can be achieved. A small part of the carpet in the ruck is temporarily disconnected from the floor. The ruck marks the location where the slip takes is taking place. This situation is geometrically equivalent to a so-called *edge dislocation* in the lattice of the material, which is the line in the lattice where a halfplane of atoms ends. Just like the ruck is being swept across the carpet, the dislocation is moved through the lattice, which increases the plane across which atoms have been slipped. Figure 3.4 further clarifies the slip by dislocation propagation.

With edge dislocations, the Burgers vector  $\vec{b}$  is perpendicular to the orientation of the dislocation line. A geometrically more complicated dislocation line is a *screw dislocation*, in which the dislocation line and  $\vec{b}$  are parallel. This geometry is also illustrated in Figure 3.4. In this case, the slipped and non-slipped atoms are connected as a helix, similar to a spiral staircase. Dislocations can sweep or *glide* across a crystal with rather low energy thresholds, but considerably more energy is necessary to let a dislocation *climb* out of their slip plane into the next one. This is sometimes necessary if an edge dislocation meets an impurity. This climb is a stress- and diffusion-controlled process, and is proportional to both  $\sigma$  and  $D_{SD}$ . The microscopic movement of dislocations is related to the macroscopic deformation rate or strain rate through Orowan's equation:

$$\dot{\epsilon} = \rho_m v b \quad (3.36)$$

where  $\rho_m$  is the density of mobile dislocations,  $v$  their velocity, and  $b$  the magnitude of the Burgers vector. Each dislocation is an irregularity in the lattice that creates a stress field around it. The total stress field of all dislocations is balanced with the externally applied stress field. Therefore the number of dislocations, or dislocation density, is related to (the square of) the stress field. The density of mobile dislocations is less than the total dislocation density, but with the same relation to the stress field, and estimated to be:

$$\rho_m \propto \left(\frac{\sigma}{\mu b}\right)^2 \quad (3.37)$$

## 3.4 Steady state creep models

According to the former paragraph, a number of macroscopic processes in a deforming crystal can take place, and by combining different diffusion processes, having edge- and screw dislocations, and both climbing and gliding

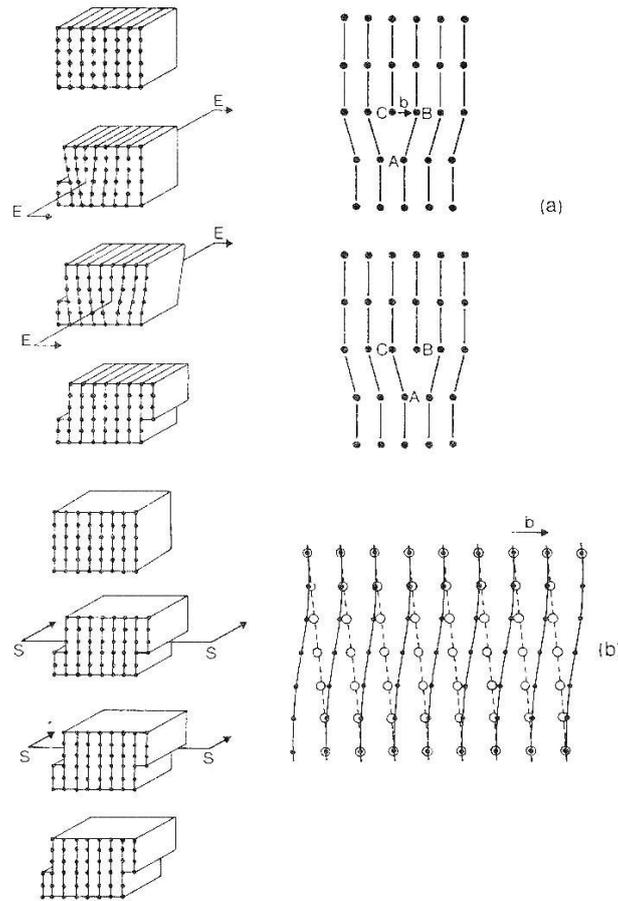


Figure 3.4: a) Slip by propagation of an edge dislocation EE, equivalent to an extra halfspace in the lattice; b) slip by propagation of a screw dislocation SS, with atoms forming a helix around the dislocation line. After [4, ].

of dislocations, the number of possible deformation mechanisms is very large. However, only a few of those turn out to be weak enough, and therefore important enough to play a significant role in the Earth.

### 3.4.1 Dislocation creep

*Climb-controlled dislocation glide* or simply referred to as *powerlaw creep* or *dislocation creep* is based on the fact that the rate of deformation by gliding of dislocations is controlled by the rate at which those dislocations can climb in the lattice to circumvent impurities. This climb is a diffusion controlled mechanism, and creep mechanism is usually written as:

$$\dot{\epsilon} \sim \frac{D_{SD}\mu b}{kT} \left( \frac{\sigma}{\mu} \right)^n \quad (3.38)$$

with  $k = 1.32 \times 10^{-23}$  J/mol referring here to the Boltzmann constant. In the special case of Weertman creep,  $n = 3$ , theoretically, if Equation 3.37 is assumed to be valid, which shows that this creep mechanism is non-Newtonian. This creep mechanism is assumed to be independent on grainsize.

### 3.4.2 Diffusion creep

Direct diffusion of vacancies from low to high pressure regions within a crystal is the other significant source of deformation, called *diffusion creep*. The atoms move in opposite direction of the vacancy (high to low pressure), and the diffusion speed is directly proportional to the applied deviatoric stress. The creep equation is:

$$\dot{\epsilon} \sim \frac{D^* \mu \Omega}{kT d^2} \left( \frac{\sigma}{\mu} \right) \quad (3.39)$$

with  $\Omega$  the atomic volume. The inverse square power of the grain size  $d$  can be derived as follows: the macroscopic deviatoric stress (i.e. difference between maximum and minimum stress) is present within each grain, so the stress gradient within a single grain is proportional to  $\sigma/d$ , so the vacancy flux through the grain is also proportional to  $\sigma/d$ . Each vacancy has a volume  $\Omega$ , so the vacancy volume flux is  $\sigma\Omega/d$ , or, in other words, a layer of thickness  $\sigma\Omega/d$  is diffused away from the high-stress side of the grain, and transported to the low-stress side. Dividing by the total size of the grain gives the approximate deformation per unit of time, or strainrate, proportional to  $\sigma\Omega/d^2$ . If diffusion of vacancies occurs primarily through the grain (bulk diffusion),  $D^* = D_{SD}$ , and the deformation mechanism is called *Nabarro-Herring creep*. If diffusion mainly occurs along the grain boundary,  $D^* = \frac{\pi\delta}{d}D_{GB}$ , where  $\delta$  is the grain boundary 'thickness', and  $D_{GB}$  the grain boundary diffusion coefficient. Note that in the latter case, which is called *Coble creep*, the grain size dependence is to the inverse third power. Diffusion creep is (at least in theory) Newtonian, so the viscosity is independent on the ambient deviatoric stress.

### 3.4.3 Observations

The strength of mantle and crustal rocks have been examined experimentally in the laboratory. By far the best studied material is olivine, which is the major component in the upper mantle. To a much lesser extent continental crust is examined. The main uncertainty of these laboratory measurements arises due to the extrapolation from high-strainrate conditions in the laboratory ( $\dot{\epsilon} \sim 10^{-4} - 10^{-8}$  to geologically relevant strainrates ( $\dot{\epsilon} \sim 10^{-14} - 10^{-15}$ ). It is also difficult to perform the experiments under the p-T conditions in the Earth. To reliably do the extrapolation it is important to know the deformation mechanism. [3] give a review of experimental results for activation energy and -volume, grainsize parameters, and stress-strain powerlaw value  $n$ . A widely used set of experimental results for dry and wet olivine is by [2]. However, values are constantly under revision. The effect of water is poorly known, as is the amount of water present in the mantle. In general, the uncertainty is much larger for crust than for mantle materials, partly because olivine is better examined, but partly also because the effect of water is an even larger unknown in the crust than it is in the mantle.

Over all, Coble creep is expected to be dominant at low pressure and temperature, while Nabarro-Herring creep is a low-pressure, high temperature mechanism. For higher stresses, powerlaw creep becomes important. Other creep mechanisms might be important under specific conditions: *grain boundary sliding* and *pressure solution* lead to Newtonian creep, while low-temperature plasticity or Peierls mechanism may be the dominant mechanism at the brittle-ductile transition [1].

Another source of observation for the rheology of the mantle comes from indirect geophysical measurement of the dynamics of the Earth. Especially post-glacial rebound studies have provided us with an estimate of the viscosity of the mantle. This topic involves the flow dynamics of the mantle, and will be further elaborated in the next Chapter.



## Bibliography

- [1] C. Kameyama, D. A. Yuen, and S. Karato. Thermal-mechanical effects of low temperature plasticity (the peierls mechanism) on the deformation of a viscoelastic shear zone. *Earth and Planetary Science Letters*, 168:159–162, 1999.
- [2] S. Karato and P. Wu. Rheology of the upper mantle: a synthesis. *Science*, 260:771–778, 1993.
- [3] S. H. Kirby and A. K. Kronenberg. Rheology of the lithosphere: Selected topics. *Review of Geophysics and Space Physics*, 25(6):1219–1244, 1987.
- [4] G. Ranalli. *Rheology of the earth*. Chapman and Hall, London, 2nd edition, 1995.