Chapter 1

Thermal and compositional structure of the Mantle and Lithosphere

1.1 Primordial heat of the Earth

The most widely accepted planetary formation theory says that the solar system accreted from a nebular cloud that became disc-like. The details of the early solar system formation are subject to debate, but broadly there is agreement that dust particles formed through condensation from the gas-cloud. Those particles accumulated through some not fully understood processes into planetesimals. Once planetesimals reach sizes of 100 km, gravitational attractions becomes important and accretion can become a self-regulating process. Computer simulations indicate that at this stage planets can be formed in relatively short timescales.

Accretion of a planet results in the release of large amounts of gravitational potential energy, which is a significant source of heat during the formation of a planet. The total potential energy E_{grav} of a spherical body with mass M and radius R is

$$E_{grav} = -\frac{GM^2}{R} \tag{1.1}$$

where $G = 6.7 \times 10^{-11}$ m^{3}kg^{-1}s^{-2} is the universal constant of gravitation. Since for a spherical body $M \propto R^3$, $E_{grav} \simeq -GR^5$. Dust particles are so much smaller in radius than a planet that the total potential energy before accretion of a planet is negligible.

$$E_{grav}^{dust} = 0 \tag{1.2}$$

Once the planet is formed, however, a large amount of potential energy is released. Since the first law of thermodynamics states that no energy should be lost, this energy should be compensated by other sources of energy. It can be shown that thermal energy is the most important one in this case.

$$E_{grav}^{dust} = E_{grav} + E_{th} \tag{1.3}$$

The increase in thermal energy is thus

$$E_{th} = +\frac{GM^2}{R} \tag{1.4}$$

Since $E_{th} = MC_p\Delta T$ (with C_p being the heat capacity, and ΔT the increase in temperature), the increase in temperature is given by

$$\Delta T = \frac{GM}{RC_p} \tag{1.5}$$

For a body with the size of the earth, $\Delta T \sim 30'000$ K (assuming $C_p \simeq 1000$Jkg^{-1}K^{-1}. Whereas this is only an order-of-magnitude estimate, and we have ignored effects such as cooling of the body upon accretion and transfer of potential in kinetic energy, it clearly illustrates that the Earth was hot in the beginning (and therefore most likely had a magma ocean). Other sources of heat include the formation of the iron core (the release in potential energy resulted in ~ 2000 K) and the decay of radioactive elements ~ 2000 K.

Ever since it's formation the Earth has been loosing heat. There are three ways through which heat can be lost:

- Radiation
- Conduction
- Convection

Radiation is an important process at high temperatures (> 5000K) and in vacuum, but plays most likely a minor role on the Earth. Conduction is the main heat transfer process in solids and convection is the dominating process in liquids. We will see later that large parts of the Earth behave on geological timescales as liquids and that therefore convection is the dominating process through which the Earth looses it's heat. Conduction, however, remains an important process in the boundary layes of the Earth (such as the lithosphere). In later lectures we will discuss convection in more detail; in this chapter we will discuss thermal conduction.

1.2 Energy equation

The variation of temperature with time is described by the energy equation:

$$\begin{array}{ccccccccccccc} I & & II & & III & & IV & & V & & VI & & VII \\ \rho C_p \frac{\partial T}{\partial t} & = & \nabla \cdot k \nabla T & + & A & - & \rho C p \mathbf{v} \cdot \nabla T & (+ & \sigma : (\dot{\varepsilon} - \dot{\varepsilon}^{\mathbf{el}}) & + & \alpha T \mathbf{v} \cdot \nabla P & + & \cdots) \end{array} \tag{1.6}$$

where ρ is density, T is temperature, t is time, $\mathbf{v}$ is velocity, σ is the stress tensor, $(\dot{\varepsilon} - \dot{\varepsilon}^{\mathbf{el}})$ the (non-elastic) strain rate tensor and P is pressure. The equation further contains several thermal material parameters:

k	thermal conductivity (W/m/K)
A	heat production (W/m^3)
C_p	heat capacity or specific heat at constant pressure (J/kg/K)
α	thermal expansion coefficient (1/K)

The terms in the energy equation describe the following:

I	the change in temperature with time
II	heat transfer by conduction (and radiation)
III	heat production (e.g., by radioactive decay)
IV	heat transfer by advection
V	heat generated by non-recoverable (or non-elastic) deformation, shear heating
VI	heat generated by adiabatic compression
VII	other terms, e.g., latent heat generated or absorbed at phase transitions (solid $\rightarrow$ fluid or solid $\rightarrow$ solid)

We will focus in the following on term I through IV, which are the most important terms for mantle temperatures and take a closer look at each of them.

1.3 Conduction

Conduction is the transfer of heat by the transfer of kinetic energy between atoms or molecules, either by the movement of electrons (important in metals, e.g., in the core), or by the movement of phonons (discrete packages of lattice vibrational energy, important in mantle rocks). Conduction is described by Fourier's law:

$$q = -k \nabla T \tag{1.7}$$

where q is heat flow (W/m^2). This equation says that heat flows down the temperature gradient (from hot to cold), the strength of the heat flow being controlled by thermal conductivity. The change in heat content of a small volume V of material through conduction can be derived as follows. Let's for simplicity assume conduction in horizontal direction only. Then conduction adds $[q - (q + dq)]Sdt$ heat to the volume through the sides with area S in time interval dt. This changes the heat content of the volume by $\rho V C_p dT$, and changes its temperature by the amount dT. Thus

$$\rho V C_p dT = -Sdt \frac{\partial q}{\partial x}, \tag{1.8}$$

or

$$\rho C_p \frac{\partial T}{\partial t} = \frac{k \frac{\partial T}{\partial x}}{\partial x}, \tag{1.9}$$

In 3-D this gives

$$\rho C_p \frac{\partial T}{\partial t} = \nabla \cdot k \nabla T \tag{1.10}$$

1.3.1 Temperatures of the oceanic lithosphere

This equation can be used to determine temperatures of the oceanic lithosphere, assuming that oceanic lithosphere is formed by cooling of the mantle as it moves away from a spreading ridge and cools conductively dominantly in vertical direction. The heat conduction equation then simplifies to:

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2} \tag{1.11}$$

where $\kappa = \frac{k}{\rho C_p}$ is another thermal material parameter, thermal diffusivity (unit m^2/s).

For the conditions that at time $t = 0$ the temperature is equal to that of the upwelling mantle, $T(z,0) = T_m$ and that surface temperature $T(0,t) = 0$ always, the solution to this equation is given by:

$$T(z,t) = T_m \mathrm{e}rf \frac{z}{2\sqrt{\kappa t}}, \tag{1.12}$$

where the error function is $\mathrm{e}rf(x) = (2/\sqrt{\pi})\int_0^x \exp(-x^2)dx$. An elaborate derivation of Equation **??** is given in (Chapter 4-15, Turcotte and Schubert 2002). The surface heat flow for this temperature distribution is given by:

$$q(0,t) = \left[-k\frac{\partial T}{\partial z}\right]_{z=0} = \left[\frac{-kT_m}{2\sqrt{\kappa t}}\left(\frac{2}{\sqrt{\pi}}\exp[\frac{z^2}{4\kappa t}]\right)\right]_{z=0} = \frac{-kT_m}{\sqrt{\pi\kappa t}}. \tag{1.13}$$

Thus surface heat flow changes with $1/\sqrt{t}$, i.e. inversely with the square root of lithospheric age. This is in quite good agreement with observations of oceanic heat flow. Furthermore the cooling of the oceanic lithosphere will lead to an increasing density with age. Assuming isostasy, the increasingly heavier lithosphere must be compensated by an increasingly thicker layer of light seawater above. The total weight of a column of lithosphere is obtained by integrating its density $\rho = \rho_m(1 - \alpha * (T - T_m))$ over depth, where T is substituted by Equation **??**. The integral of the error function is given by:

$$\int_0^\infty erfc(x)dx = \frac{1}{\sqrt{\pi}} \tag{1.14}$$

Under the assumption of isostasy, a sinking of the oceanic lithosphere with the $\sqrt{t}$ is also expected. This fits observations of sea floor bathymetry well up to ages of 70 to 80 Ma. Older lithosphere lies shallower than predicted by this relation, i.e. there appears to be an additional heat input. Most likely this heat input is there for all ages but becomes apparent at older lithosphere because the heat loss due to cooling has significantly decreased by then. The source of this heat input is still being debated, small scale convection under the plates or a cumulative heat input by plumes are proposed mechanisms.

1.3.2 Temperatures of the continental lithosphere

Heat produced by radiogenic elements is important in the long-term heat budget of the Earth's mantle. It is also a very important heat source in continental lithosphere. Continental crust contains about a factor of 100 more heat producing elements then oceanic crust, and a factor 1000 more than the upper mantle. Much of the continental lithosphere is old (100's Ma and older) and its thermal structure can therefore be described as being in a steady state ($\frac{\partial T}{\partial t} = 0$). Assuming constant material properties with depth, a surface temperature T_s and a surface heat flux q_s, the solution to the heat conduction equation with heat production (term III) becomes:

$$T(z) = T_s + \frac{q_s z}{k} - \frac{Az^2}{2k} q(z) = q_s - Az \rightarrow q_s = q_m + AD, \tag{1.15}$$

where the subscript s stands for surface, the subscript m for mantle and D is the thickness of the layer with heat producing elements. In reality the concentration of heat producing elements decreases with depth in the crust and is often approximated with $A(z) = A_0 \exp(-z/D)$, where A_0 is about $2.5 \cdot 10^{-6}$ W/m^3 and D on the order of 10 km. With such a distribution of heat sources it has been found that about 50% of the continental surface heat flow is produced in the upper crust and 50% of the heat flow is derived from deeper levels, i.e., lower crust and mantle. Both A and q_s in continents have been found to be decrease with thermotectonic age (although this trend is much less clear then in the oceans).

1.4 Radiation

Heat transport through radiation occurs by electromagnetic waves in the infrared part of the spectrum and does not require a medium to travel in. Experiments suggest that the effect of radiation becomes important only at temperatures larger than about 1000 K. In calculating temperatures in the mantle it is usually included as an extra term in the conductivity $k = k_L + k_R$, where k_L describes conductivity through lattice vibrations and k_R through radiation. Since k_R typically is temperature-dependent, this effect results in a non-linear temperature equation. There is however still some debate on whether radiation is important or not. Laboratory experiments on k_R have been performed on single crystals. Rocks, however, are mixtures of various minerals and the effect of this could be that EM waves get scattered much more, making it more difficult to transport energy through radiation.

1.5 Advection

We know that due to the low conductivity of mantle rocks and the large thickness of the mantle, heat conduction is a very inefficient way to cool the mantle. In spite of the mantle's large viscosity, convection turns out to be a much more efficient way to lose mantle heat. We distinguish advection from convection:

Advection
Heat transport by mass motion, regardless of what drives the motion

Convection
Heat transport by advection, where the mass motion is driven by internal buoyancy.

If we assume there is a mass flow in horizontal direction through a small volume V, we can calculate how this modifies the heat content of this volume. The flow transports $\rho C_p T \mathbf{v} S dt$ heat into the volume through the side with area S in time interval dt. On the other side $\rho C_p (T + dT)\mathbf{v} S dt$ is transported out. Thus the net change in heat is $S dT \rho C_p \mathbf{v} \frac{\partial T}{\partial x}$. Thus,

$$\rho C_P \frac{\partial T}{\partial t} = -\rho C_p v_x \frac{\partial T}{\partial x} \tag{1.16}$$

or in 3-D,

$$\rho C_P \frac{\partial T}{\partial t} = -\rho C_p \mathbf{v} \cdot \nabla T. \tag{1.17}$$

1.5.1 Mantle adiabat

In the interior of a convecting medium temperatures follow an adiabatic profile. At the top and bottom of a convecting layer thermal boundary layers with large thermal gradients form. The interior is thermally well mixed and therefore essentially isothermal, with a slight increase of temperatures with depth due to the effect of pressure. For example in the Earth's mantle the geothermal gradient $\partial T/\partial z$ is about 20°C/km near the surface and about 0.3°C/km in the interior of the mantle. This small gradient in the interior is the adiabatic gradient. If a small volume of material is moved to shallower depth is experiences a slight increase in volume due to the decreasing pressure and associated with this a slight decrease in temperature. This change in temperature is the adiabatic temperature change.

The adiabatic gradient can be determined from the thermodynamics relation between entropy per unit mass S, temperature T, and pressure P:

$$dS = \left[\frac{dS}{dT}\right]_P dT + \left[\frac{dS}{dP}\right]_T dP = \frac{C_p}{T} dT - \frac{\alpha}{\rho} dP \tag{1.18}$$

In case of a reversible adiabatic process the entropy change is zero, and so the adiabatic gradient is:

$$\left[\frac{\partial T}{\partial P}\right]_S = \frac{\alpha T}{\rho C_p}. \tag{1.19}$$

The gradient can also be expressed in terms of depth, remembering that $dp = \rho g dz$ in a hydrostatic fluid:

$$\left[\frac{\partial T}{\partial z}\right]_S = \frac{\alpha g T}{C_p}. \tag{1.20}$$

Thus to determine the adiabatic gradient one needs values of α and C_p with depth. These are obtained from laboratory experiments. And one needs an estimate of density as a function of depth. Density is generally determined from seismology (will be discussed later). Integration of the adiabatic gradient in terms of pressure then gives temperature as a function of pressure. Temperature as a function of depth is obtained by integrating the density distribution to obtain g as a function of depth. Several of the thermal parameters can be obtain in one parameter, the thermodynamic Grüneisen parameter, $\gamma = \frac{\alpha K_S}{\rho C_P}$ (a dimensionless parameter).

The advantage of this is that γ is close to 1 throughout the mantle. The adiabatic gradients then become:

$$\left[\frac{\partial T}{\partial P}\right]_S = \frac{\gamma T}{K_s}, \tag{1.21}$$

and

$$\left[\frac{\partial T}{\partial z}\right]_S = \frac{\gamma g T}{\phi}. \tag{1.22}$$

Here K_S is the adiabatic bulk modulus or incompressibility (units Pa), which is given by $K_S = \rho\left[\frac{\partial P}{\partial \rho}\right]_S = V\left[\frac{\partial P}{\partial V}\right]_S$. The bulk modulus is again a parameter that seismic velocities (that depend on bulk modulus, shear modulus and density) provide constraints for. The same applies for the squared bulk sound velocity $\phi = K_s/\rho = V_P^2 - (4/3)V_S^2 = V_\phi^2$. Thus seismological data provide important constraints on the thermal (and compositional) state of the mantle. This will be discussed more in detail later.

With the definition for K_S the change in temperature and density can be related directly and the usefulness of the Grüneisen parameter is demonstrated further:

$$\frac{dT}{T} = \frac{\gamma d\rho}{\rho}. \tag{1.23}$$

This gives:

$$T = T_0(\rho/\rho_0)^\gamma, \tag{1.24}$$

a relation that can be used to calculate temperatures from density given a starting density ρ_0 and temperature T_0 in an adiabatic layer. The relation is not valid when phase or chemical transitions are crossed. The melting temperature of mid-oceanic ridge basalts provides a starting temperature at the top of the mantle. The melting temperature of iron provides starting temperature for the outer and inner cores. Laboratory data provide estimates of starting densities.

This concludes a summary of constraints on the one-dimensional thermal profile of the mantle. The uncertainties in the temperatures below the lithosphere will become clearer in the discussion of the seismological constraints. Even variations in this 1-D profile on the order of $100 - 200$°C are important because mantle rheology is extremely temperature dependent. Furthermore, lateral deviations from a 1-D temperature distribution reflect the dynamics of the mantle (up- and downwellings).

1.6 Heat budget

To understand the thermal state of the mantle we need to cover one more topic. Especially the thermal evolution of the mantle depends on what the sources of heat are and how they are distributed. As mentioned before radiogenic heat production is an important source of mantle heat. Estimates of the amount of radiogenic heat produced are linked with estimates of the composition of the Earth, namely with estimates of the amount of radiogenic heat producing elements. Other constraints on the total heat budget and the different sources come from the surface heat flow.

1.6.1 Surface heat flow

Let's start with the constraints from surface heat flow. The lithospheric geotherms discussed before matched surface heat flow observations and can be used to extrapolate to areas where no measurements are available. The oceanic cooling models provide surface heat flow estimates everywhere where the age of the ocean floor is known (which is all of the ocean floor with the exception of a few areas with very thick sediment cover). The averaged observed oceanic surface heat flow is 57 mW/m^2. The average modeled heat flow, which in particular corrects for the effect of hydrothermal circulation near the ridges, is about 107 mW/m^2. The modeled heat flow gives a total heat flow through the ocean floor of 3.2×10^{13} W. While the oceans cover about 59% of the surface area, they contribute about 70% of the total surface heat flux. In a similar way continents can be divided into heat flow provinces based on thermotectonic age and crustal type. The average continental heat flow is about 67 mW/m^2, and they contribute a total of 1.4×10^{13} W. Thus in spite of the high heat production in the continental crust, continents contribute only about 30% of the total surface heat flow. The total amount of heat flux through the surface of the Earth is estimated in this way to be 46 +/- 3 TW (10^{12} W). About 15% of this total is generated in the crust, 85% is derived from the mantle.

1.6.2 Radioactive heat in the mantle

Radiogenic heat production has been estimated to contribute 33% of the total heat loss from the mantle. This heat is predominantly generated by isotopes with a long half life of 1-10 Ga, the important ones being $_{238}U$, $_{235}U$, $_{232}Th$ and $_{40}K$. The amount present is estimated from compositional models of the Earth, which will be discussed in the next chapter.

1.7 Heat sources

From these compositional models the amount of heat producing elements are estimated. If all of the mantle were like a pyrolitic upper mantle only 10-15% of the total surface heat flux would be due to radiogenic heat production. However, chondritic models predict a much higher concentration of heat producing elements in the mantle. It is therefore often assumed that the deeper (lower?) mantle much be more enriched in heat producing elements. The enrichment would be either a primordial feature and/or due to re-enrichment due to recycling of oceanic crust.

A smaller portion of the surface heat flow is assumed to be from the core. This contribution is estimated to be about 20%, predominantly due to crystallization of the inner core and to a minor component due to primordial heat and radioactive elements in the (rapidly convecting isentrope) outer core.

The remaining source of mantle heat must be cooling of the mantle since the formation of the Earth. Estimates of its contribution are generally in the 45 (+/-15)% range, but not well constrained as they depend on estimates of the original heat, heat released by core formation and models of the thermal evolution of the mantle.

Other sources, such as for example frictional dissipation (due to Earth-Moon interaction) are thought to be minor contributors. Latent heat released by phase transitions in the mantle and viscous dissipation associated with mantle flow mainly redistribute heat in the mantle.

1.8 Further reading

[**?**] - Chapter 7
[**?**] - Chapter 7
[**?**] - Chapter 4.2
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[**?**] - Chapter 7
[**?**] - Chapter 6
[**?**] - Chapter 4
[**?**] - Chapter 4
[**?**] - Chapter 6 (By Jaupart, Labrosse and Mareschal)