Thermal Structure of the Earth

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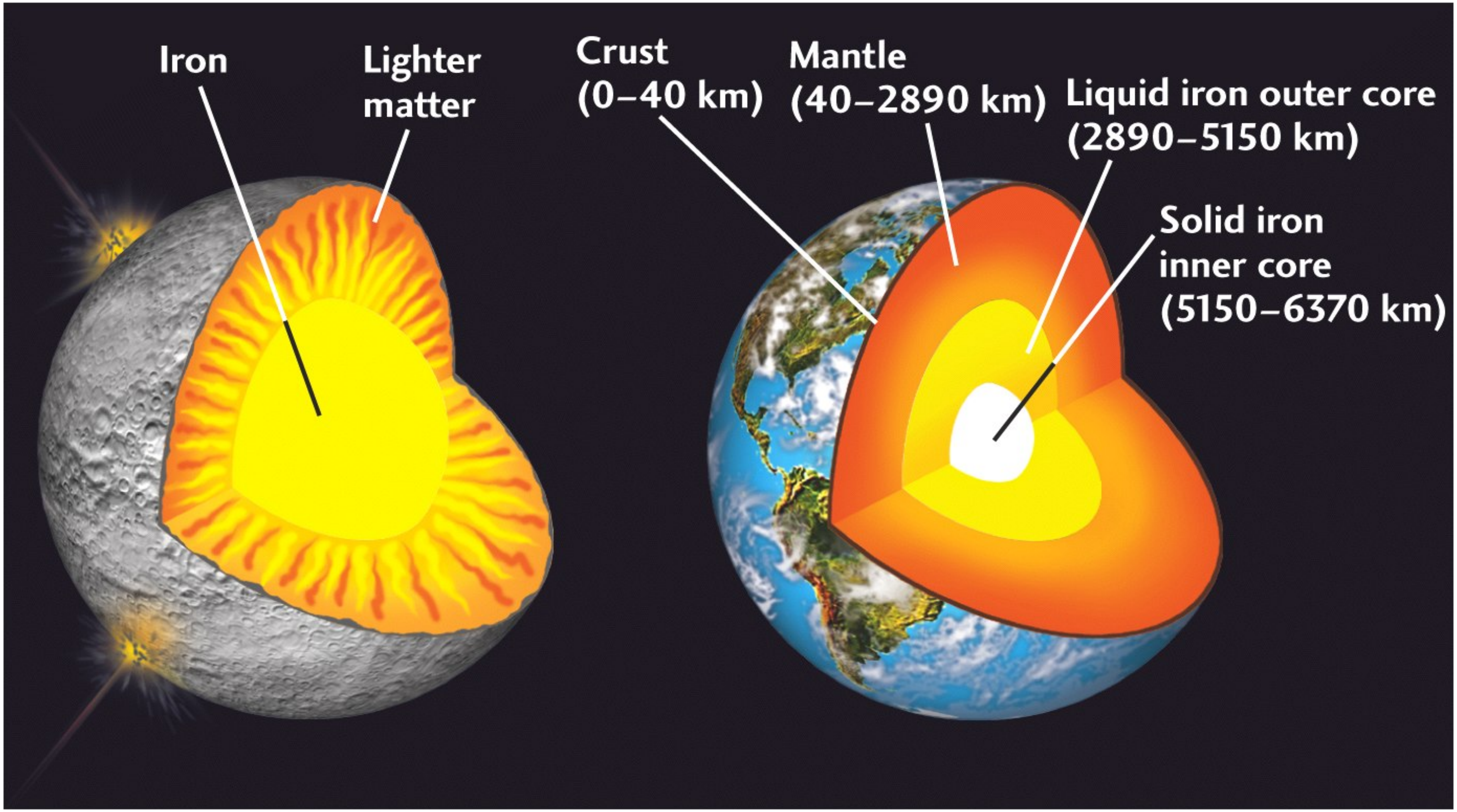
Units

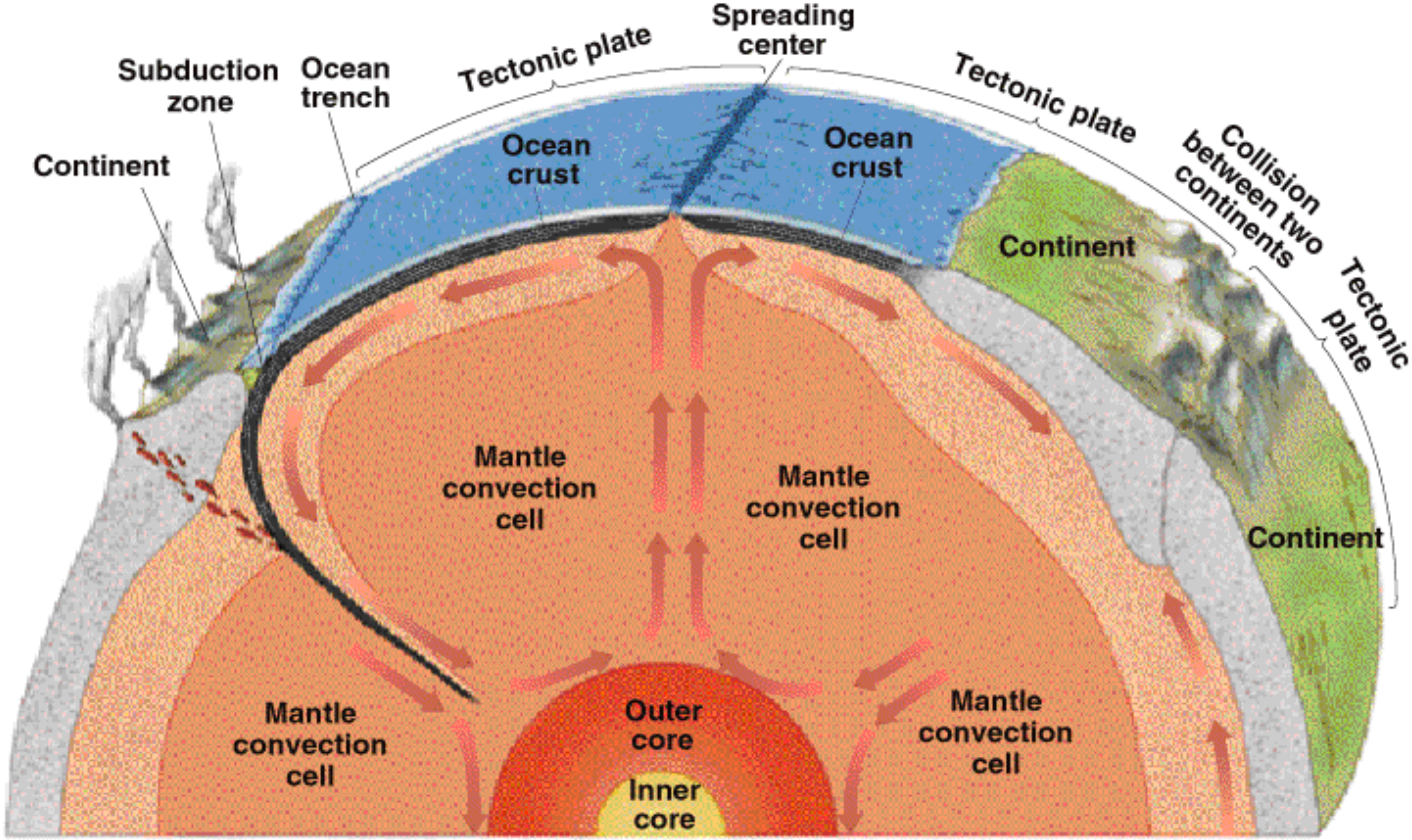
- All physical quantities must have their units specified to be meaningful.
- We will always used SI units.
- We must be comfortable with conversions between different scales.

General View of the Earth



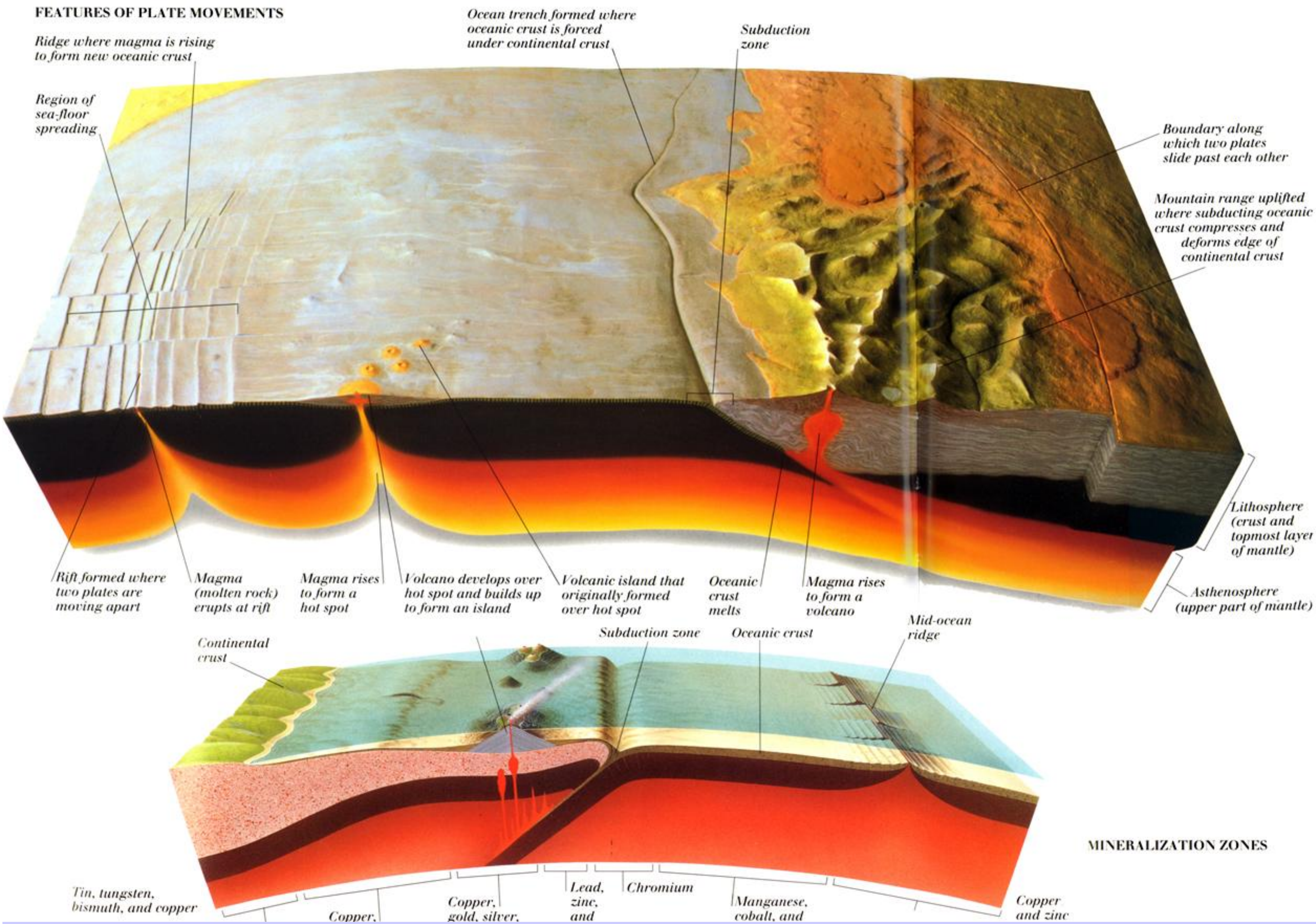
General View of the Earth

Thompson and Turk: Earth Science and the Environment, 2/e
Figure 5.12

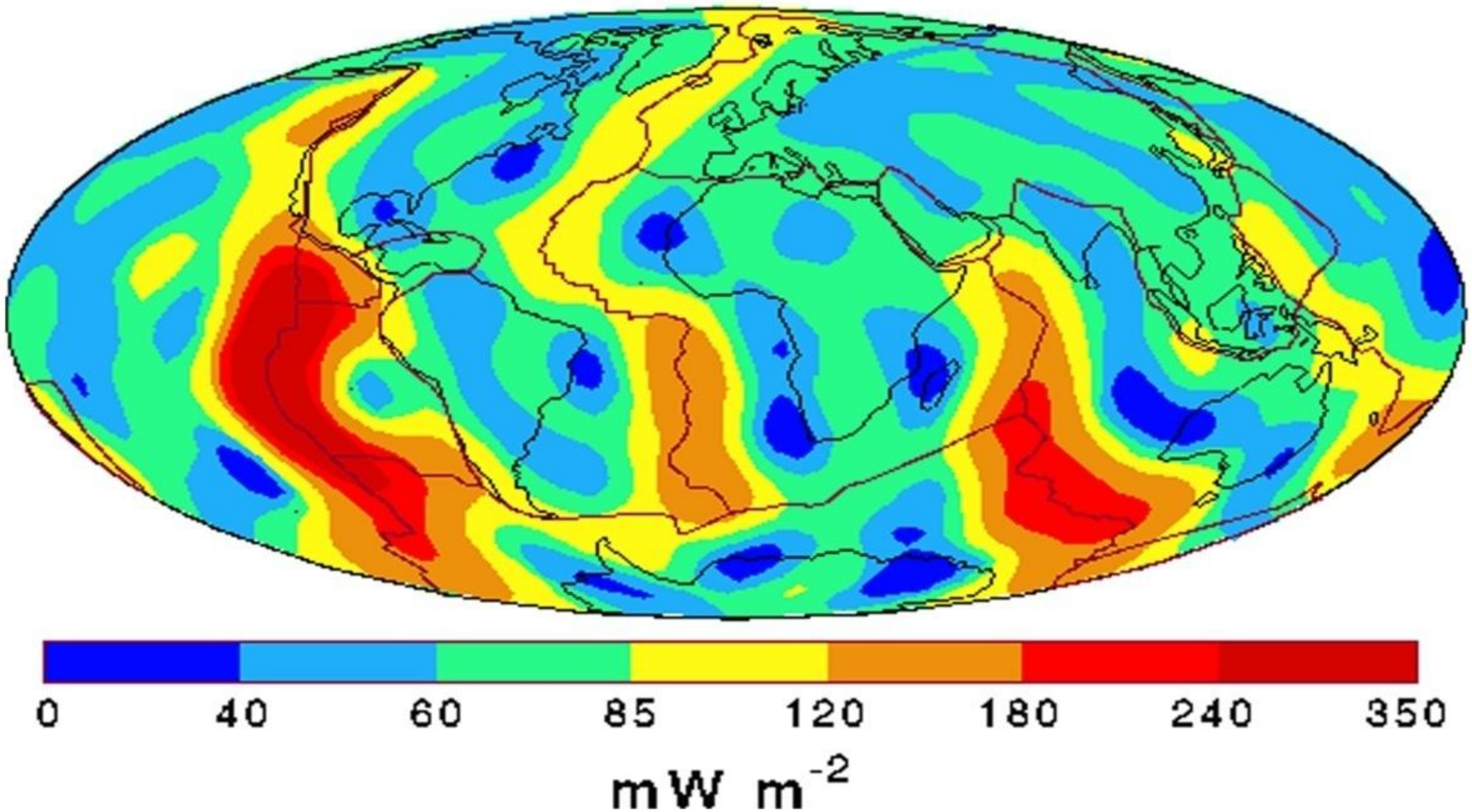


Saunders College Publishing

General View of the Earth



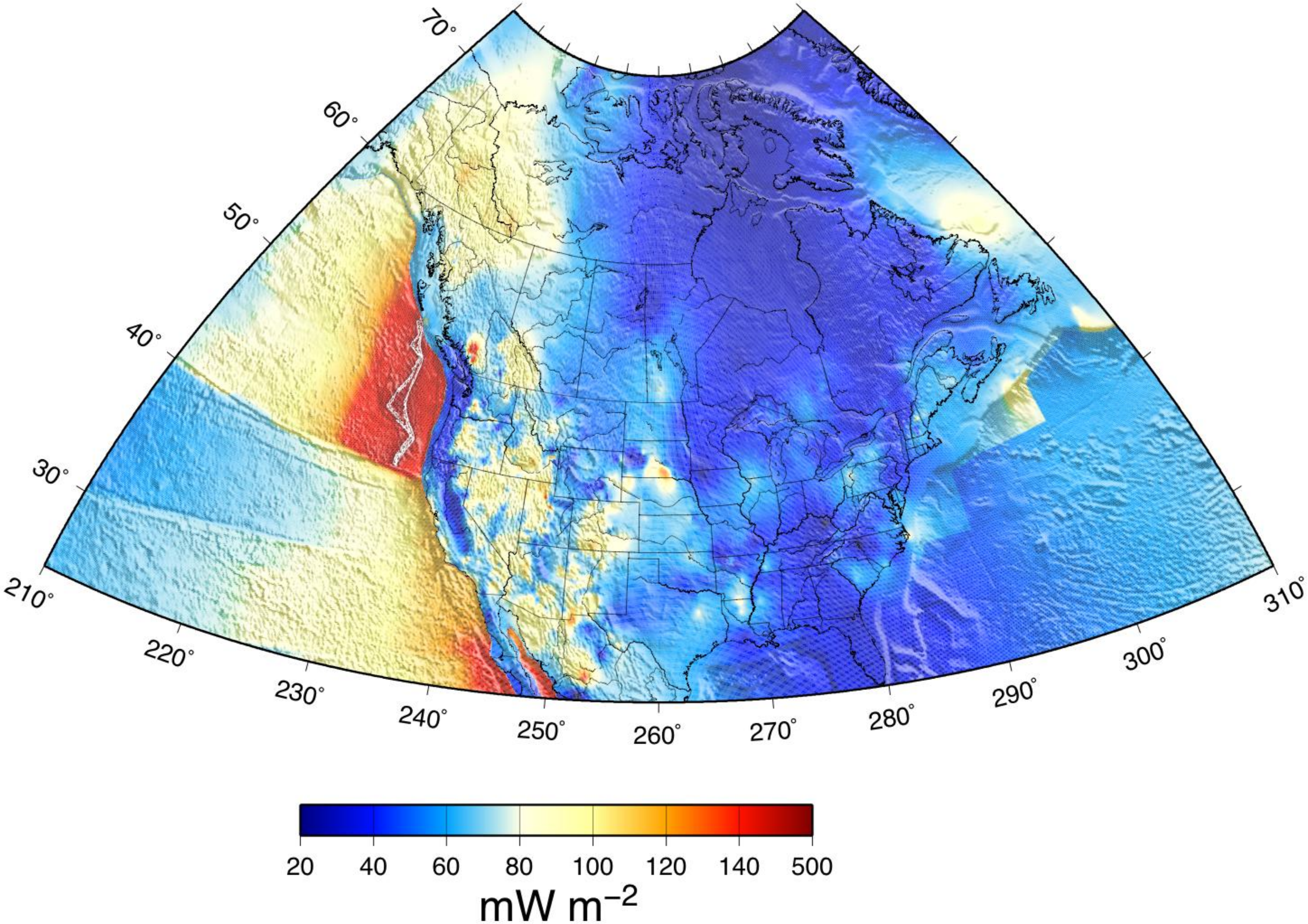
Global Heat Flow Map



Continental average: ~65 mW/m^2
Oceanic average: ~100 mW/m^2

Continental Heat Flow Map

North America

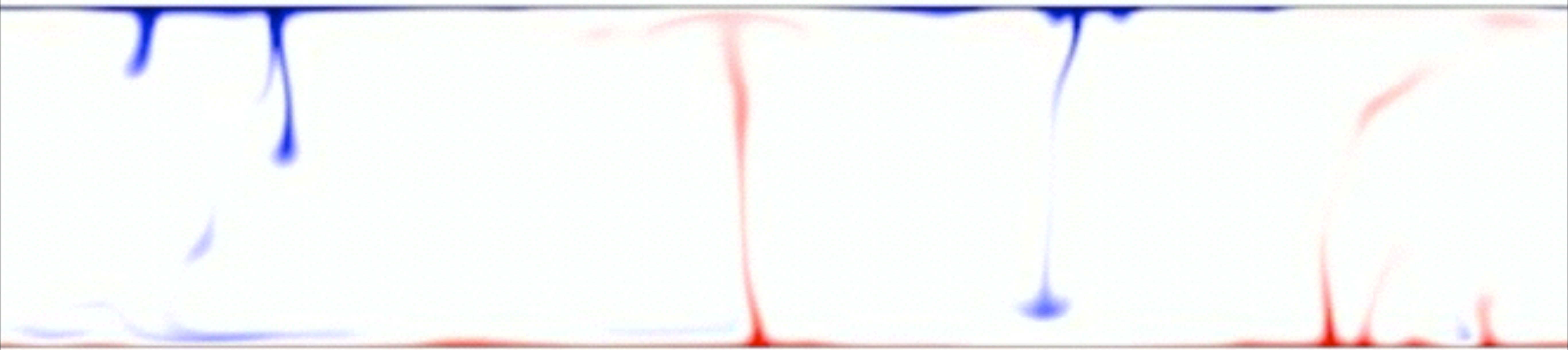


Importance of Thermal Effects

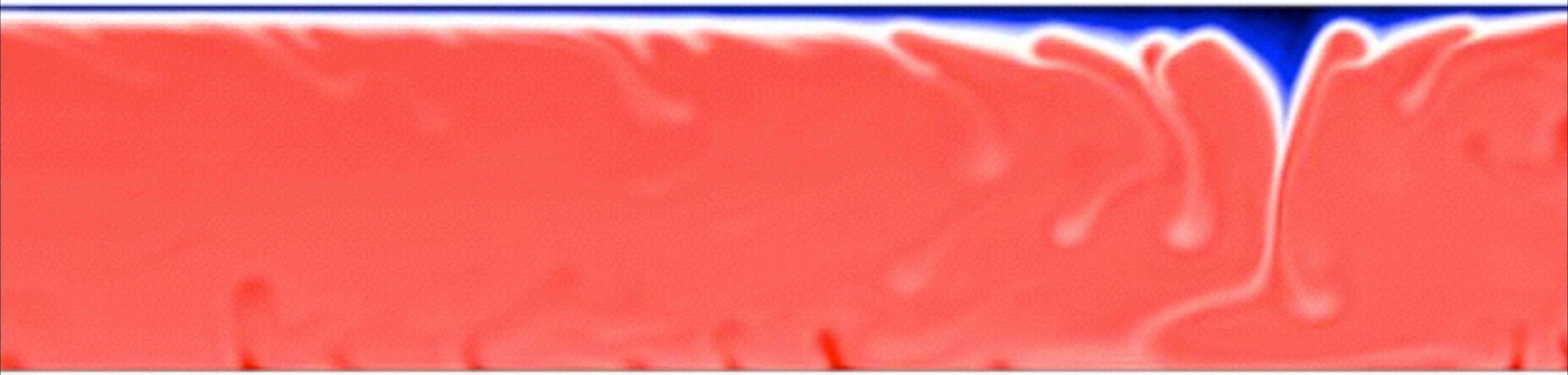
- Surface heat flow provides information about the amount of heat produced within the Earth's interior.
- Material properties are a strong function of temperature
- Thus, the dynamics of a material is thus a strong function of temperature.
- For example, the viscosity of the mantle is highly temperature dependent, e.g.

$$\eta \sim \exp\left(-\theta T\right)$$

Constant viscosity



Temp. dependent viscosity
(cold material is 10^5 times more viscous)



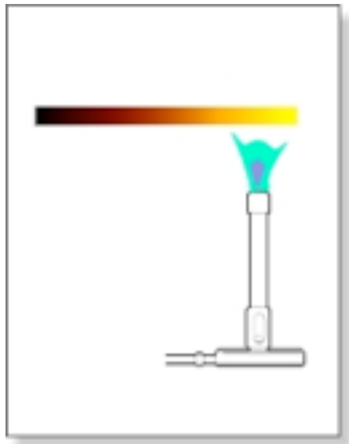
http://mcnamara.asu.edu/content/educational/mantle_convection_tutorial_01/index.html

Heat Transfer

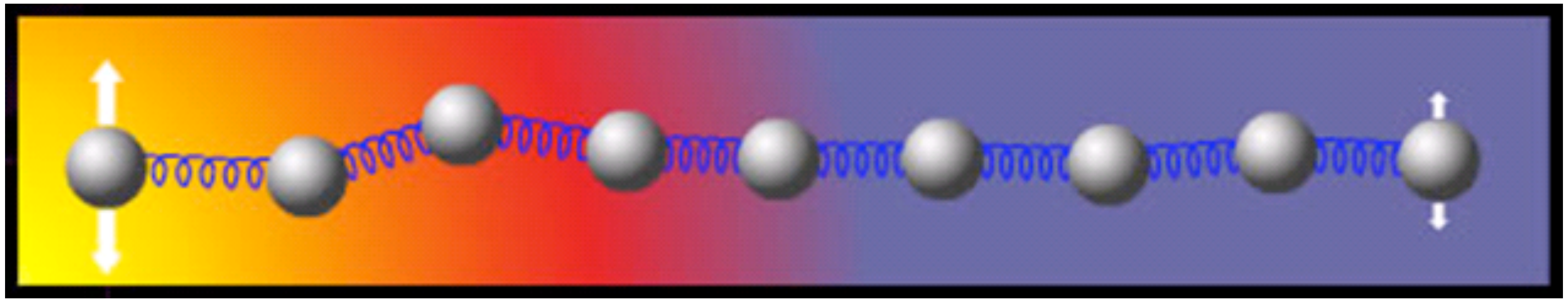
- The science which predicts how energy transfer may occur between materials as a result of a temperature difference
- Three modes of heat transfer

1. **Conduction**
2. **Convection**
3. **Radiation**

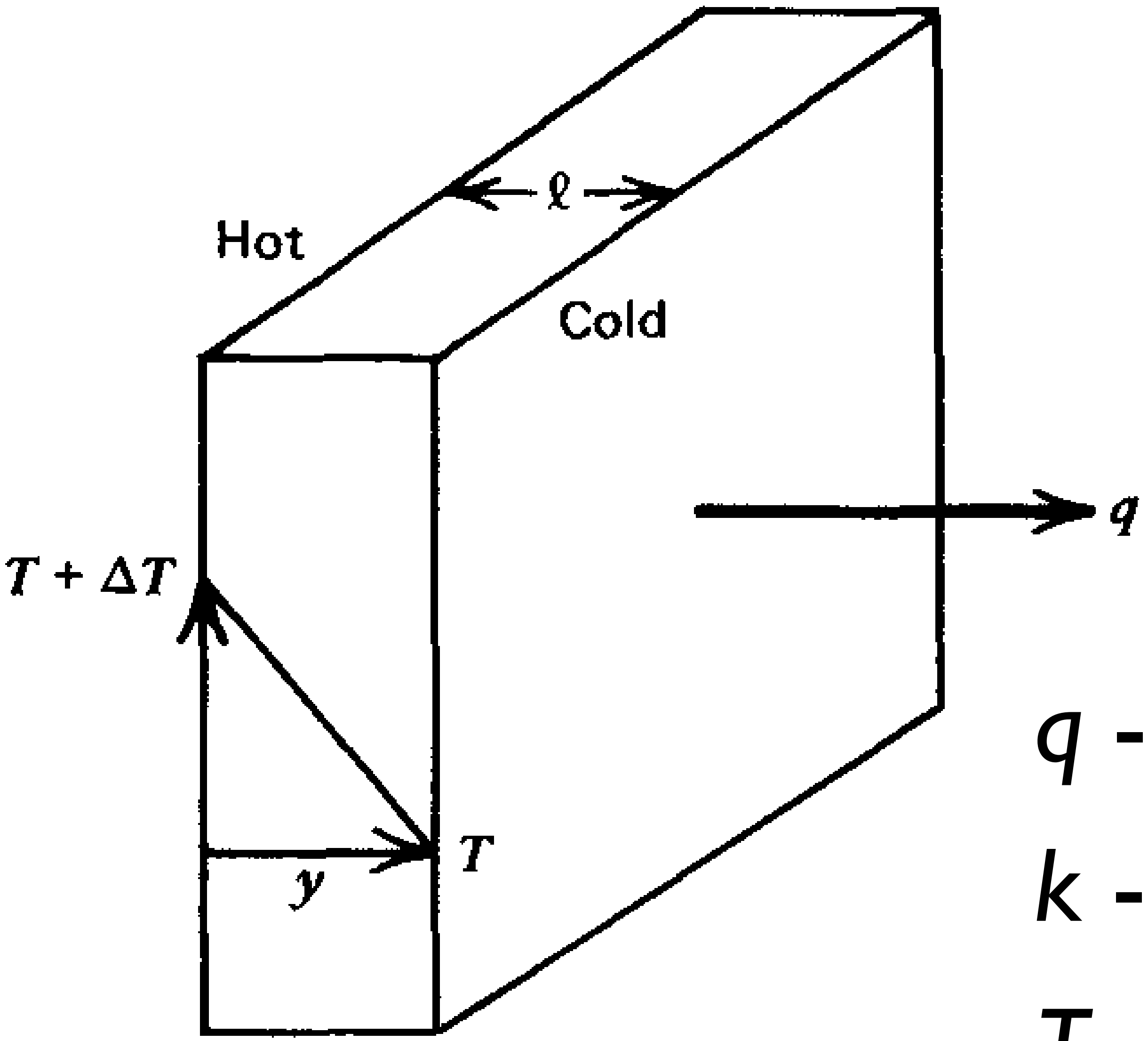
Conduction



- Heat transfer occurs via net effect of molecular collisions. Molecules transmit kinetic energy through these collisions.
- Essentially a diffusion process.
- Heat conduction occurs through a *stationary* medium across which there is a variation in temperature.



Conduction



Fourier's Law of Heat Conduction

$$q = -k\frac{dT}{dy}$$

q - heat flux (W/m^2)

k - thermal conductivity (W/m/K)

T - temperature (K)

y - position (m)

dT/dy - thermal gradient (K/m)

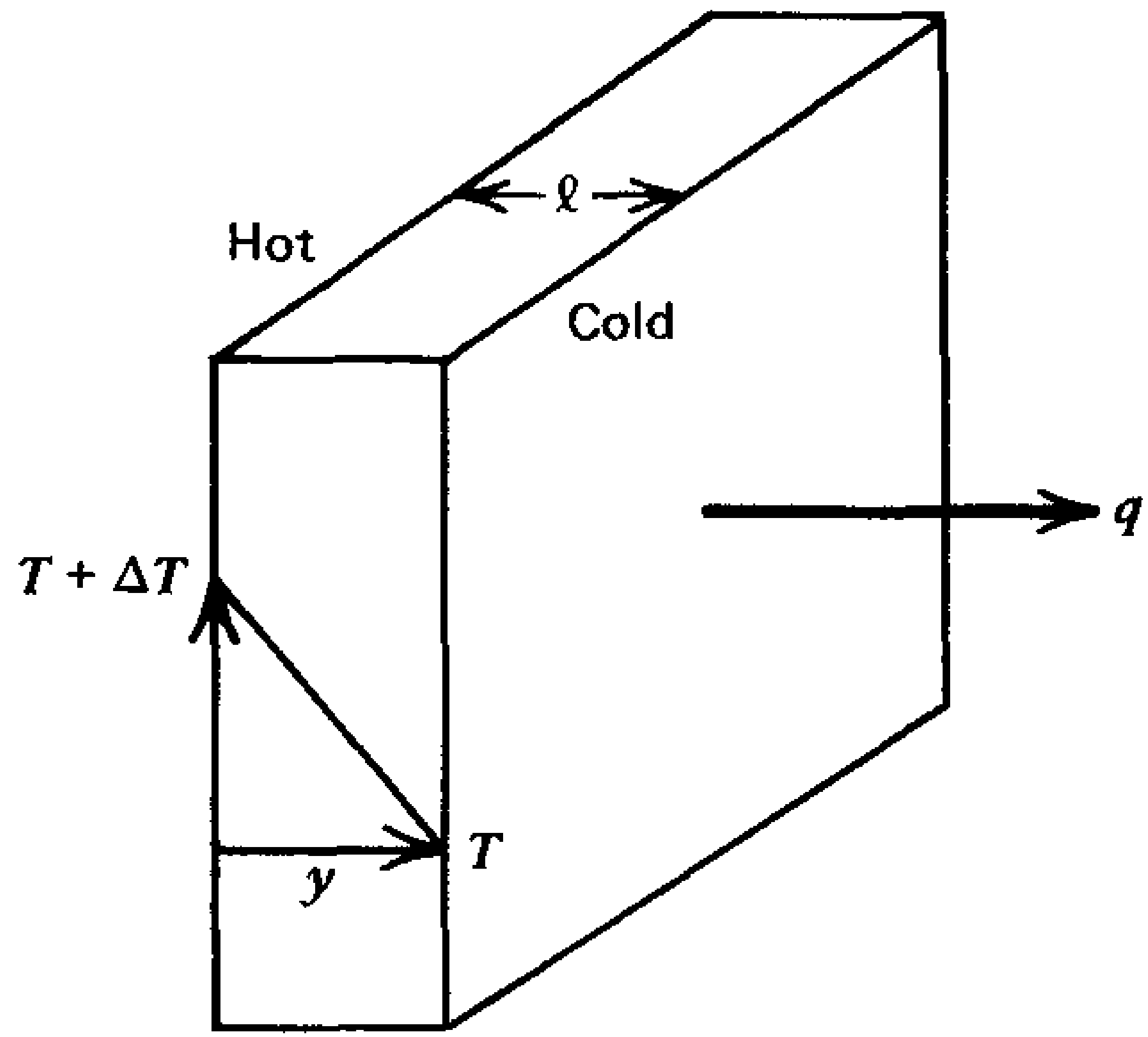
Conduction

$$q = -k\frac{dT}{dy}$$

Positive heat flows in the direction of *decreasing* temperature

Simplified form

$$q = k\frac{\Delta T}{l}$$



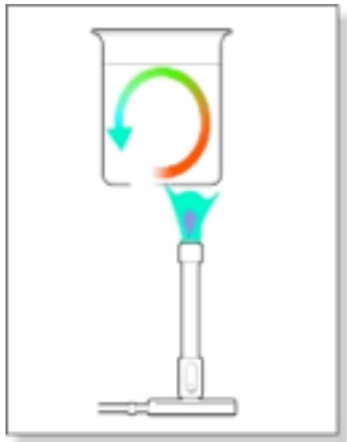
Conductivities

TABLE 4–1 **Temperatures Between Layers of Rock Types**

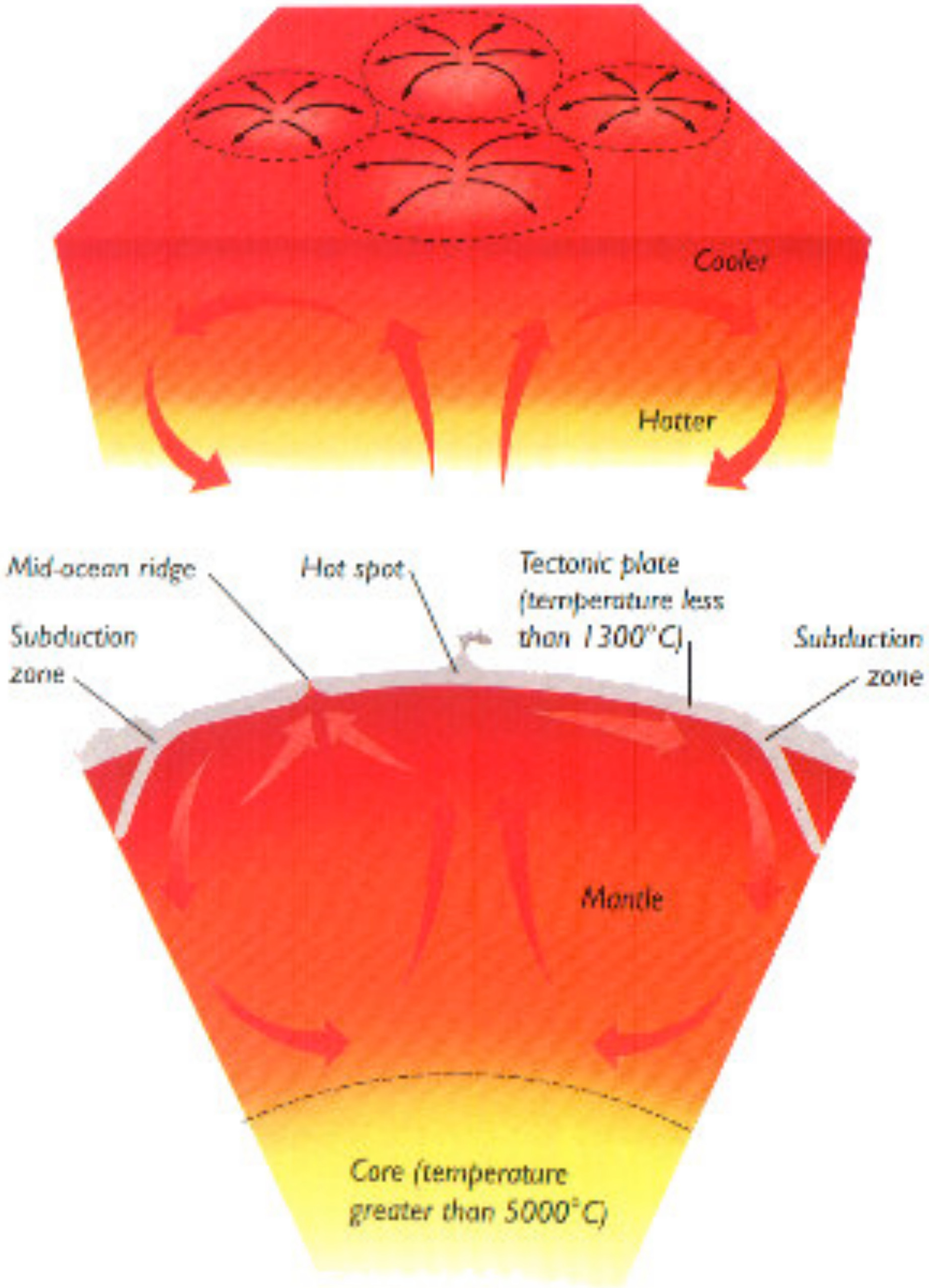
Depth (m)	Temp. (°C)	Rock Type	k ($Wm^{-1}K^{-1}$)
380	18.362		
		Sandstone	3.2
402	18.871		
		Shale	1.7
412	19.330		
		Sandstone	5.3
465	20.446		
		Salt	6.1
475	20.580		
		Sandstone	3.4
510	21.331		
		Shale	1.9
515	21.510		

	(W/m/K)
Water:	0.556
Diamond:	2300
Quartz:	41.6
Marble:	2.08-2.94
Ice:	2.22
Iron:	73
Aluminium:	202
Copper:	401

Convection



- Heat transport associated with *motion* of the medium

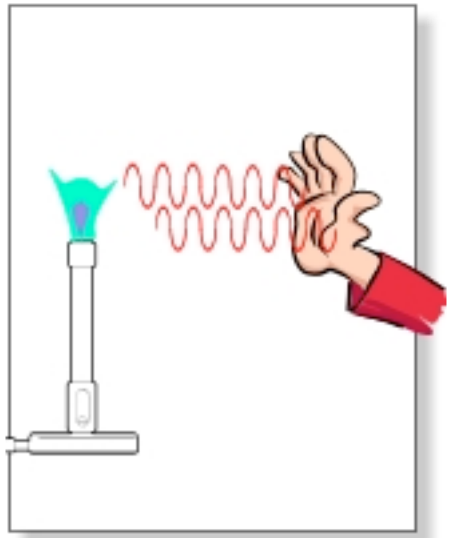


Hot fluid flows into cold region, resulting in heating

Cold fluid flows into hot region, resulting in cooling

We'll discuss this in detail in the mantle convection lectures

Radiation



- Electro-magnetic radiation through a *vacuum*

$$q = \sigma A T^4$$

A - area (m^2)

σ: "Stefan-Boltzmann constant" (5.669×10^{-8} $W/m^2 \cdot K^4$)

Conservation of Energy

- Assume zero internal motion within the material

$$\rho C_p \frac{\partial T}{\partial t} = -\frac{\partial q_x}{\partial x} - \frac{\partial q_y}{\partial y} - \frac{\partial q_z}{\partial z} + H$$

t - time (s)

x_i - spatial coordinate in direction *i* (m)

ρ - density (kg/m^3)

C_p - heat capacity at constant pressure (m^2/s^2/K)

q_i - heat flux in direction *i* (W/m^2)

H - volumetric heat production (W/m^3)

Heat Sources in the Earth

$$H = H_r + H_s + H_a + H_L$$

H_s - shear heating (viscous friction)

H_a - adiabatic heating (or cooling) due to changes in pressure

H_L - latent heat production / consumption due to phase transformations of rocks (e.g. melting)

H_r - radioactive heat production due to the decay of radioactive elements present in rocks

(+ accretionary processes involved in forming the Earth)

Radioactive Elements

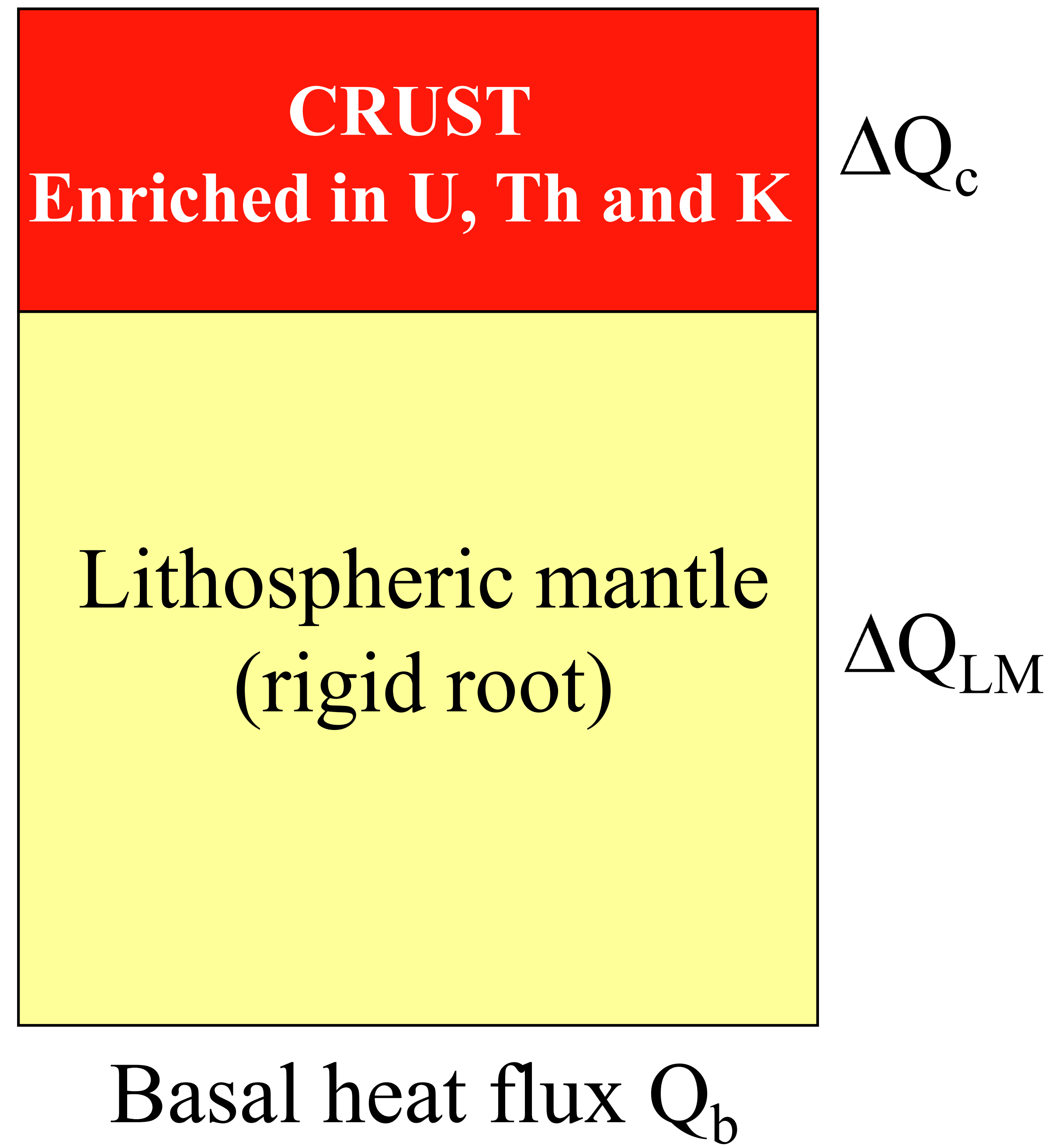
- Radioactive heating attributed to uranium (U), thorium (Th) and potassium (K) isotopes.

TABLE 4–2 Rates of Heat Release H and Half-Lives $\tau_{1/2}$ of the Important Radioactive Isotopes in the Earth's Interior

Isotope	H (W kg^{-1})	$\tau_{1/2}$ (yr)	Concentration C (kg kg^{-1})
^{238}U	9.46×10^{-5}	4.47×10^{9}	30.8×10^{-9}
^{235}U	5.69×10^{-4}	7.04×10^{8}	0.22×10^{-9}
U	9.81×10^{-5}		31.0×10^{-9}
^{232}Th	2.64×10^{-5}	1.40×10^{10}	124×10^{-9}
^{40}K	2.92×10^{-5}	1.25×10^{9}	36.9×10^{-9}
K	3.48×10^{-9}		31.0×10^{-5}

Note: Heat release is based on the present mean mantle concentrations of the heat-producing elements.

$$Q_s = \Delta Q_c + \Delta Q_{LM} + Q_b$$



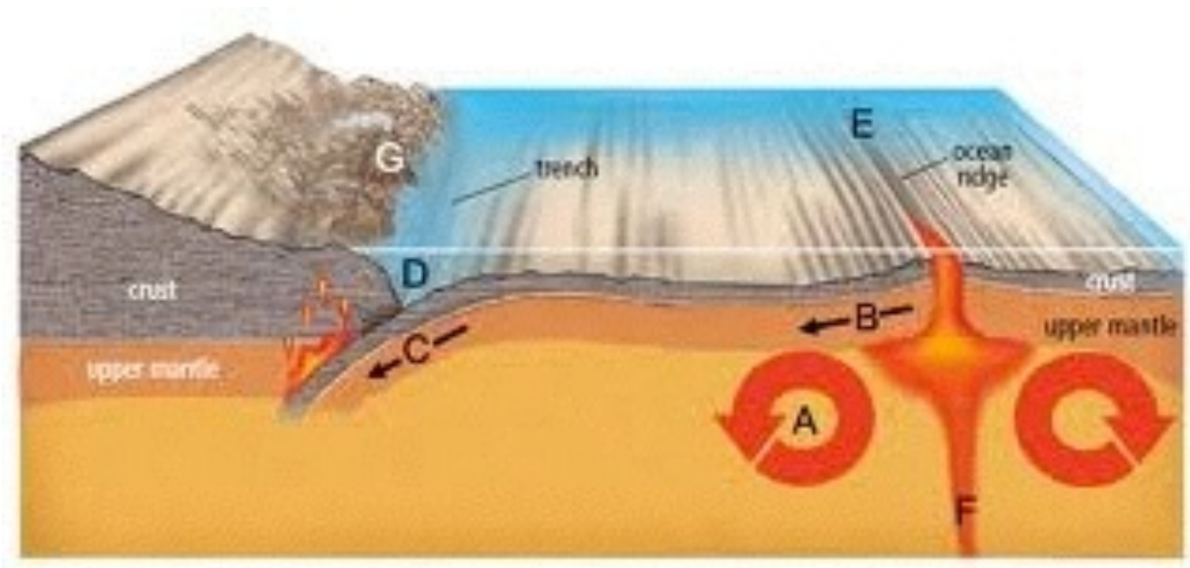
Radioactive Elements

Present within many surface rocks

Partial melting at mid ocean ridges depletes mantle rocks of U,Th,K, leading to high concentrations in basalts.

TABLE 4–3 **Typical Concentrations of the Heat-Producing Elements in Several Rock Types and the Average Concentrations in Chondritic Meteorites**

		Concentration	
Rock Type	**U (ppm)**	**Th (ppm)**	**K (%)**
Reference undepleted (fertile) mantle	0.031	0.124	0.031
"Depleted" peridotites	0.001	0.004	0.003
Tholeiitic basalt	0.07	0.19	0.088
Granite	4.7	20	4.2
Shale	3.7	12	2.7
Average continental crust	1.42	5.6	1.43
Chondritic meteorites	0.008	0.029	0.056



Processes related to the formation of continental crust (e.g. volcanism) also differentiate incompatible elements, leading to high concentrations in granitic rocks.

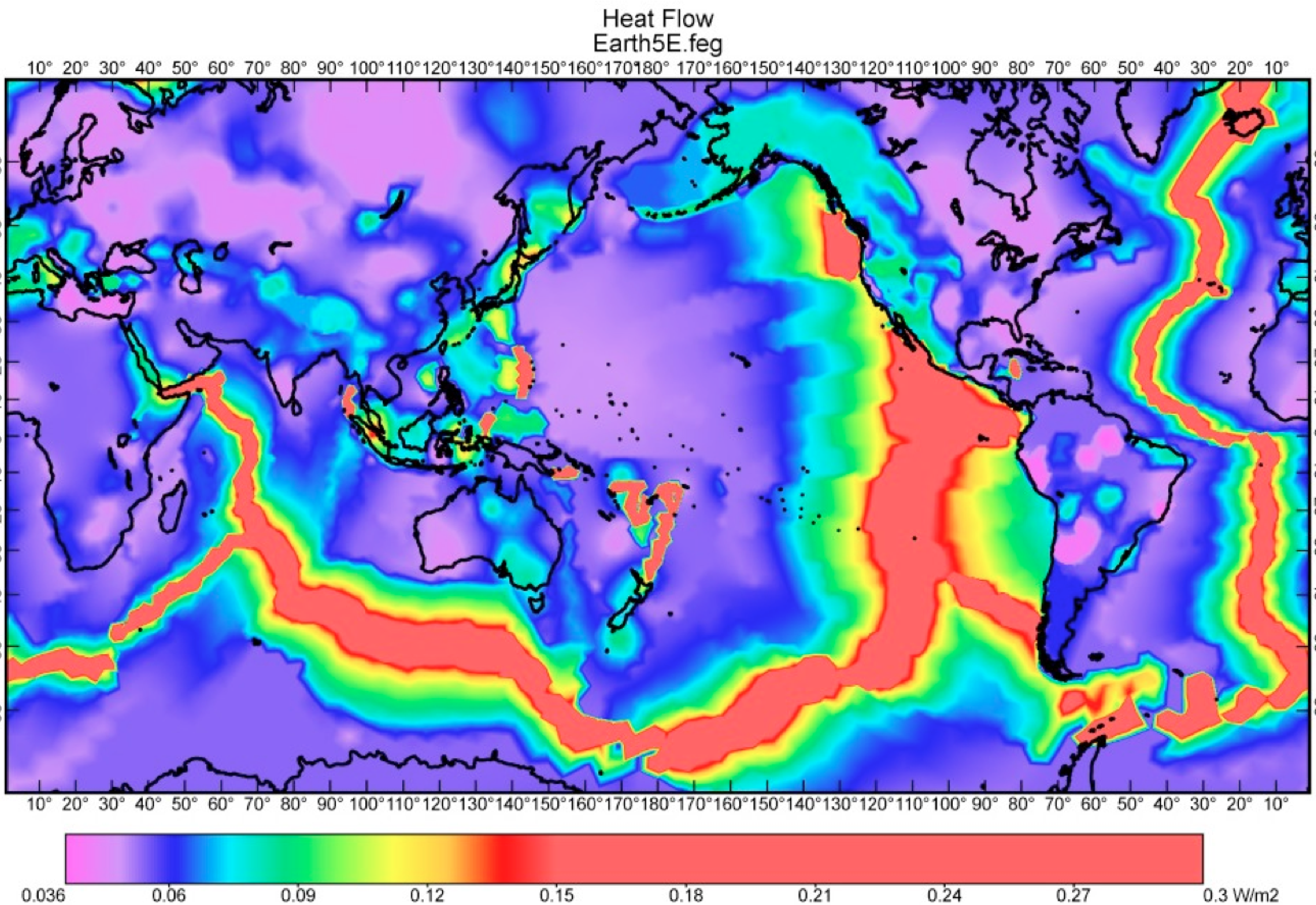
Radioactive Elements

	Granite	Tholeiitic basalt	Alkali basalt	Peridotite	Average continental upper crust	Average oceanic crust	Undepleted mantle
Concentration by weight							
U (ppm)	4	0.1	0.8	0.006	1.6	0.9	0.02
Th (ppm)	15	0.4	2.5	0.04	5.8	2.7	0.10
K (%)	3.5	0.2	1.2	0.01	2.0	0.4	0.02
Heat generation (10^{-10} W kg^{-1})							
U	3.9	0.1	0.8	0.006	1.6	0.9	0.02
Th	4.1	0.1	0.7	0.010	1.6	0.7	0.03
K	1.3	0.1	0.4	0.004	0.7	0.1	0.007
Total	9.3	0.3	1.9	0.020	3.9	1.7	0.057
Density (10^3 kg m^{-3})	2.7	2.8	2.7	3.2	2.7	2.9	3.2
Heat generation (μWm^{-3})	2.5	0.08	0.5	0.006	1.0	0.5	0.02

continental crust

oceanic crust

Heat Budget for the Earth



Continental average: ~65 mW/m^2
Oceanic average: ~100 mW/m^2

Does the Budget Balance?

Oceans

59% surface area of Earth
Average heat flux = 107 mW/m^2
Total Q = 32 TW (70% of total)

Continents

41% surface area of Earth
Average heat flux = 67 mW/m^2
Total Q = 14 TW (30% of total)

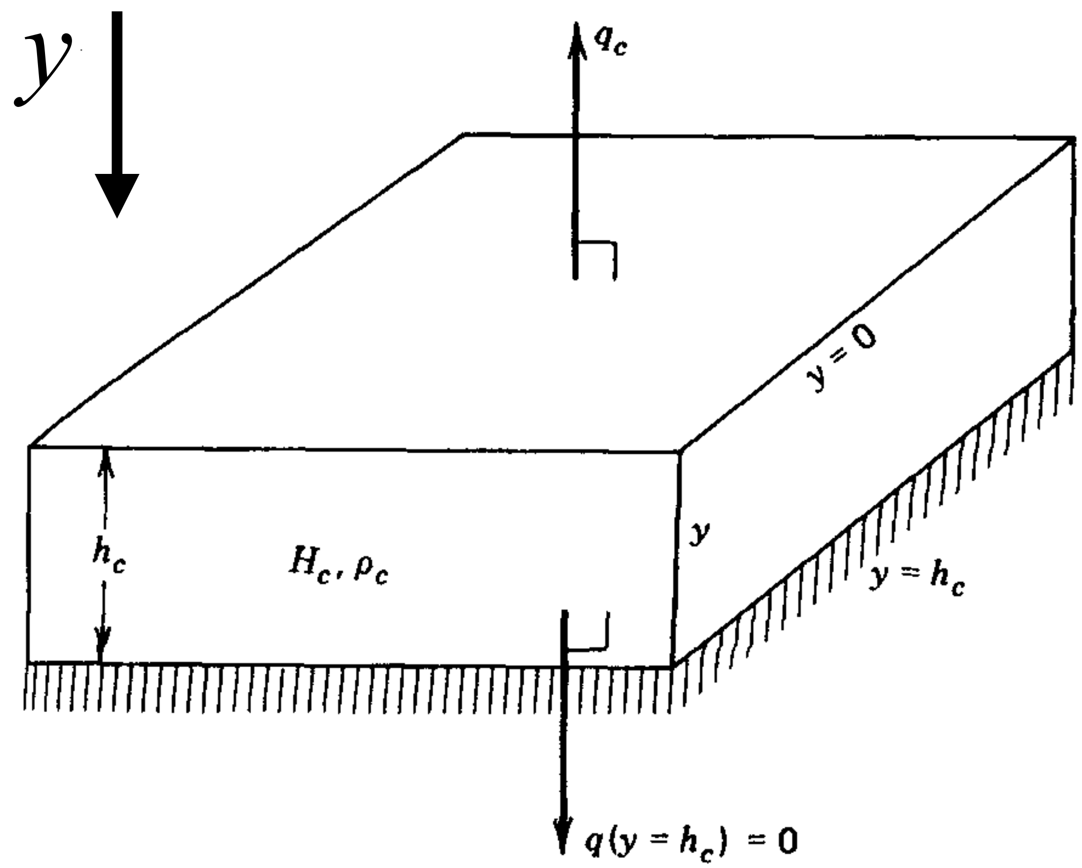
Question: Can we account for the heat flow observed at the surface?

i) considering only conductive heat transfer

ii) considering radioactive heat sources only

iii) assuming steady state, i.e. no time dependence

Oceanic Crust



Heat flow through the top
Insulated at the bottom
Internal heat source

$$\rho_c = 2900 \ \text{kg.m}^{-3}$$

$$h_c = 6 \ \text{km}$$ *(average oceanic crustal thickness)*

$$H_c = 2.6 \times 10^{-11} \ \text{W.kg}^{-1}$$

(heat source from predominately basalts)

$$q_c = \rho_c H_c h_c$$

$$q_c = 0.45 \ \text{mW.m}^{-2} << 100 \ \text{mW.m}^{-2}$$

Radioactive heat sources DO NOT explain the observed heat flux

Continental Crust

- Repeat calculation with properties for continental crust

$$\rho_c = 2700 \ \text{kg.m}^{-3}$$

$$h_c = 35 \ \text{km}$$ *(average continental crustal thickness)*

$$H_c = 9.6 \times 10^{-10} \ \text{W.kg}^{-1}$$ *(heat source from predominately granite)*

$$q_c = \rho_c H_c h_c$$

$$q_c = 91 \ \text{mW.m}^{-2} > 65 \ \text{mW.m}^{-2}$$

Heat flux computed is higher than that observed

Assume the heat source must decrease with depth

Continental Crust

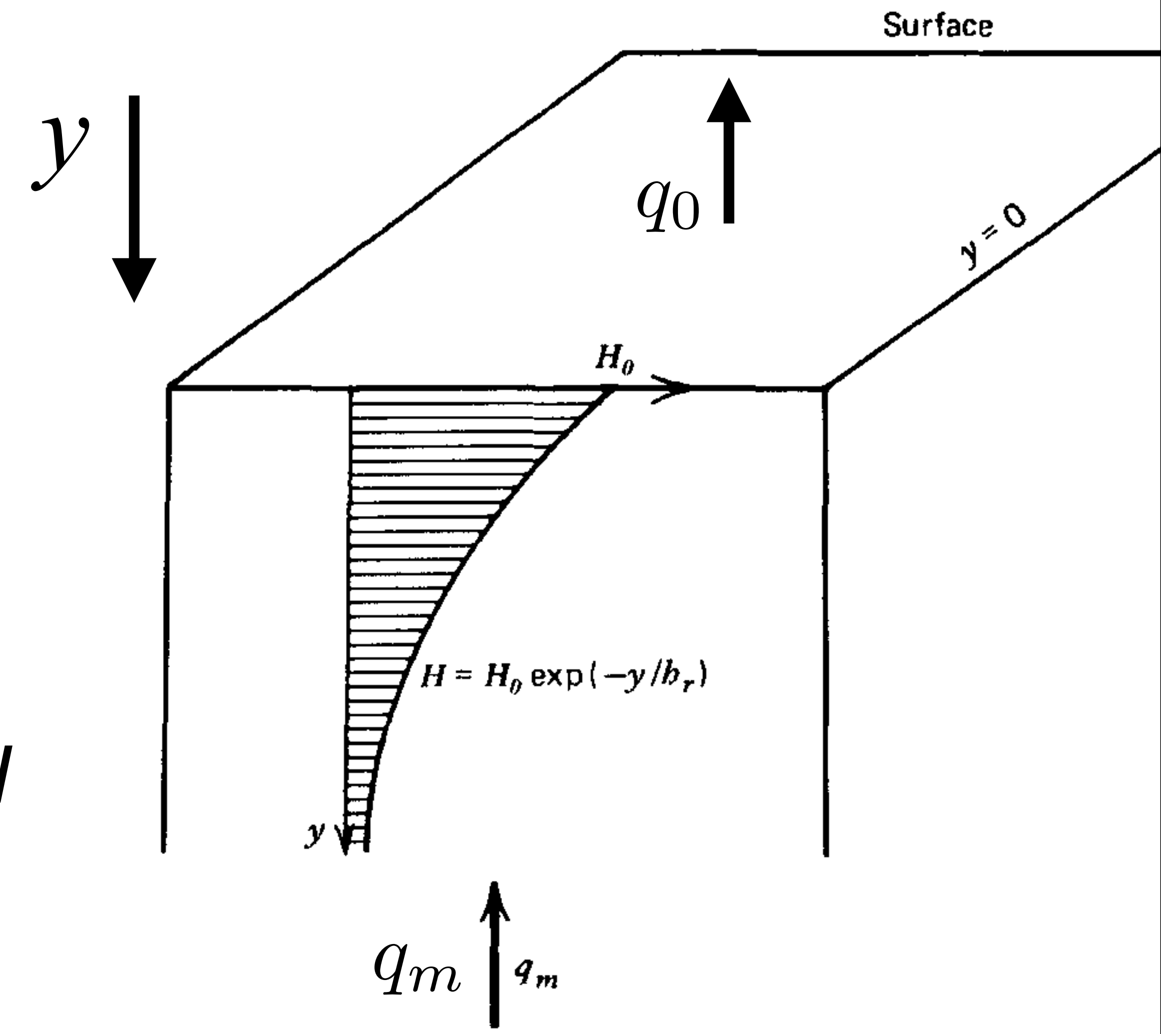
$$H = H_0 \exp\left(-y/h_r\right)$$

H_0 Surface radiogenic heat production (W/kg)

h_r Length scale for decrease in *H* with depth (m)

q_m Basal heat flux from the mantle at $y = \infty$ (W/m^2)

Experimentally determined



Heat flow through the top
Basal heat flux from the mantle
Internal heat source

Continental Crust

- Same analysis, yields

$$q = -q_m - \rho H_0 h_r \exp\left(-y/h_r\right)$$

$$-q(y=0) = q_0 = q_m + \rho H_0 h_r$$

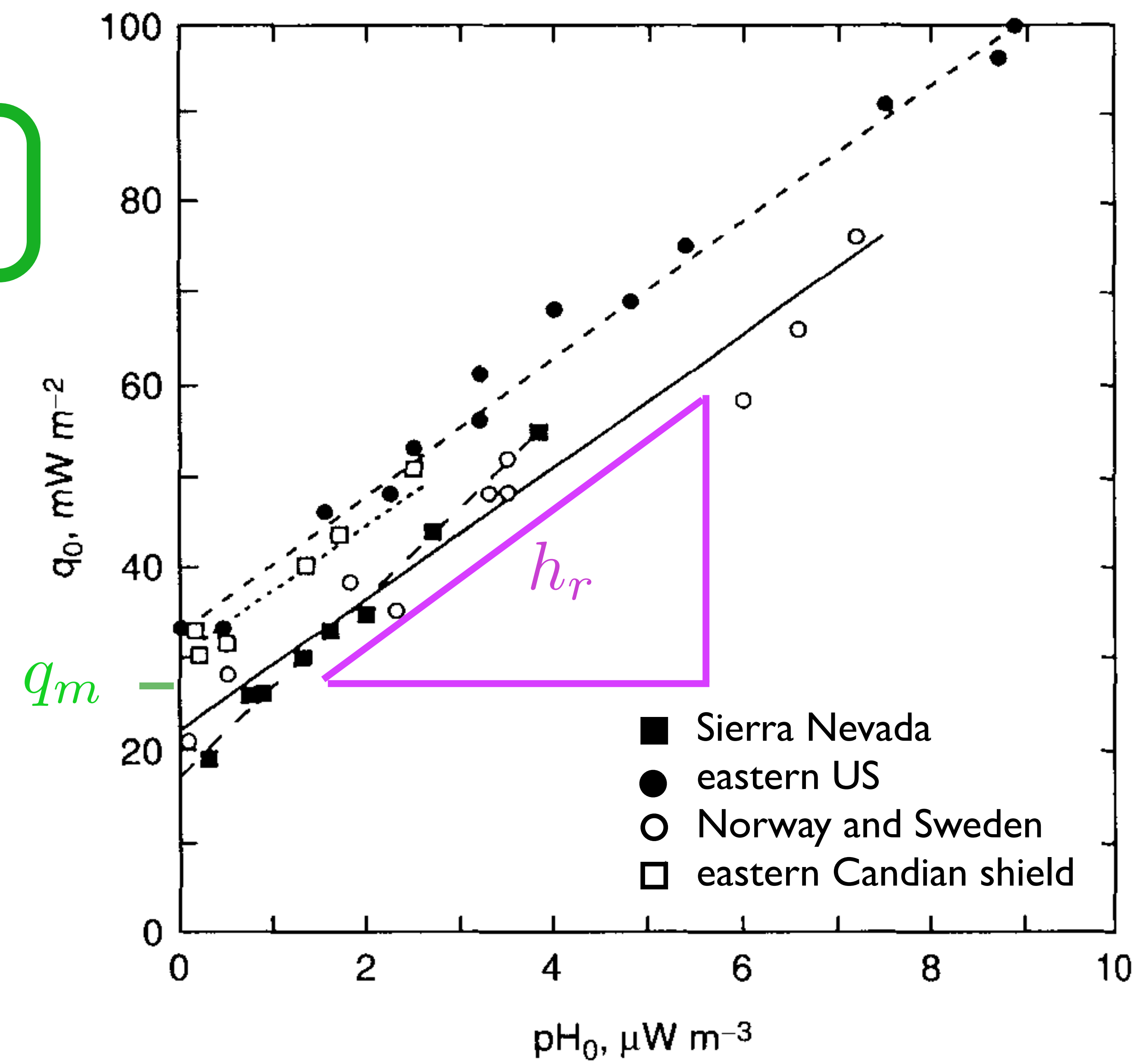
Typical values

$$h_r \sim 7.5 \ \text{km}$$

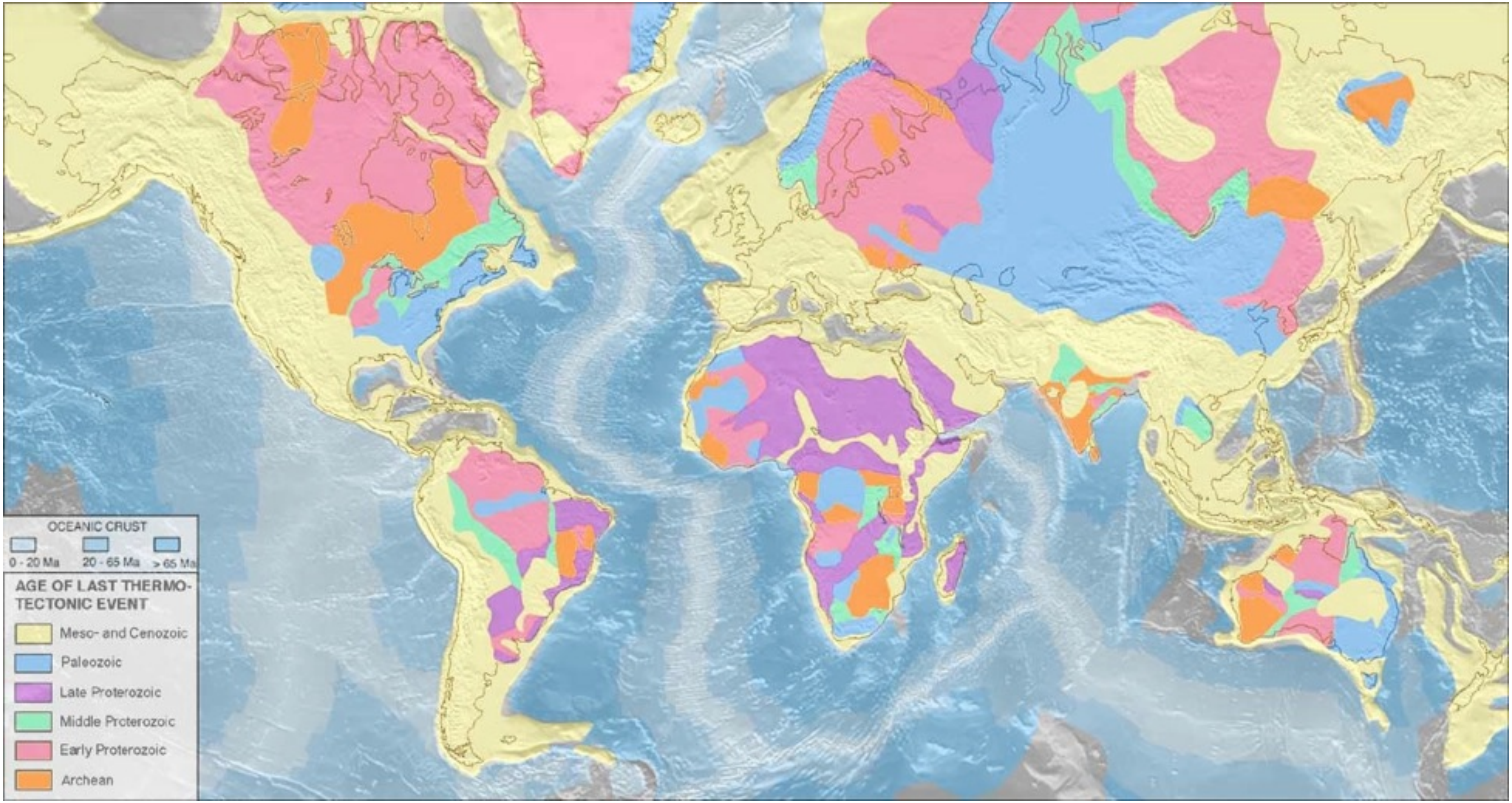
$$q_m \sim 17 - 30 \ \text{m W.m}^{-2}$$

$$36 \ \text{m W.m}^{-2} < q_0 < 49 \ \text{m W.m}^{-2}$$

$$\sim 67 \ \text{m W.m}^{-2}$$



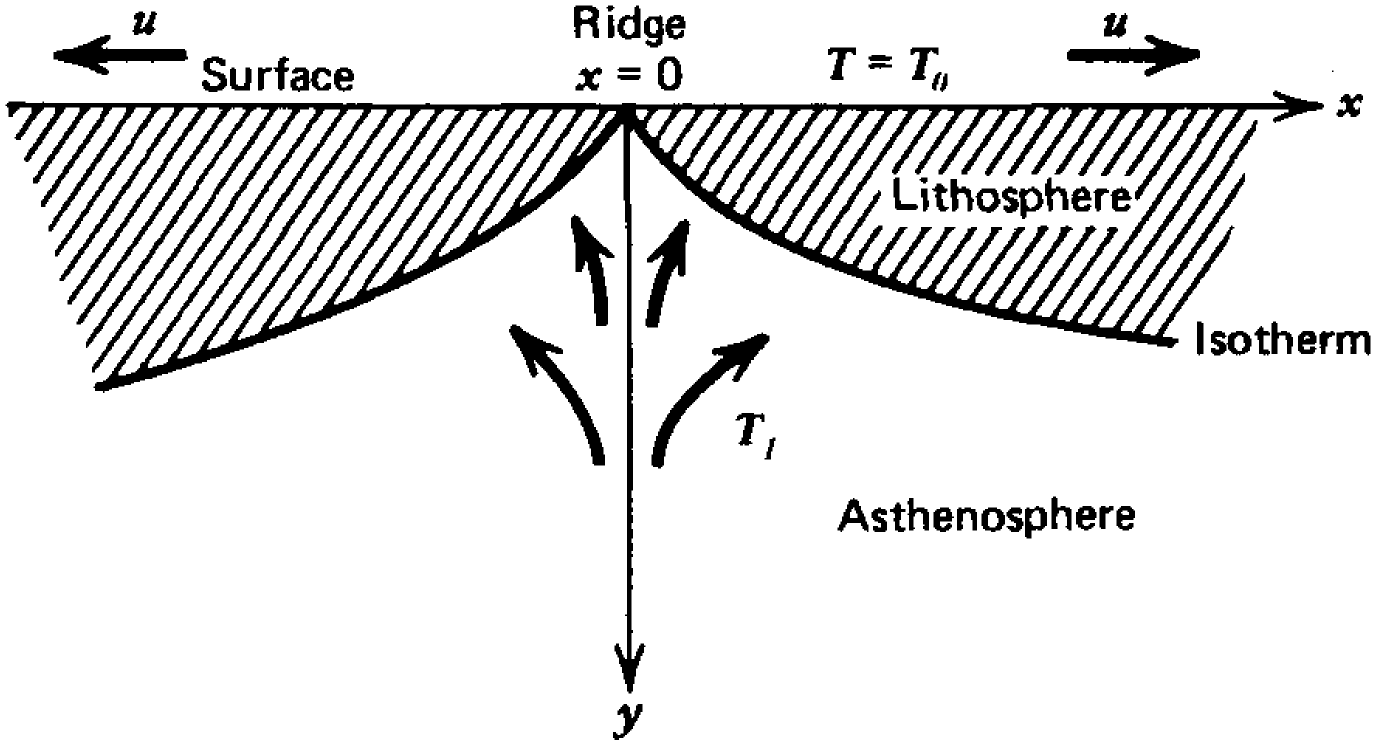
What Went Wrong?

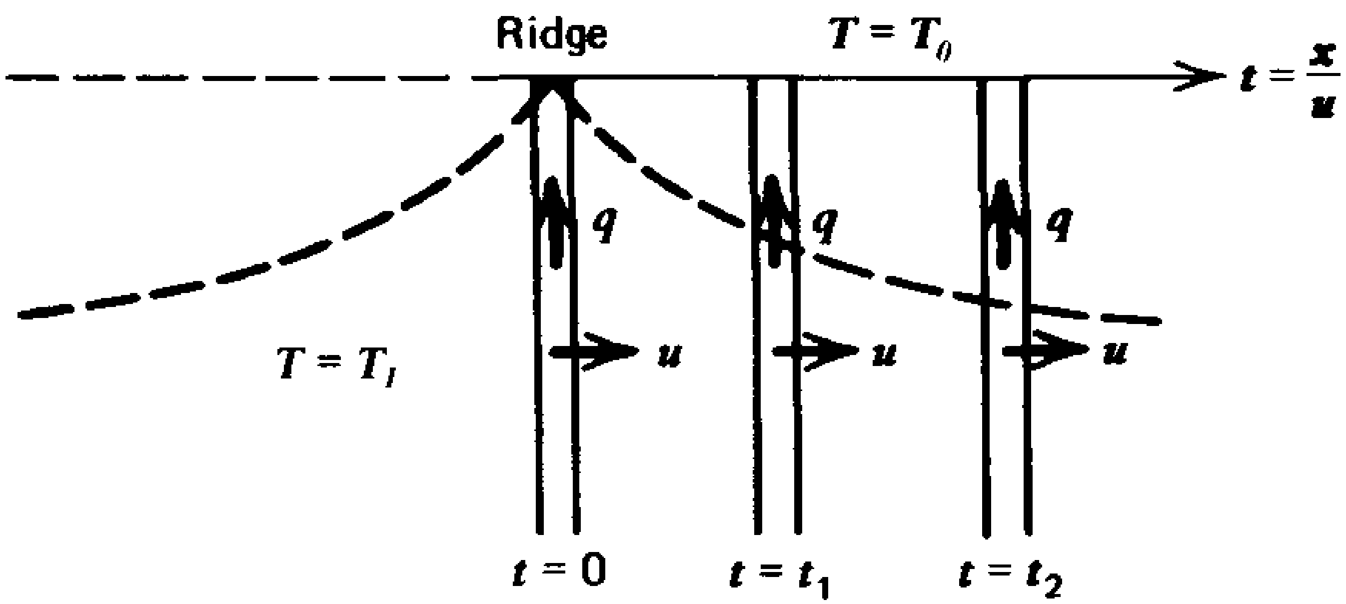


Era	Age	Time Span
Cenozoic and Mesozoic	0-245 Ma	245 m.y.
Paleozoic	245-570 Ma	325 m.y.
Late Proterozoic	500-900 Ma	400 m.y.
Middle Proterozoic	900-1600 Ma	700 m.y.
Early Proterozoic	1600-2500 Ma	900 m.y.
Archean	> 2500 Ma	--

Assumption of steady state is incorrect

Mid-ocean Ridge Model





Hot mantle rock rises.

At the ridge, the mantle rock is suddenly exposed to the cold surface temperature.

The seafloor spreads away from the ridge, losing heat to the water via conduction.

The rocks solidify as they cool, forming the oceanic lithosphere.

$$T = T_1 \quad \text{at} \quad t = 0, \qquad y > 0$$

$$T = T_0 \quad \text{at} \quad y = 0 \qquad t > 0$$

$$T \to T_1 \quad \text{as} \quad y \to \infty \qquad t > 0.$$

Time Dependent Conduction

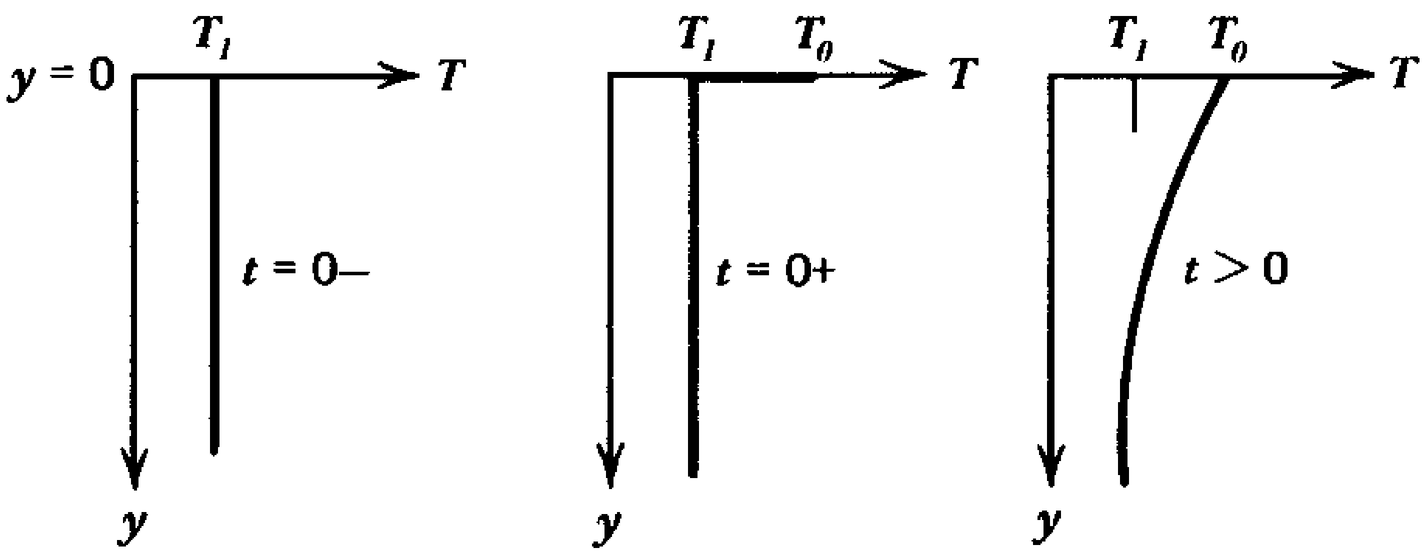
- Heating (or cooling) of a semi-infinite half space ($y > 0$)

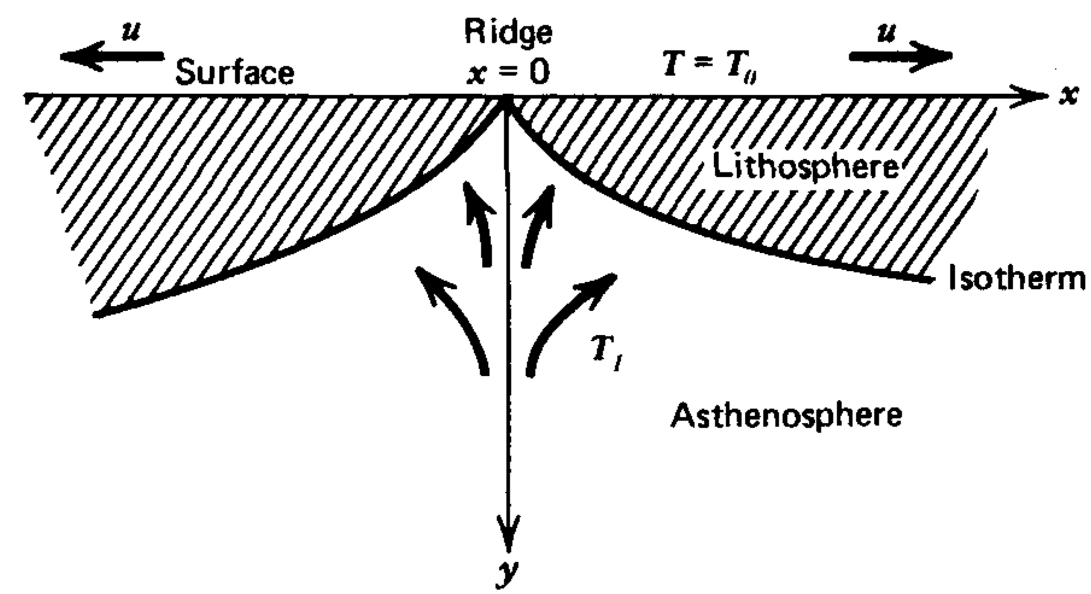
Solve $$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} \qquad \kappa = \frac{k}{\rho C_p}$$

κ *thermal diffusivity* (m^2/s)

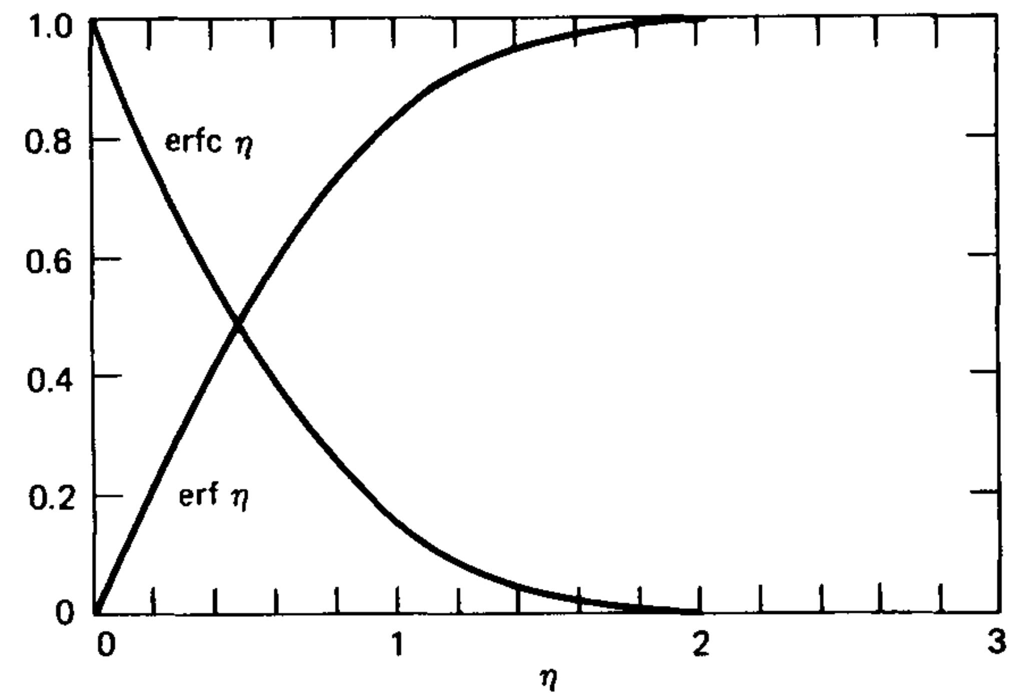
when the temperature T_1 at time = 0, is instantaneously changed to T_0.





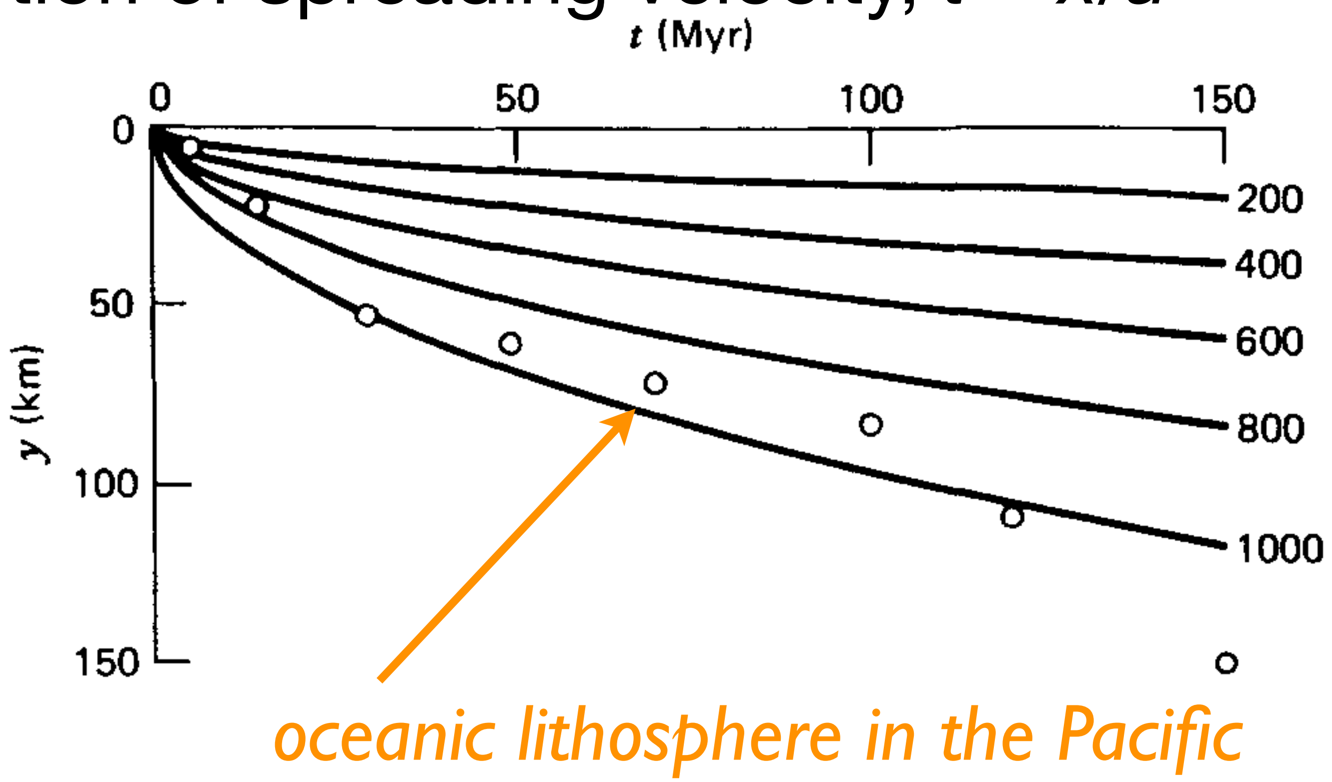
Half Space Cooling Model (T)

$$\frac{T - T_1}{T_0 - T_1} = \operatorname{erfc}\frac{y}{2\sqrt{\kappa t}}.$$



Temperature profile as function of spreading velocity, $t = x/u$

$$\frac{T - T_0}{T_1 - T_0} = \operatorname{erf}\left(\frac{y}{2\sqrt{\kappa x/u}}\right)$$

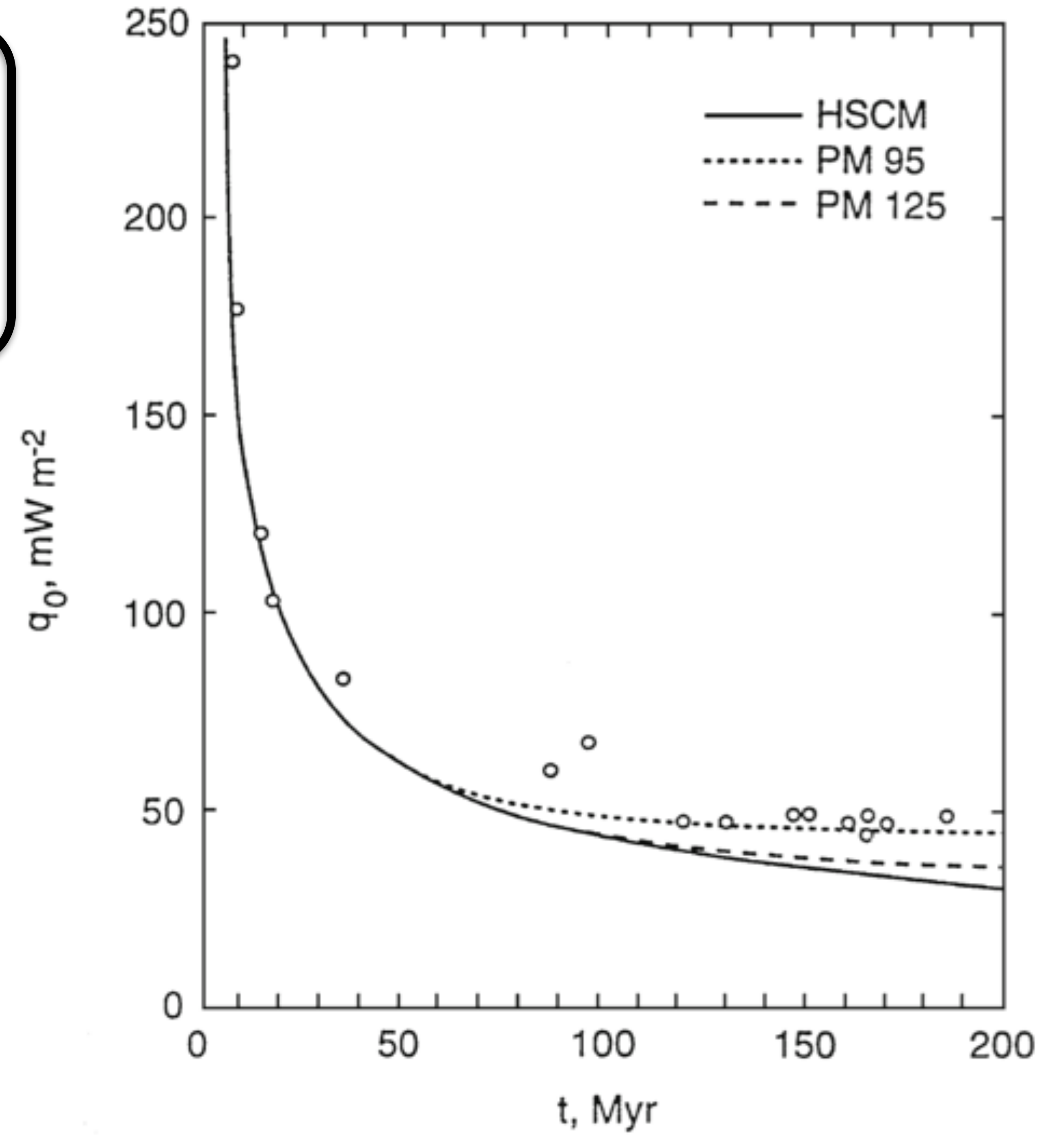


Half Space Cooling Model (q)

$$\frac{T - T_1}{T_0 - T_1} = \mathrm{erfc}\frac{y}{2\sqrt{\kappa t}}.$$

$$q = -k\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{k(T_0 - T_1)}{\sqrt{\pi \kappa t}}.$$

Heat flow as a function of the age of the ocean floor. HSCM is the "half space cooling model". Data points are from sediment covered regions of the Atlantic and Pacific oceans.



Oceanic Lithosphere

- Average age of the subducted lithosphere is ~120 Myr.
- Compute the average surface heat flux over this time period using the half space cooling solution yields

$$\bar{q}_0 = \frac{1}{\tau}\int_0^{\tau} q_0\,dt = \frac{1}{\tau}\int_0^{\tau} \frac{k(T_1 - T_0)}{\sqrt{\pi\kappa t}}\,dt = \frac{2k(T_1 - T_0)}{\sqrt{\pi\kappa\tau}}$$

$$\tau = 120 \ \mathrm{Myr}$$

$$k = 3.3 \ \mathrm{W/m/K}$$

$$\kappa = 1 \ \mathrm{mm^2/s}$$

$$T_1 - T_0 = 1300 \ \mathrm{K}$$

$$\bar{q}_0 = 79 \ \mathrm{mW.m^{-2}}$$

$$\sim 101 \ \mathrm{mW.m^{-2}}$$

Summary

- Even highly simplified, 1D representations of the Earth enable first order estimates to be made of relative importance of radiogenic heat sources.
- In considering the global heat budget for the Earth, the simplified 1D analysis indicates;

1. In continents, most heat is lost to the surface via *steady state conduction* through the crust.
2. In the oceans, most of the cooling occurs in the lithosphere.