Rheology of the Earth

(651-4008-00 G)

Schedule

- Rheology basics
  - Viscous, elastic and plastic
- Creep processes
- Flow laws
- Yielding mechanisms
- Deformation maps
- Yield strength envelopes
- Constraints on the rheology from the laboratory, geology, geophysics and numerical modelling (next time)

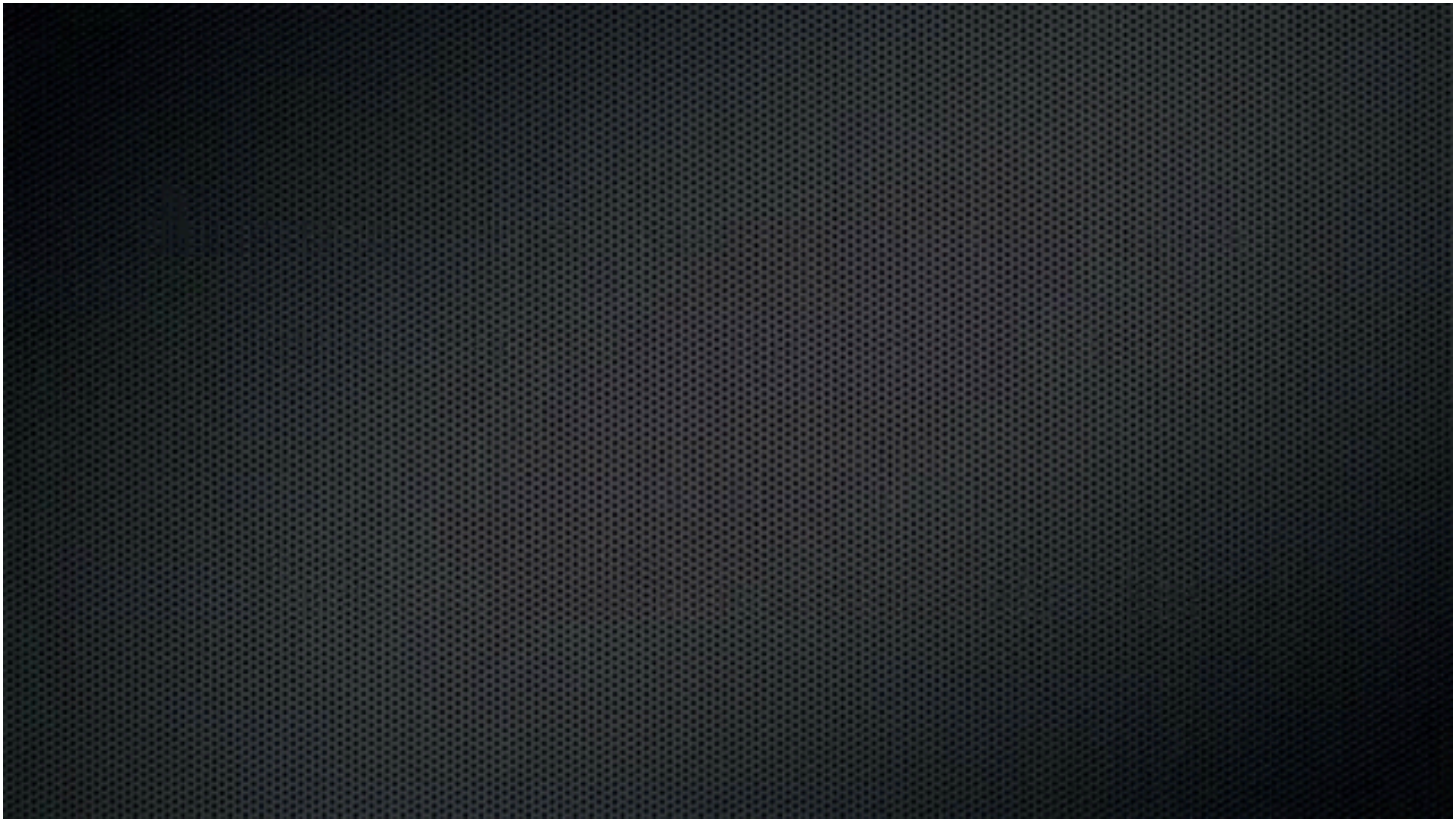
Rheology: What is it?

- The branch of science concerned with how material “flows”
- More precisely, “the response of a material to deformation (e.g. applied strain, or strain-rate, or stress)”
- Some examples

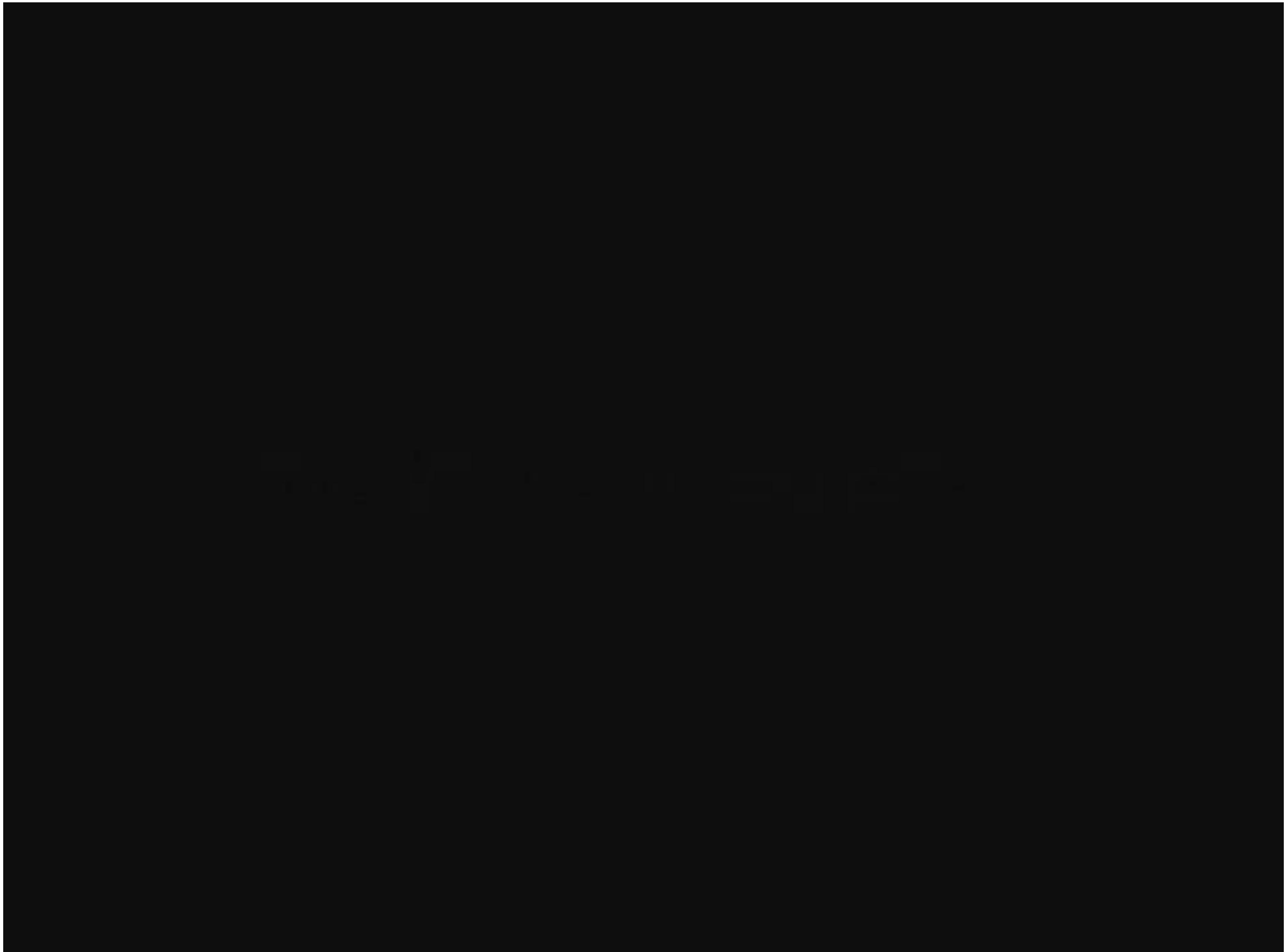
Rheology of Fluids

… steel …

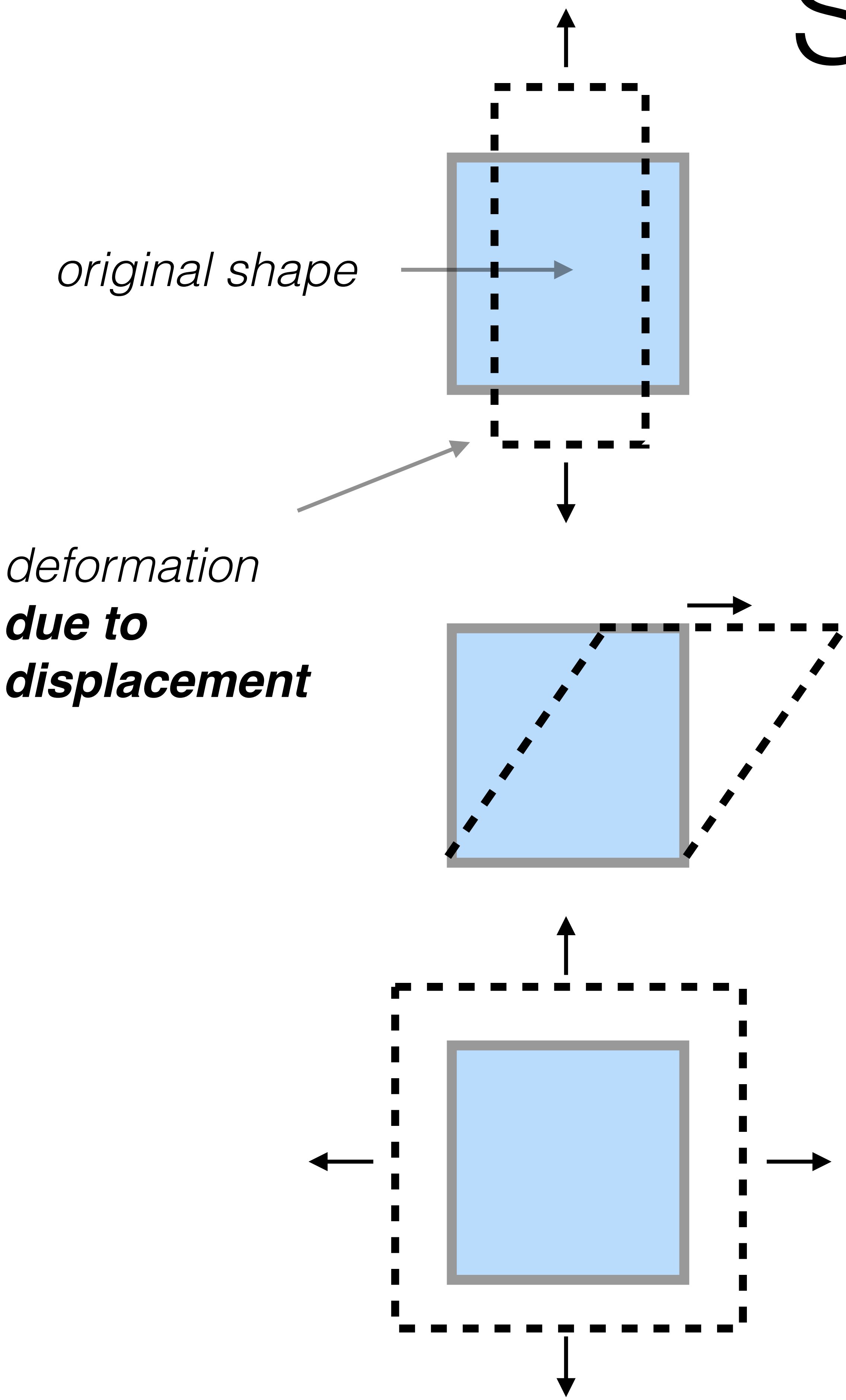
… concrete …



… visco-elastic fluids …



Strain



Tensile strain - or - direct strain

$$\epsilon_{xx}, \epsilon_{yy}$$

Shear strain

$$\epsilon_{xy}$$

Volumetric strain (dilatation)

Strain-rate

- Analogous to the strain tensor, but involves *gradients of velocity* (and not displacement)

$$\dot{\epsilon}_{xx}, \dot{\epsilon}_{yy}, \dot{\epsilon}_{xy} \quad [1/\mathrm{s}]$$

$\boldsymbol{x} = (x, y)$ position

$\boldsymbol{v} = (v_x, v_y)$ velocity

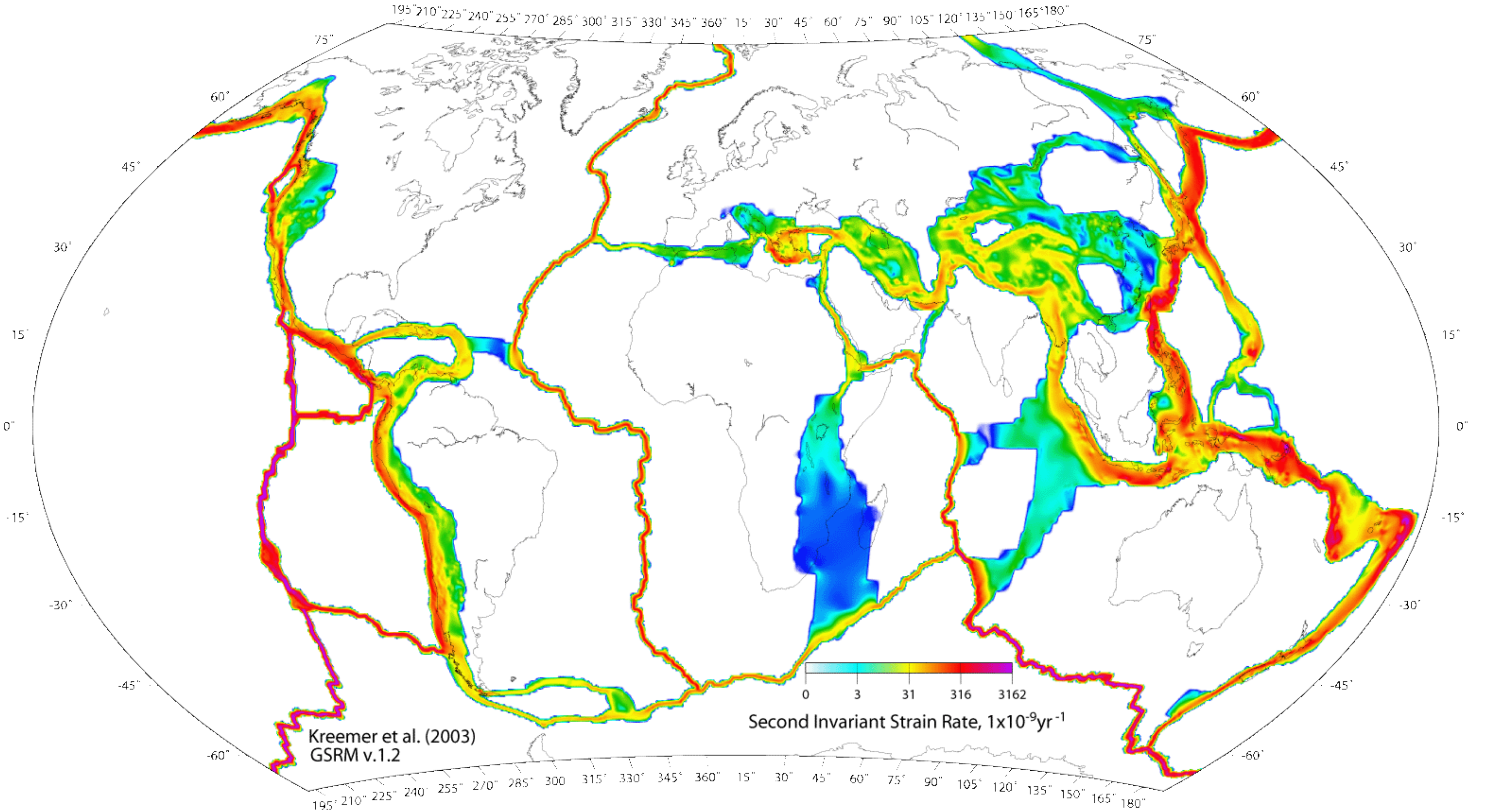
$$\dot{\epsilon}_{xx} = \frac{\partial v_x}{\partial x} \qquad \dot{\epsilon}_{yy} = \frac{\partial v_y}{\partial y} \qquad \dot{\epsilon}_{xy} = \frac{1}{2}\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$\dot{\epsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

- Strain-rate *invariant:* a scalar measure of the magnitude of a tensorial quantity

$$\dot{\epsilon}_{II} = \sqrt{\tfrac{1}{2}\dot{\epsilon}_{ij}^2}$$

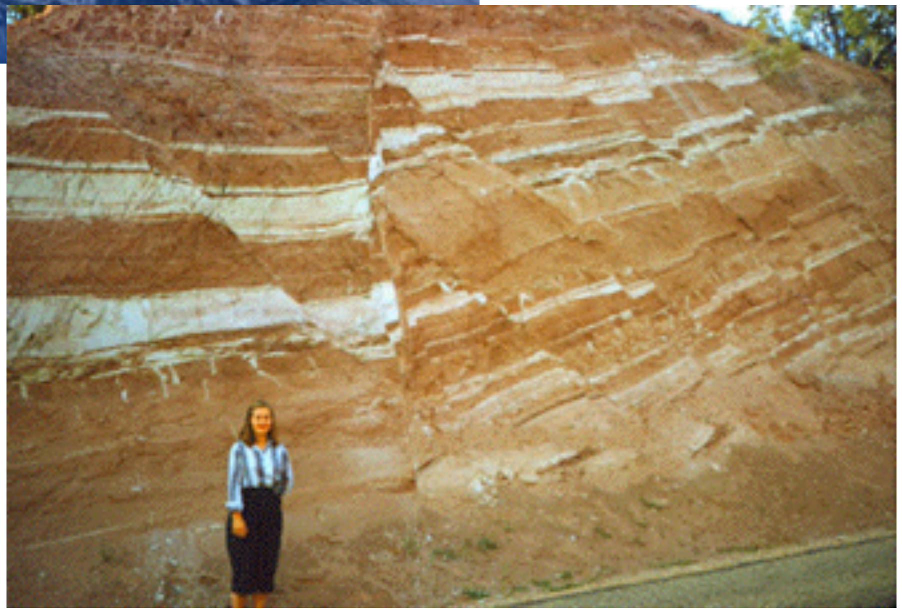
World strain-rate map



Three basic rheologies

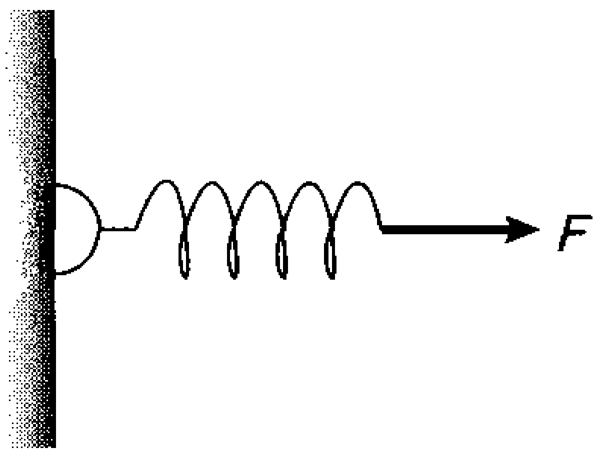
- Elastic
- Viscous
- Plastic





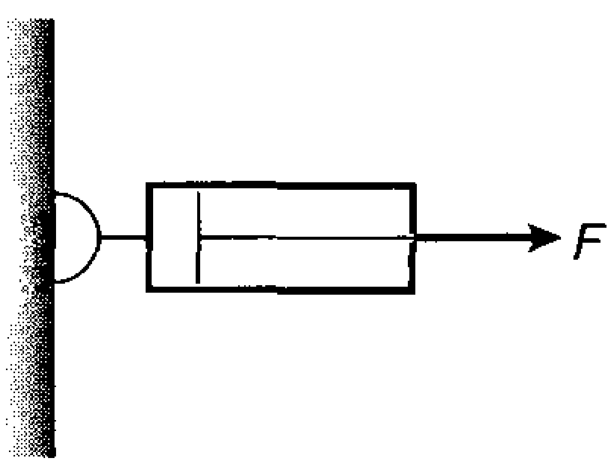
0D elastic and viscous media

Elastic

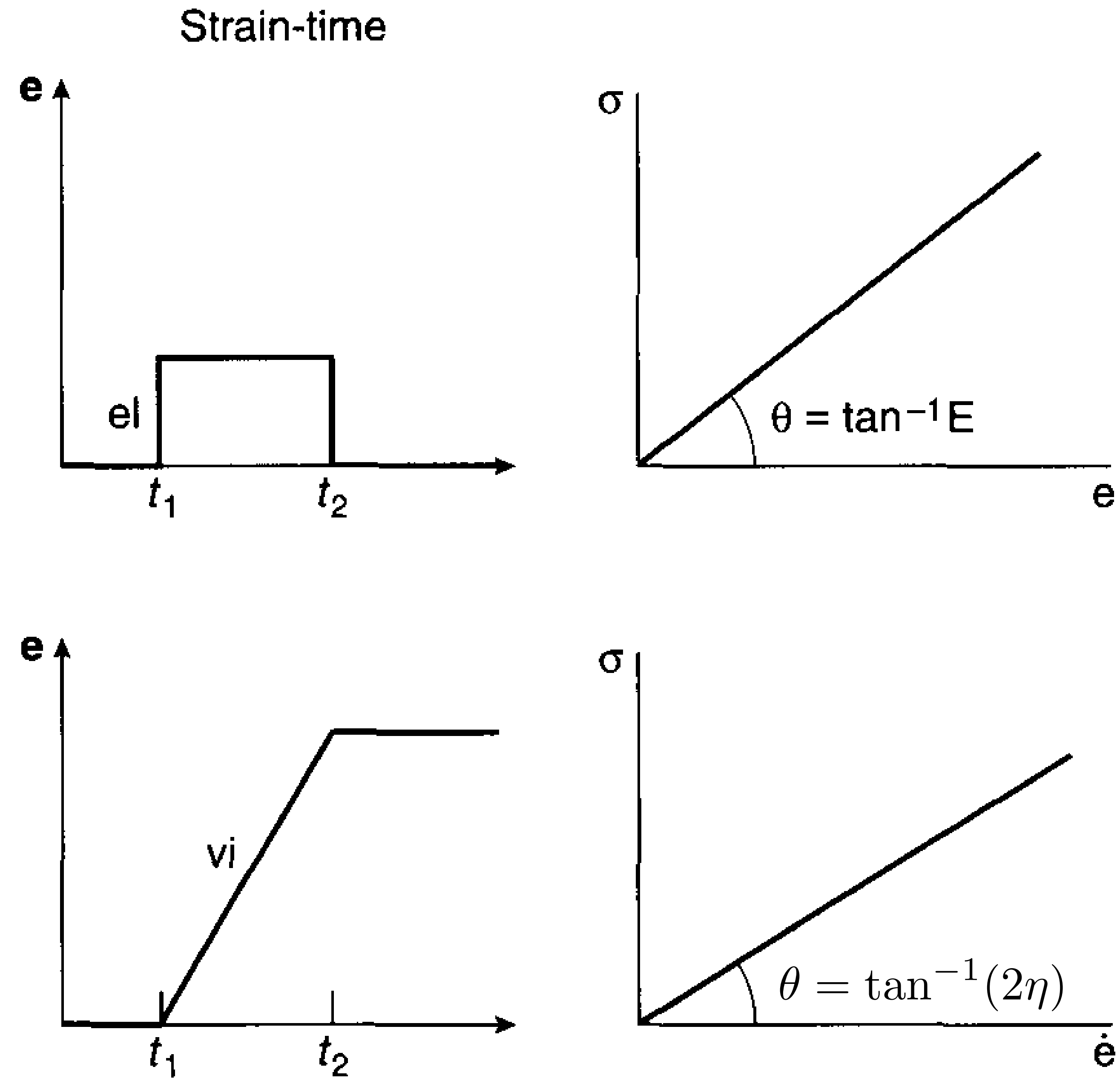


$$\sigma = E\epsilon$$

Viscous



$$\sigma = 2\eta\dot{\epsilon}$$



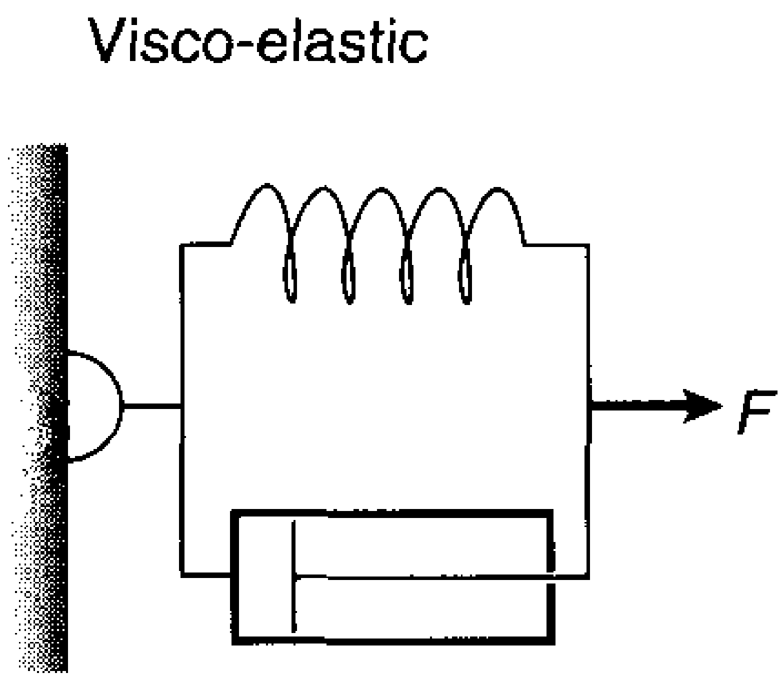
Characteristic values

Table 5.5 Some Representative Viscosities (in Pa · s)

Air	10^{-5}
Water	10^{-3}
Olive oil	10^{-1}
Honey	4
Glycerin	83
Lava	10–10^{4}
Asphalt	10^{5}
Pitch	10^{9}
Ice	10^{12}
Rock salt	10^{17}
Sandstone slab	10^{18}
Asthenosphere (upper mantle)	10^{20}
Lower mantle	10^{21}

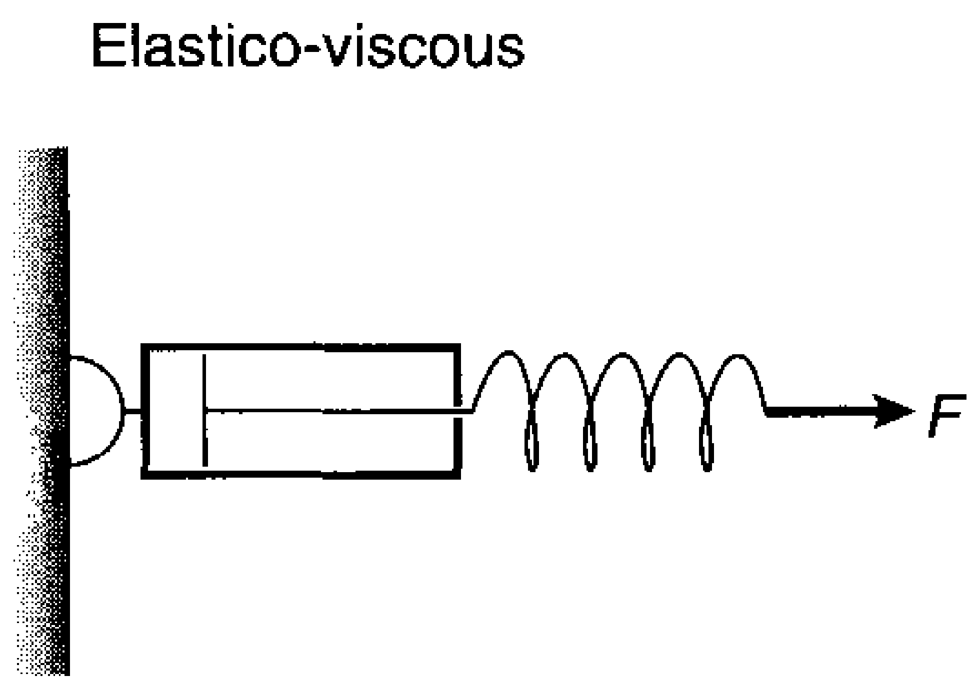
Sources: Several sources, including Turcotte and Schubert (1982).

Visco-elasticity



Kelvin-Voigt body

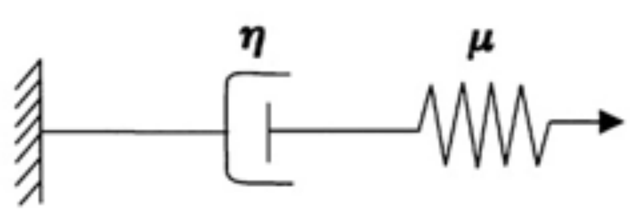
$$\sigma = E\epsilon + 2\eta\dot{\epsilon}$$



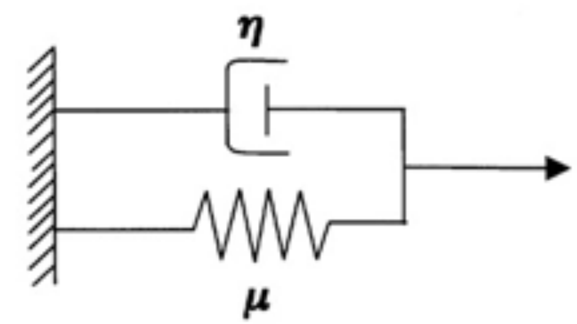
Maxwell body

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{2\eta}$$

Visco-elasticity

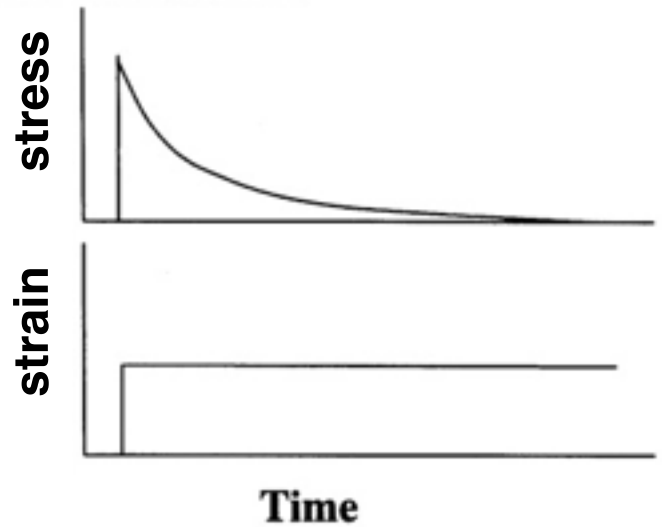


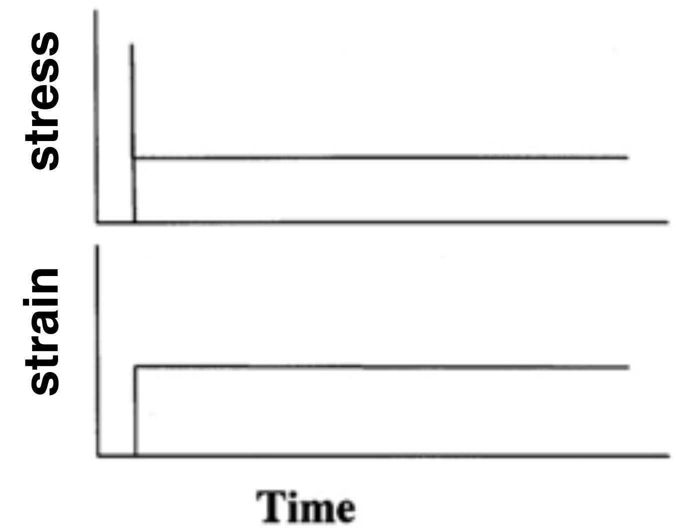
A) Maxwell model $\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{2\eta}$



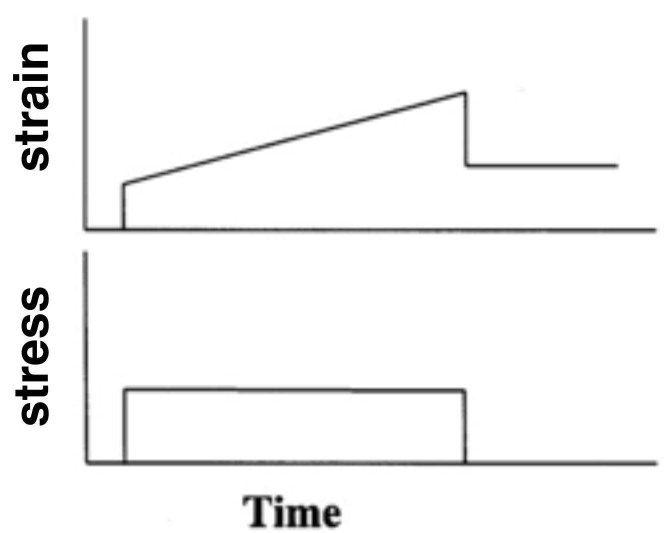
B) Voigt model $\sigma = E\epsilon + 2\eta\dot{\epsilon}$

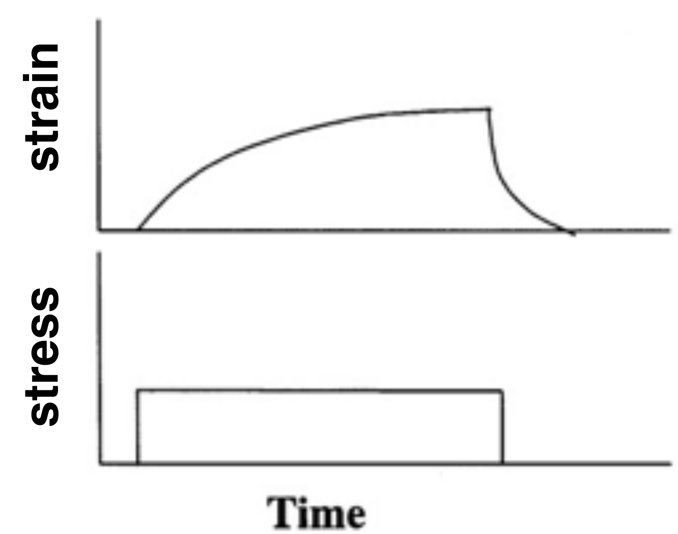
Stress relaxation



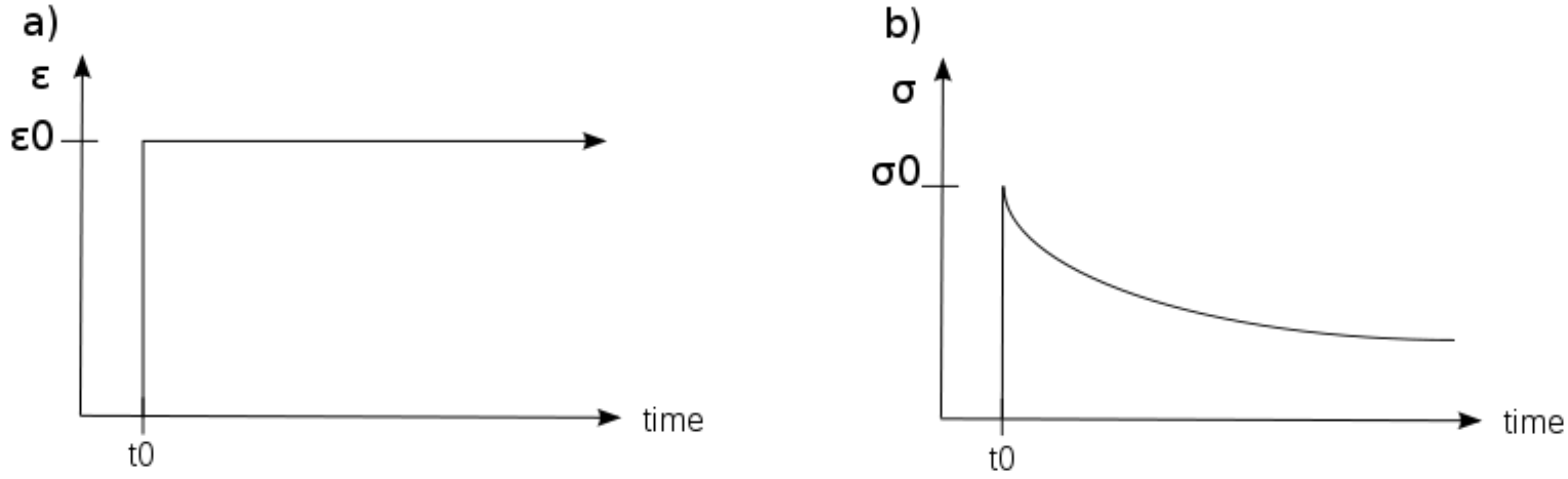


Creep





Maxwell relaxation time



Maxwell relaxation time: $t_m = 2\eta/E$

Time required for initial stress to reduce by 1/*e*

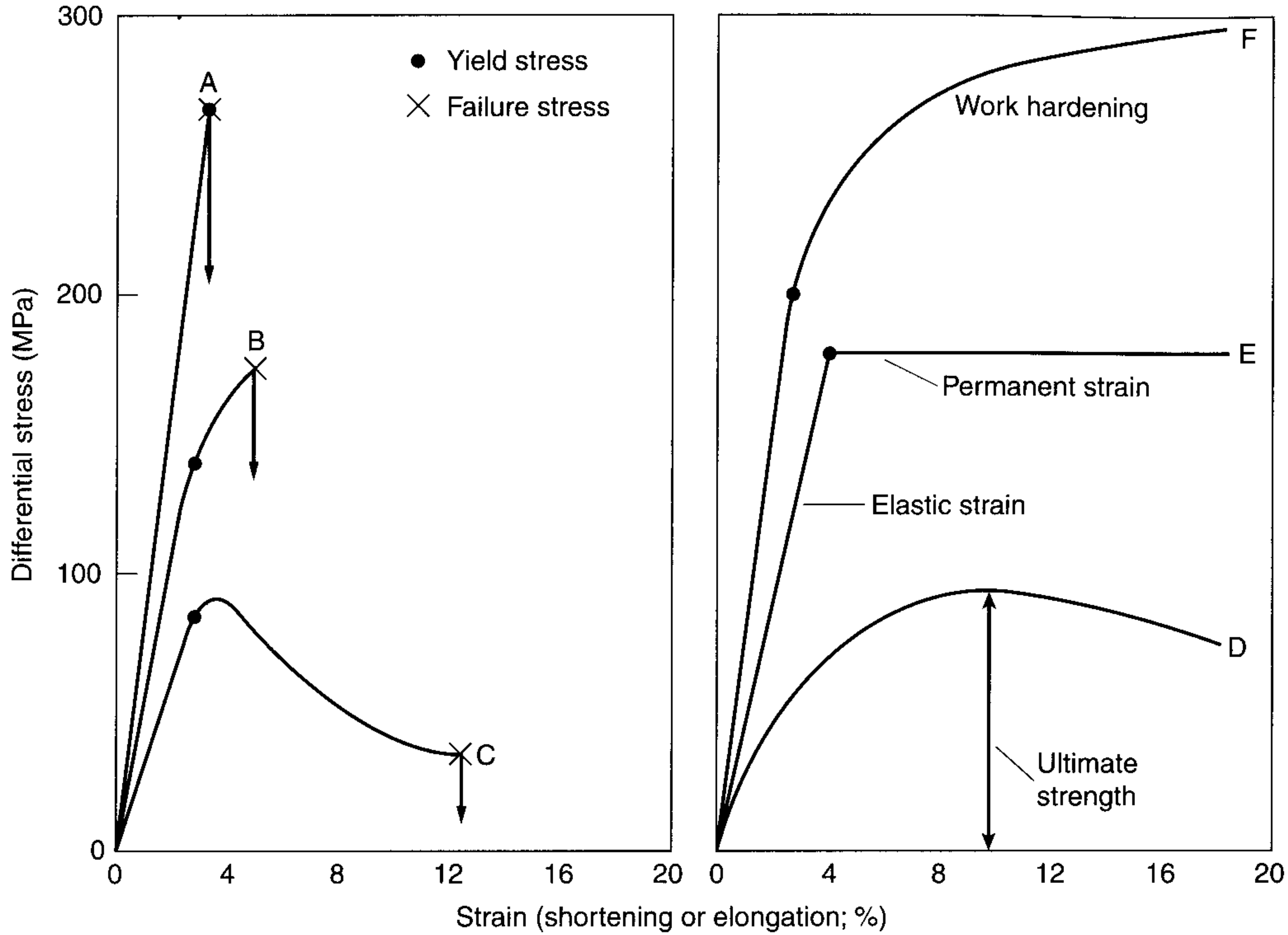
Parameters values: $E \sim 10^{10}$-10^{11} Pa, $\eta = 10^{17}$-10^{27} Pa s

Implying t_M = 11 days – 3000 million years

Material behaviour classification

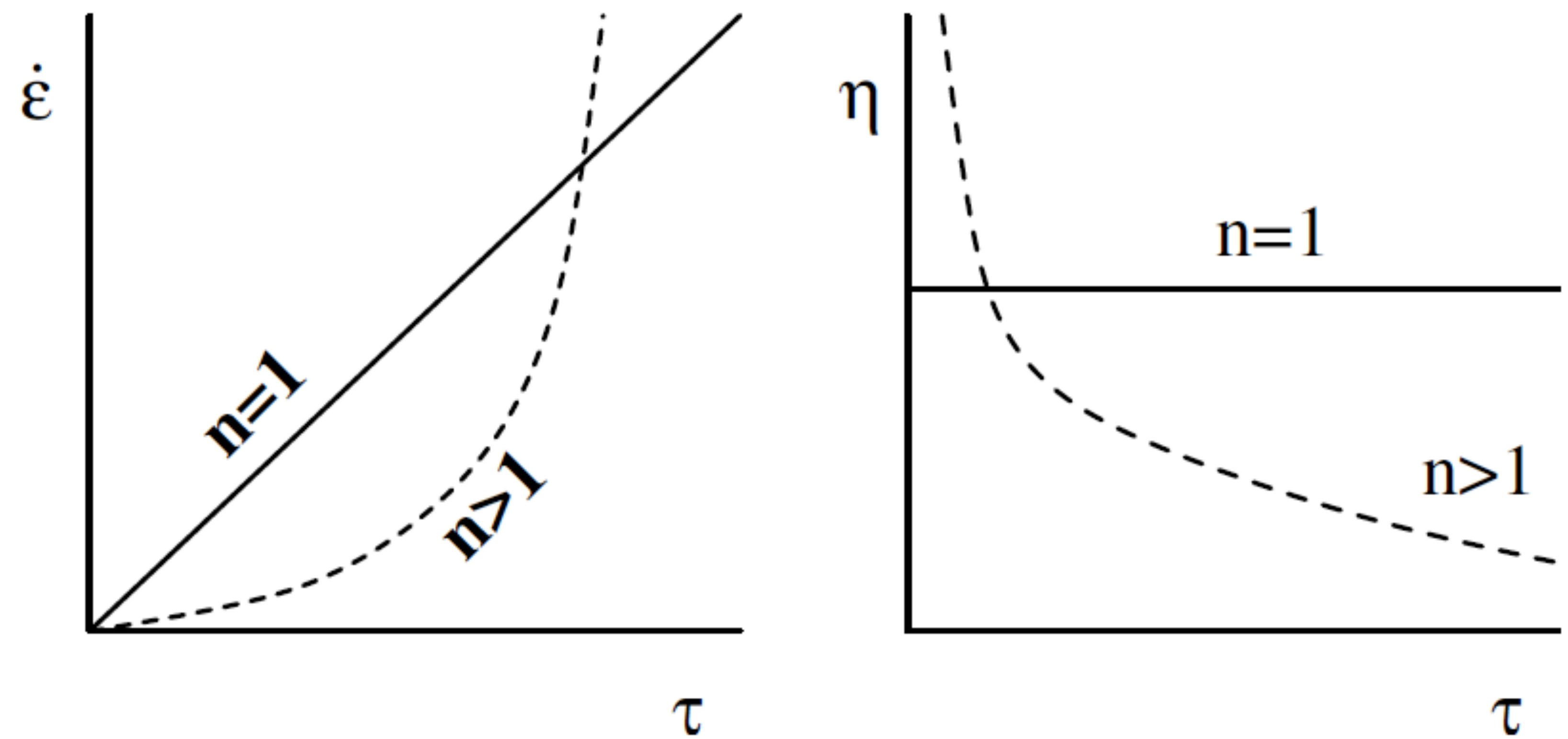
Stress and strain (strain-rate) relationships

- Linear
- Non-linear
- Ductile
- Plastic
- Brittle



Non-linear (non-Newtonian) behaviour

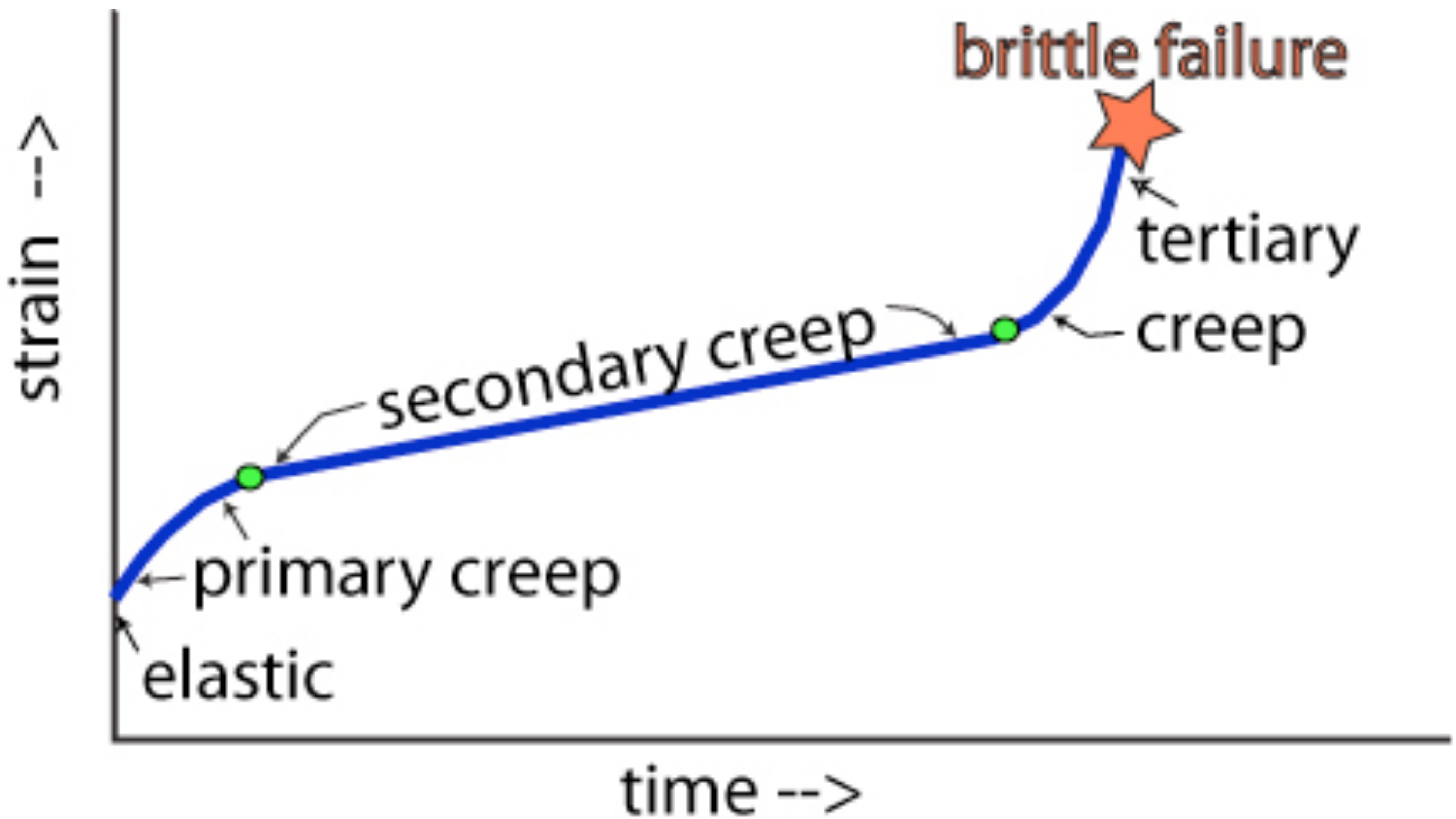
$$\dot{\epsilon} \propto \tau^n$$



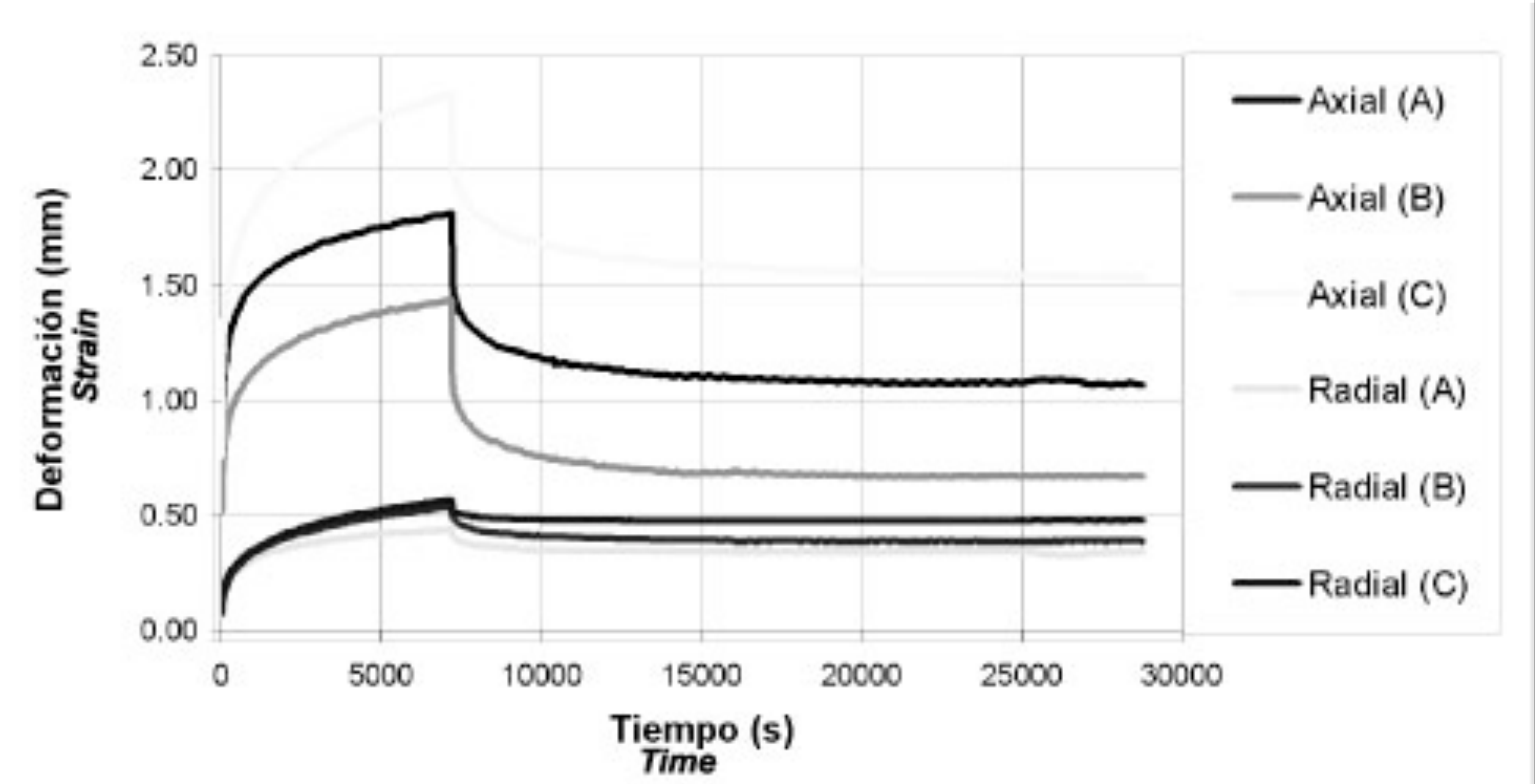
- General stress - strain-rate relation
  - *n* = 1: Newtonian
  - *n* > 1: non-Newtonian (shear thinning)
  - *n* infinite: pseudo-brittle
- Local slope: Effective viscosity
- Application: Different viscosities within the lithosphere due to different absolute plate motions

Rocks are not rheologically simple: Loading experiment

- Simple compression experiment
- Constant stress loading
- Many effects observed (viscous, elastic, brittle)



Rocks are not rheologically simple: Unloading experiment



Combined effects

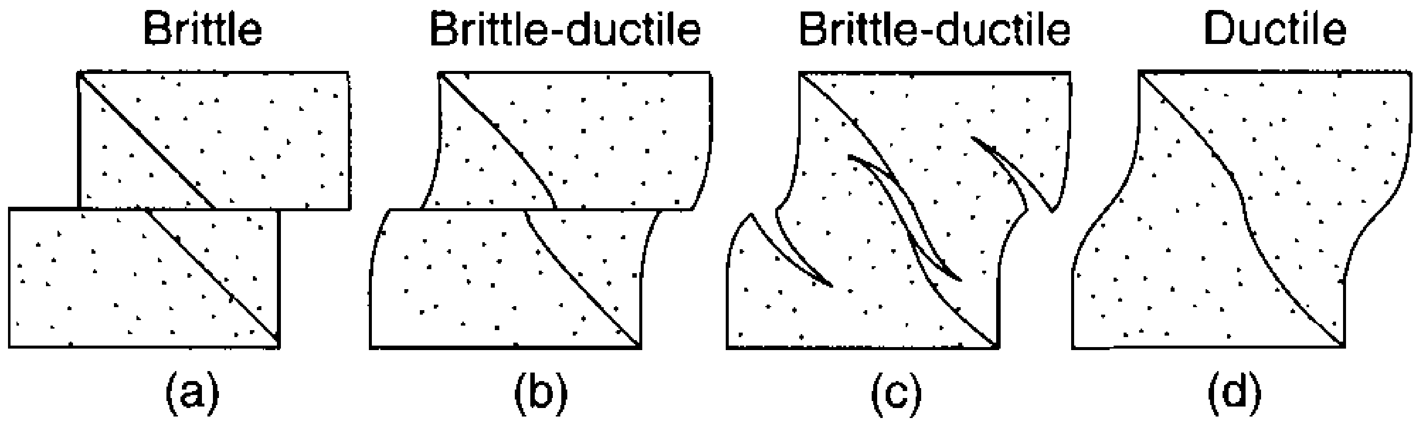
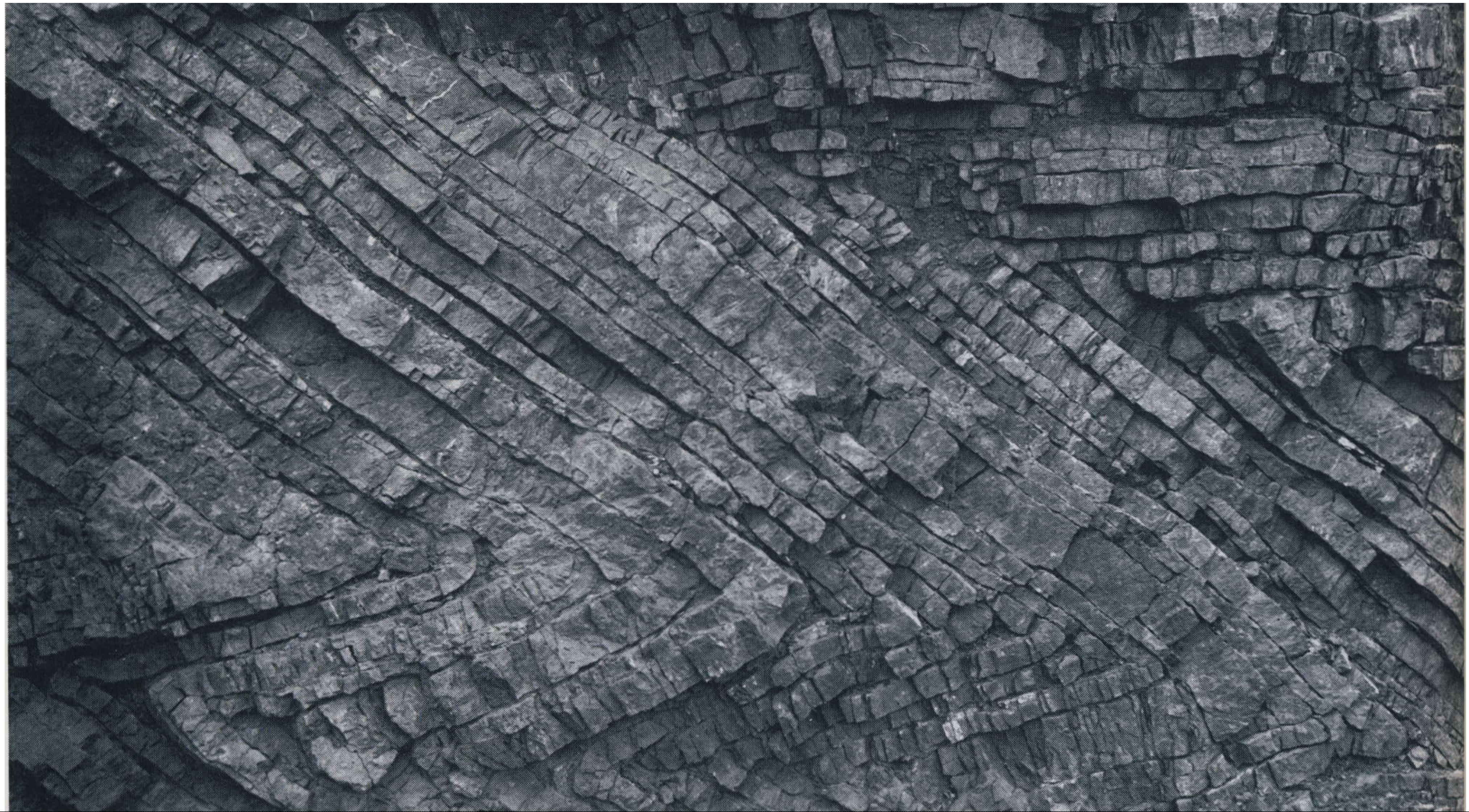


Figure 5.19 Brittle (a) to brittle-ductile (b, c) to ductile (d) deformation, reflecting the general subdivision that is used in the subsequent chapters.



Pressure-temperature effects

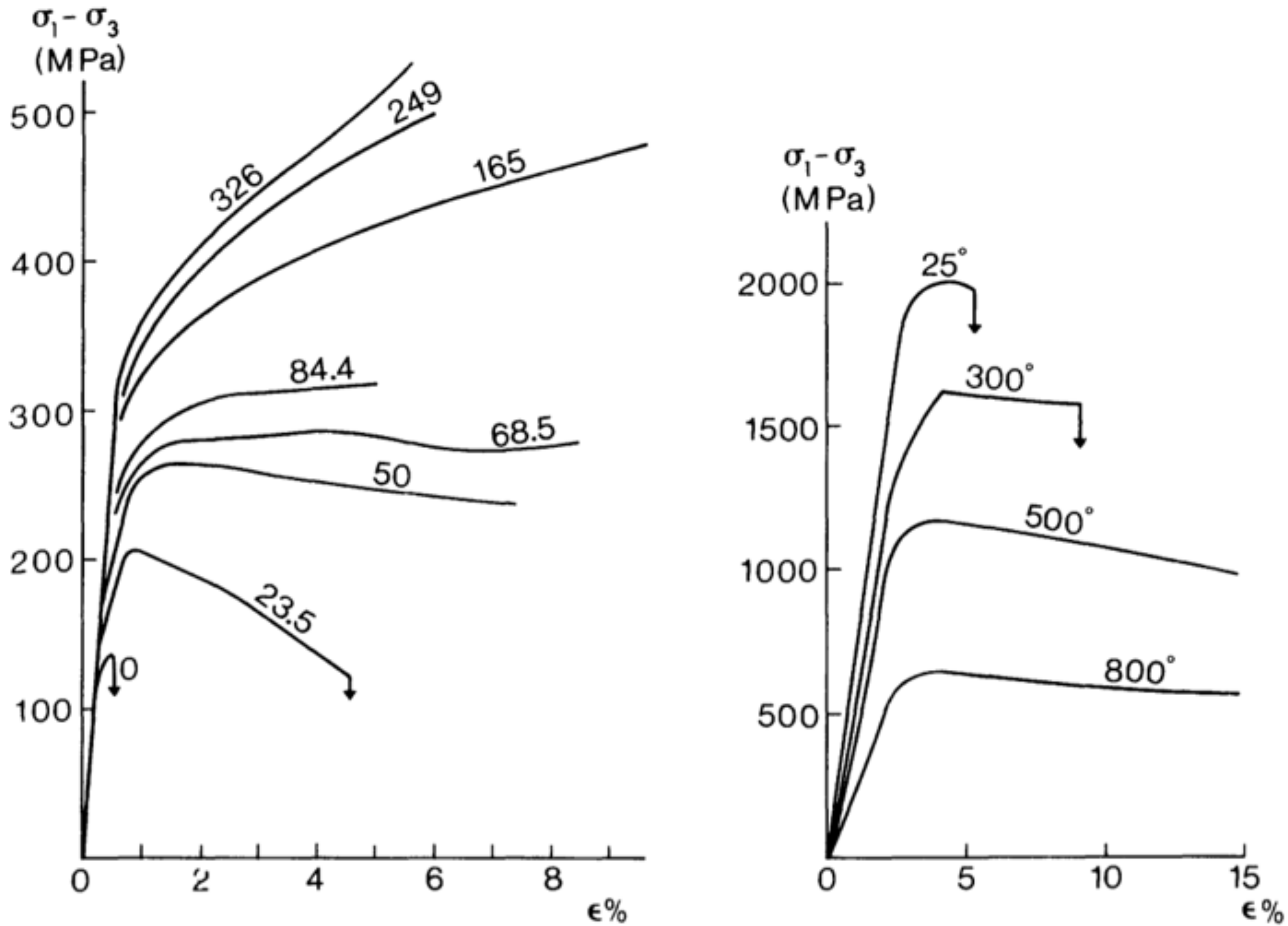


Figure 5.3 Effects of pressure (left: Carrara marble) and temperature (right: granite) on the stress–strain behaviour of rocks. Numbers on curves give confining pressure (in MPa) and temperature (in °C), respectively (from Jaeger and Cook 1979).

(Ranalli, Rheology of the Earth, 1995)

Temperature dependence

- SHIVA test
- High speed, torsion experiment
- Gabbro test sample
- Axial load of ~ 8 MPa
- Rotational velocity ot 5 m/s



Creep

- The slow, continuous deformation of a material over time
- Mechanism occurs under applied stress, due to thermally activated motion of atoms and ions associated with crystal defects
- Thermally activated diffusion process
- Viscous behaviour (strain-rate)
- Flow law

$$\dot{\epsilon} = f(\sigma, t, T, ...)$$

- Solid state creep is a *major deformation mechanism* in the Earth's crust and mantle

Creep processes

- **Diffusion creep**
- Migration of atoms through the (i) interior of the crystalline lattice (Herring-Nabarro), or (ii) along grain boundaries (Coble)

$$\dot{\epsilon} = A_{\mathrm{diff}}\, \tau$$

Function of:

* crystal grain size (*d*)
* pressure (*P*)
* temperature (*T*)

- **Dominant at low stresses**
- **Linear (Newtonian) flow law**

Creep processes

- **Dislocation creep**
- Migration of defects (dislocations) within the crystalline lattice. Dislocations may assume line or point geometries

$$\dot{\epsilon} = A_{\mathrm{disc}}\, \tau^{n}$$

Function of:
* crystal grain size (*d*)
* pressure (*P*)
* temperature (*T*)

- **Dominant at high stresses**
- **Non-linear (non-Newtonian) flow law**

Viscous creep law

- Experimental data
- The viscosity of rocks is strongly dependent on

  pressure, temperature, stress (strain-rate), grain size, water content, melt and mineralogy, ...

Arrhenius flow law

$$\dot{\epsilon} = A\,\tau^n\,d^{-p}\,C_{OH}^s\,\exp(-\alpha\phi)\,\exp\left[\frac{-(E+PV)}{RT}\right]$$

Effective viscosity

$$\eta_{\text{effective}} = \frac{\tau}{2\dot{\epsilon}}$$

Viscous creep laws typically used

$$\dot{\epsilon}_{ij} = \frac{1}{2\eta_{\mathrm{eff}}}\tau_{ij}$$

$$\eta_{\mathrm{eff}} = B\left(\tau_{II}\right)^{1-n}\,\exp\left[\frac{E+pV}{RT}\right], \qquad \tau_{II} = \sqrt{\tfrac{1}{2}\tau_{ij}\tau_{ij}}$$

- *B* depends on grain size (in the linear domain)
- *n* = 1 —> diffusion creep
- *n* > 1 —> dislocation creep
- Common simplification —> Frank-Kamenetskii approx.

$$\eta \propto \exp(-\theta T)$$

Satisfactory for a limited *p,T* range

Upper mantle

$$\dot{e} = A \left(\frac{\tau}{\mu}\right)^n \left(\frac{b}{d}\right)^m \exp\left[-\frac{(E^* + pV^*)}{RT}\right]$$

Table 5.3. ***Parameter Values for Diffusion Creep and Dislocation Creep in a Dry Upper Mantle***[a]

Quantity	Diffusion Creep	Dislocation Creep
Pre-exponential factor A (s^{-1})	8.7×10^{15}	3.5×10^{22}
Stress exponent n	1	3.5
Grain size exponent m	3	0
Activation energy E^* ($kJ\,mol^{-1}$)	300	540
Activation volume V^* ($m^3\,mol^{-1}$)	6×10^{-6}	2×10^{-5}

[a] After Karato and Wu (1993). Other relevant parameter values are $\mu_{\substack{\text{shear}\\\text{modulus}}} = 80\,\text{GPa}$, $b = 0.55\,\text{nm}$, and $R = 8.3144\,\text{J}\,\text{K}^{-1}\,\text{mol}^{-1}$.

More experimental data

Values valid for the following form of the flow law

$$\dot{\epsilon} = A\,\tau^n \,\exp\left[-\frac{E}{RT}\right]$$

Table 5.6 Experimentally Derived Creep Parameters for Some Common Rock Types

Rock type	**$^{10}\log A$ ($MPa^{-n}s^{-1}$)**	**n**	**E* ($kJ \cdot mol^{-1}$)**
Albite rock	18	3.9	234
Anorthosite	16	3.2	238
Clinopyroxenite	17	2.6	335
Clinopyroxenite (wet)	5.17	3.3	490
Diabase	17	3.4	260
Granite	6.4	3.4	139
Granite (wet)	7.7	1.9	137
Marble	33.2	4.2	427
Olivine rock	4.5	3.6	535
Olivine rock (wet)	4.0	3.4	444
Quartz diorite	11.5	2.4	219
Quartzite	10.4	2.8	184
Quartzite (wet)	10.8	2.6	134
Rock salt	–1.59	5.0	82

Source: Kirby and Kronenberg (1987).

Rock and mineral aggregates

- Diffusion creep and dislocation creep mechanisms are not independent -
- Both simultaneously occur at a given stress state
- Composite rheology

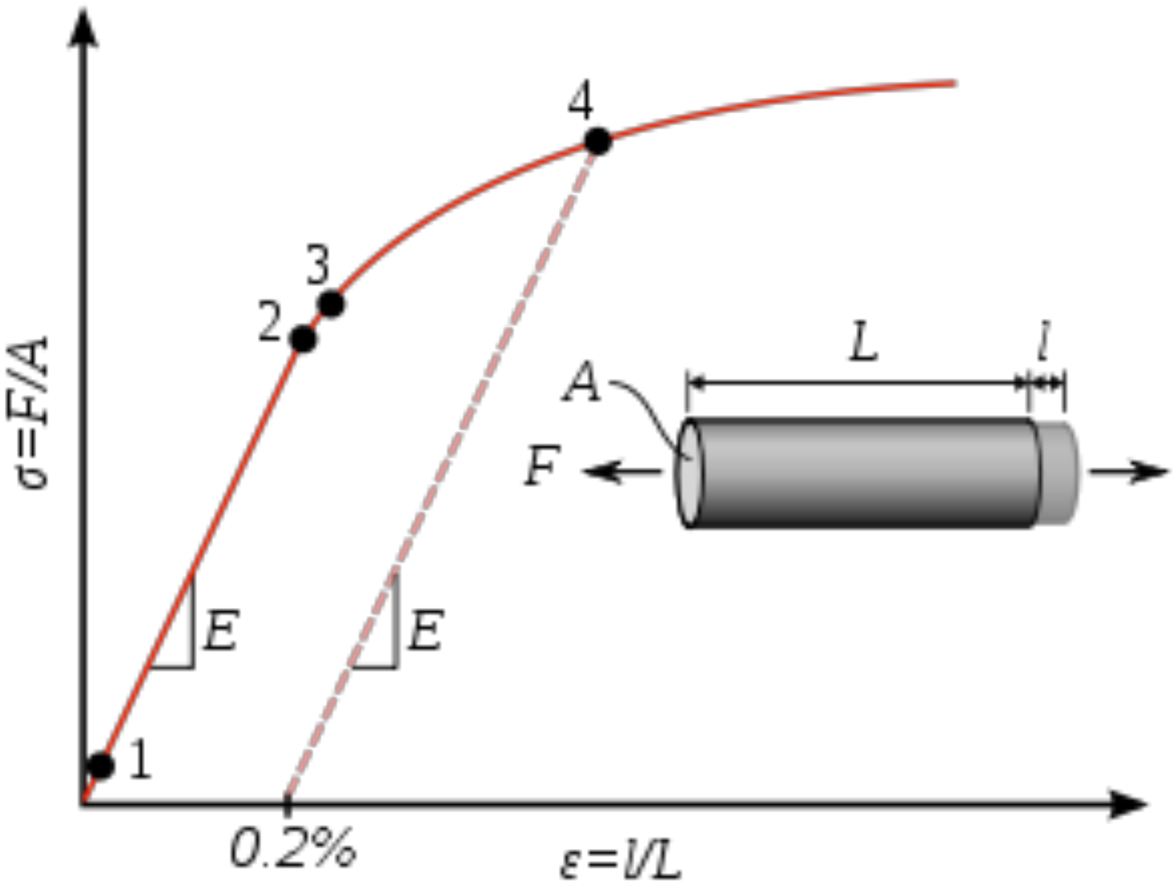
Effective composite viscosity

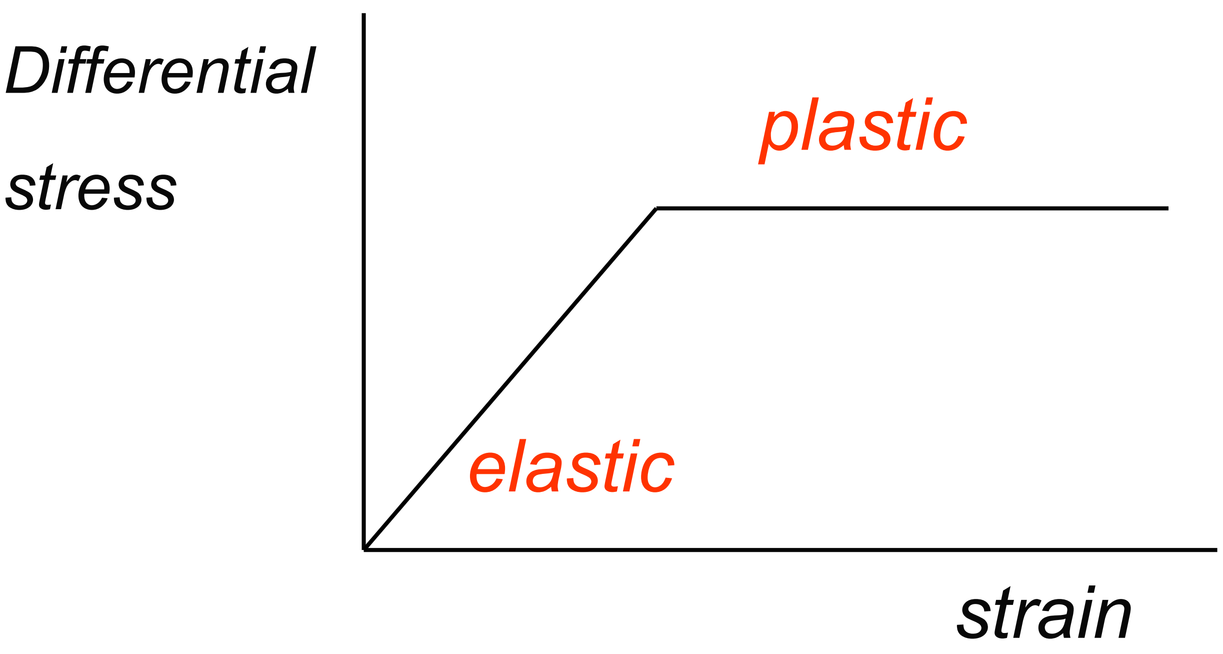
$$\frac{1}{\eta_{\mathrm{eff}}} = \frac{1}{\eta_{\mathrm{diff}}} + \frac{1}{\eta_{\mathrm{disc}}}$$

under strain rate decomposition assumption (Maxwell like)

$$\dot{\epsilon} = \dot{\epsilon}_{\mathrm{diff}} + \dot{\epsilon}_{\mathrm{disc}}$$

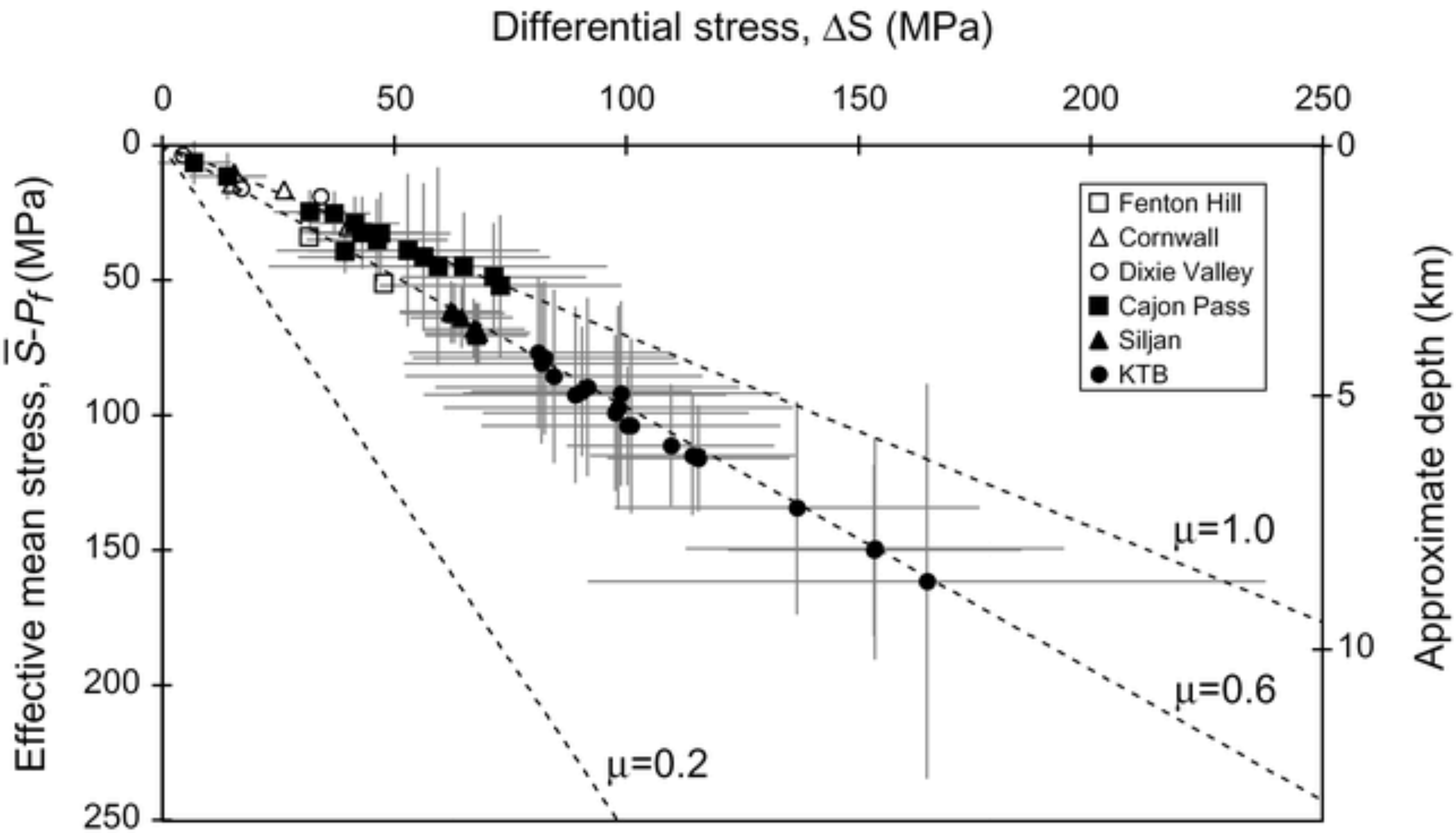
Rocks have a finite strength





- The differential stress ($\sigma_1 - \sigma_3$) is limited in nature
- Caused by micro-defects, breaking bonds between atoms, growth of micro cracks

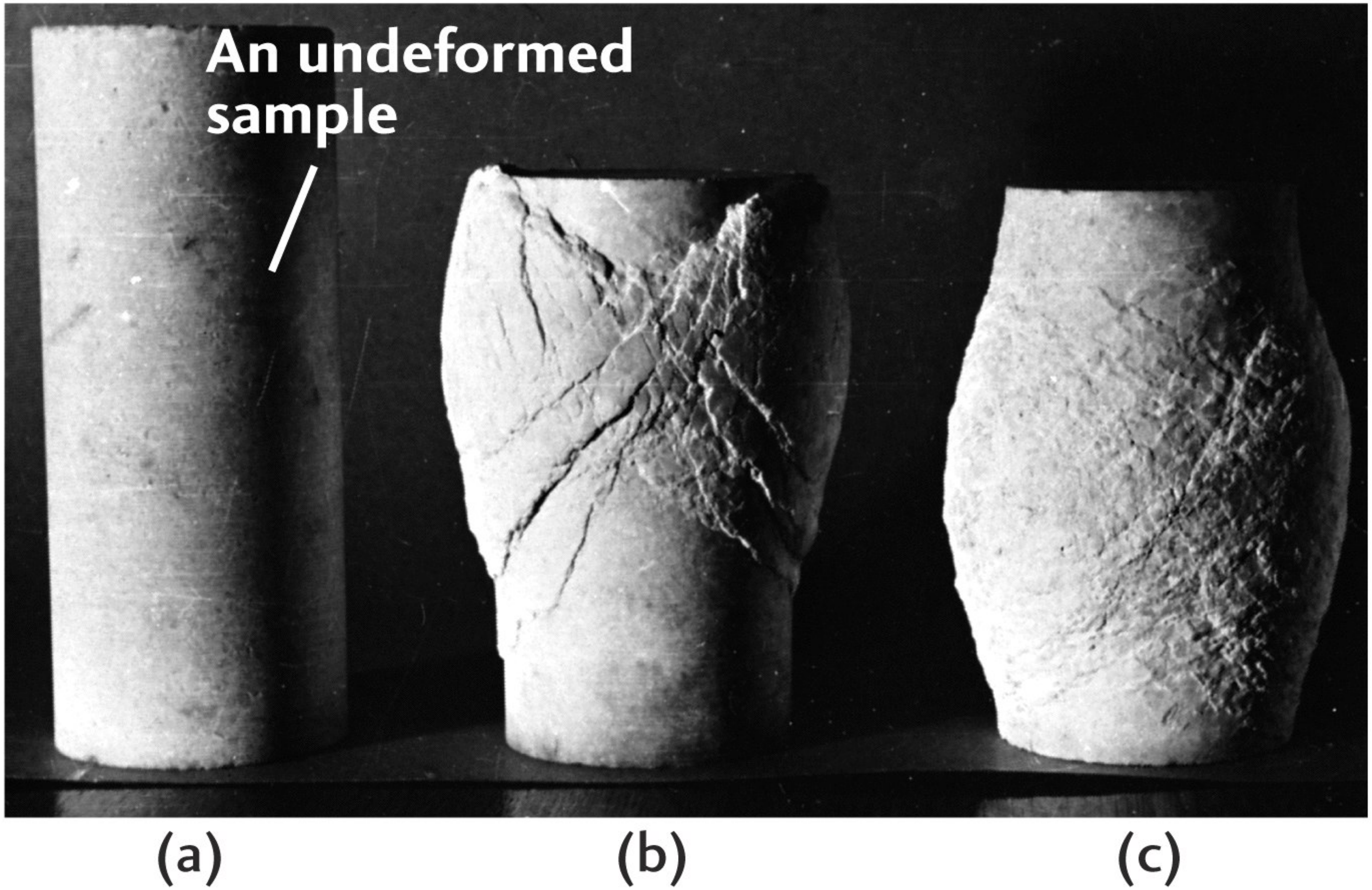
Differential stresses in the crust



$$p \propto (\sigma_1 - \sigma_3)$$

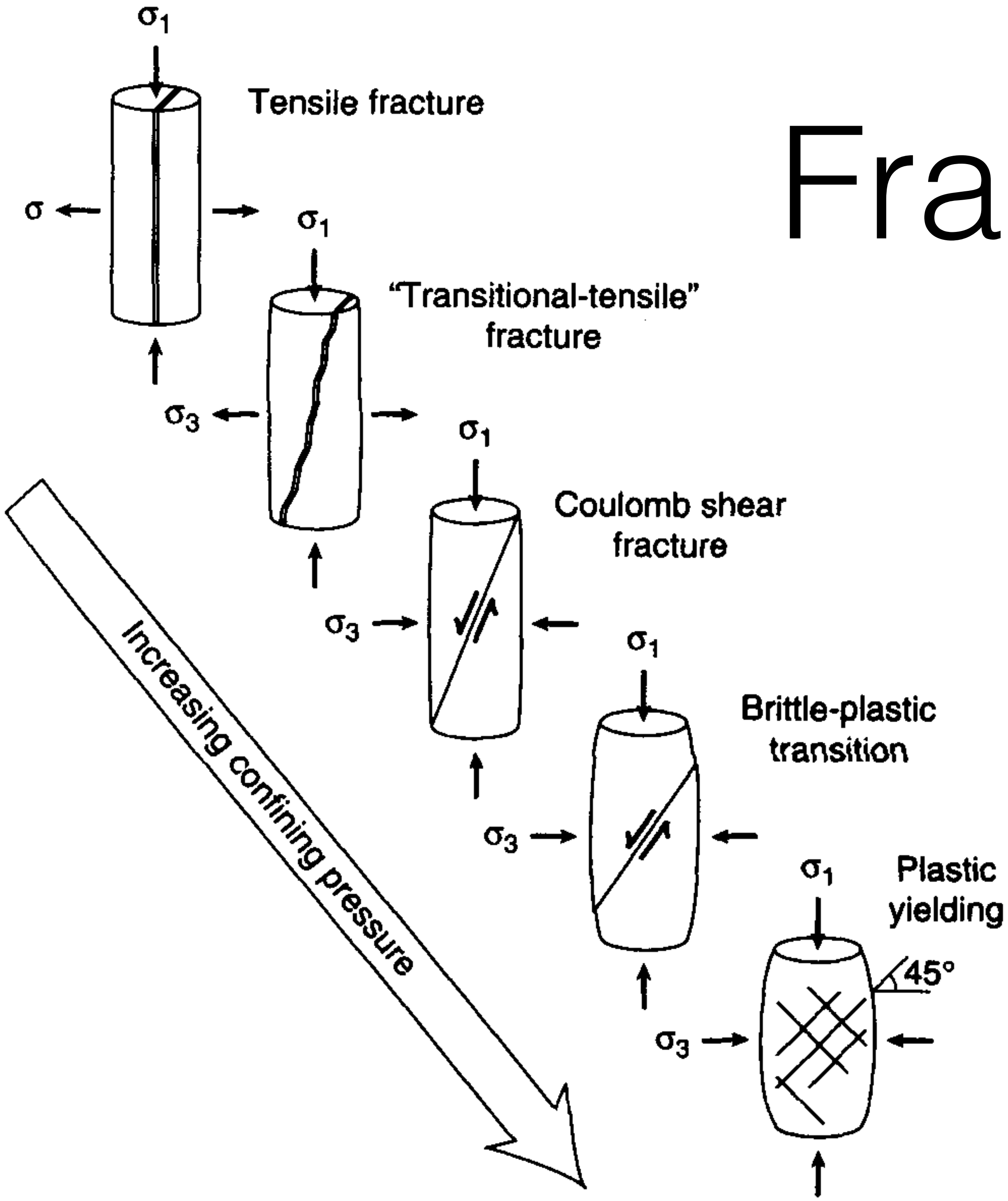
(Townend & Zoback Geology 2000)

Plastic vs. viscous deformation



brittle/plastic viscous

Fracture style as a function of confining pressure



(b)

Figure 6.25 (a) A representative composite failure envelope on a Mohr diagram. The different parts of the envelope are labeled, and are discussed in the text. (b) Sketches of the fracture geometry that forms during failure. Note that the geometry depends on the part of the failure envelope that represents failure conditions, because the slope of the envelope is not constant.

Coulomb failure criteria

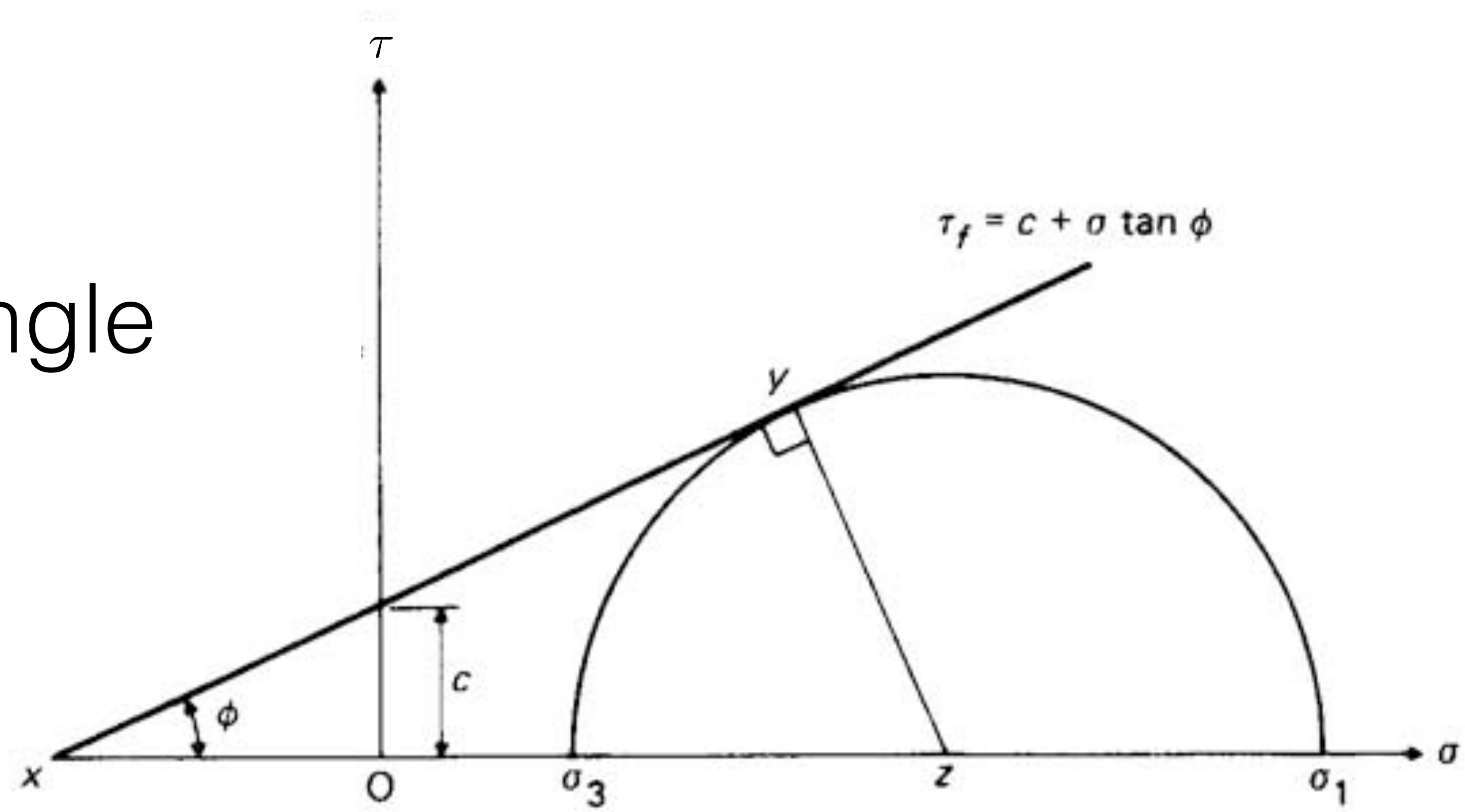
$$\tau = C + \sigma \tan(\phi)$$

friction angle

normal stress

cohesion

max shear stress



$$\sigma = \sigma_m - \tau_m \sin(\phi) \qquad \tau_m = \frac{1}{2}(\sigma_1 - \sigma_3)$$

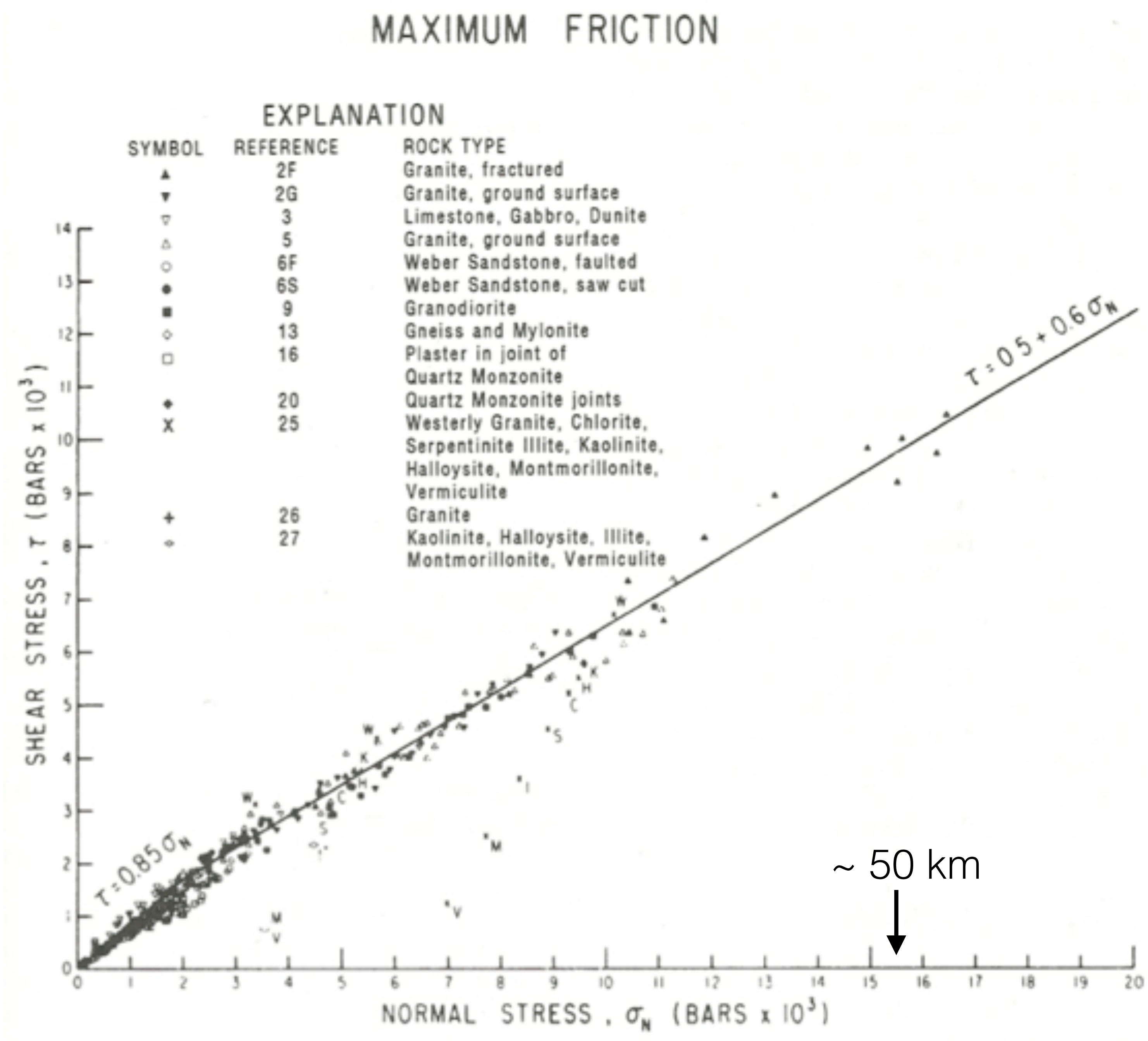
$$\tau = \tau_m \sin(\phi) \qquad \sigma_m = \frac{1}{2}(\sigma_1 + \sigma_3)$$

$$\tau_m = C\cos(\phi) + \sigma_m \sin(\phi)$$

$$\rightarrow \sigma_1 - \sigma_3 = C' + p\mu'$$

Byerlee’s law

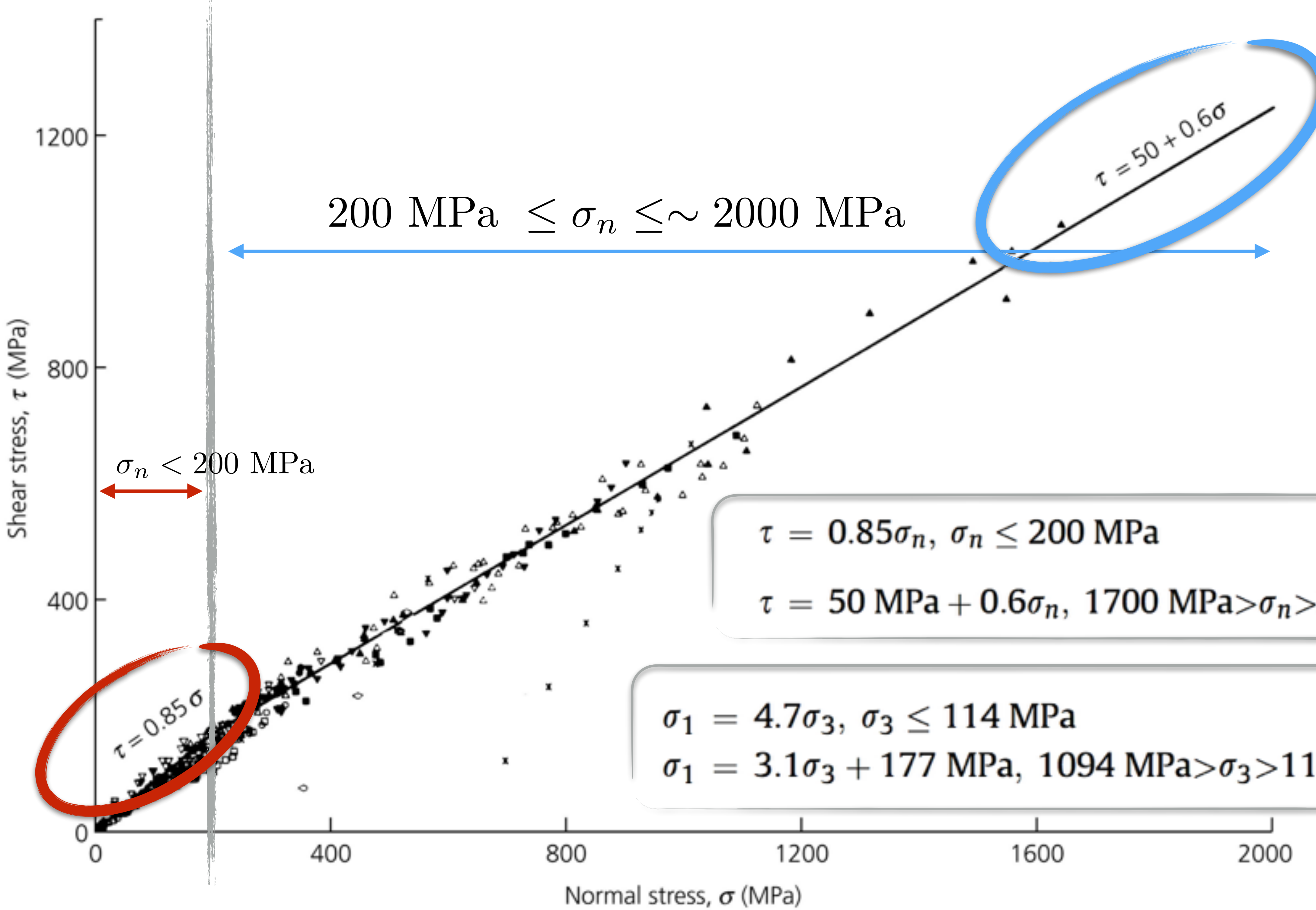
- Coulomb plasticity is empirical theory, however seems to work reasonably well for upper crustal rocks



Byerlee's law

http://geophysics.eas.gatech.edu/people/anewman/classes/Geodynamics/misc/5_7_10.jpg

Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.



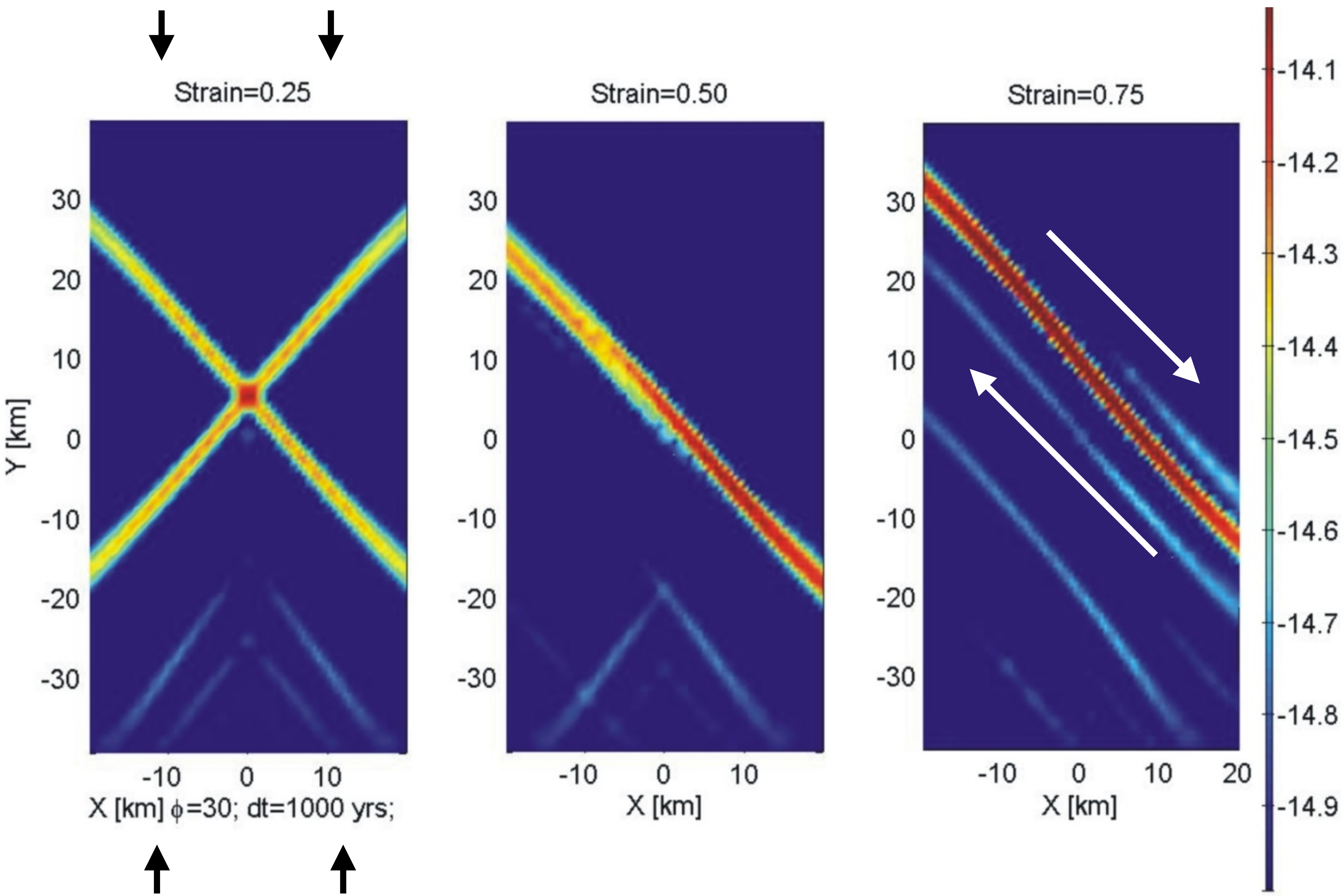
$$\tau = 0.85\sigma_n, \ \sigma_n \leq 200 \text{ MPa}$$

$$\tau = 50 \text{ MPa} + 0.6\sigma_n, \ 1700 \text{ MPa} > \sigma_n > 200 \text{ MPa}$$

$$\sigma_1 = 4.7\sigma_3, \ \sigma_3 \leq 114 \text{ MPa}$$

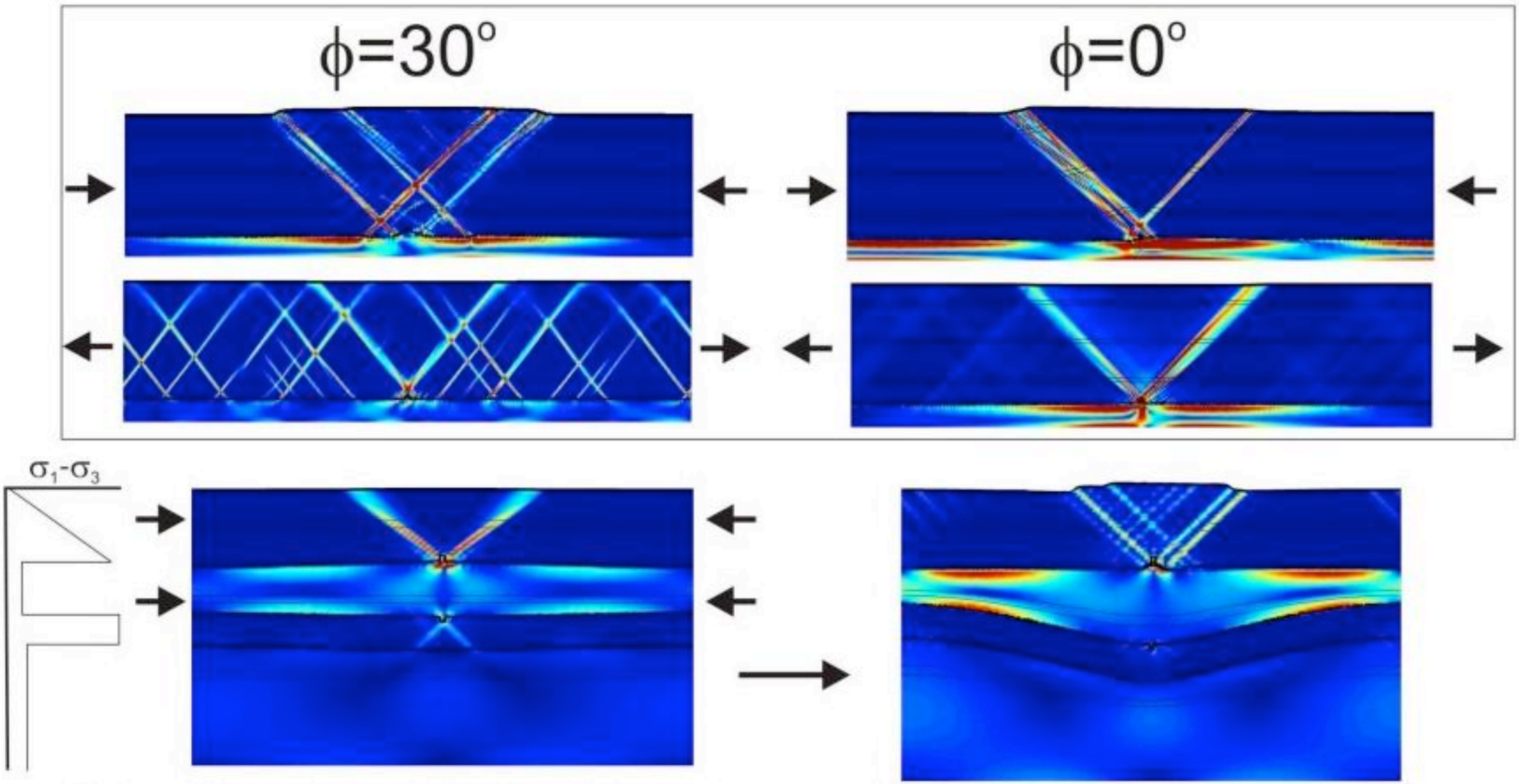
$$\sigma_1 = 3.1\sigma_3 + 177 \text{ MPa}, \ 1094 \text{ MPa} > \sigma_3 > 114 \text{ MPa}$$

Numerical models of (brittle) localisation



Numerical models of (brittle) localisation

- Brittle failure in the upper crust may result in localised zones of deformation due to Mohr-Coulomb plasticity (which mimics Byerlee's law)



Peierls creep and strength of rocks

(Kameyama et al, 2001)

- Problem:
  - Byerlee is valid for upper crustal rocks. At deeper levels, the mechanism limiting stress is not very clear
- Low temperature plasticity (Peierls creep) has been suggested based on; experiments, numerical calculations and theoretical considerations
- Consequences
  - This plasticity form does not produce localised faults as easy, requires shear-heating feedbacks to “break” the lithosphere
  - Possible explanation of the small number of earth quakes in the lithospheric mantle compared to upper crustal rocks?

Deformation maps

- Question: Is diffusion creep or dislocation creep the dominant deformation mechanism in the upper mantle?

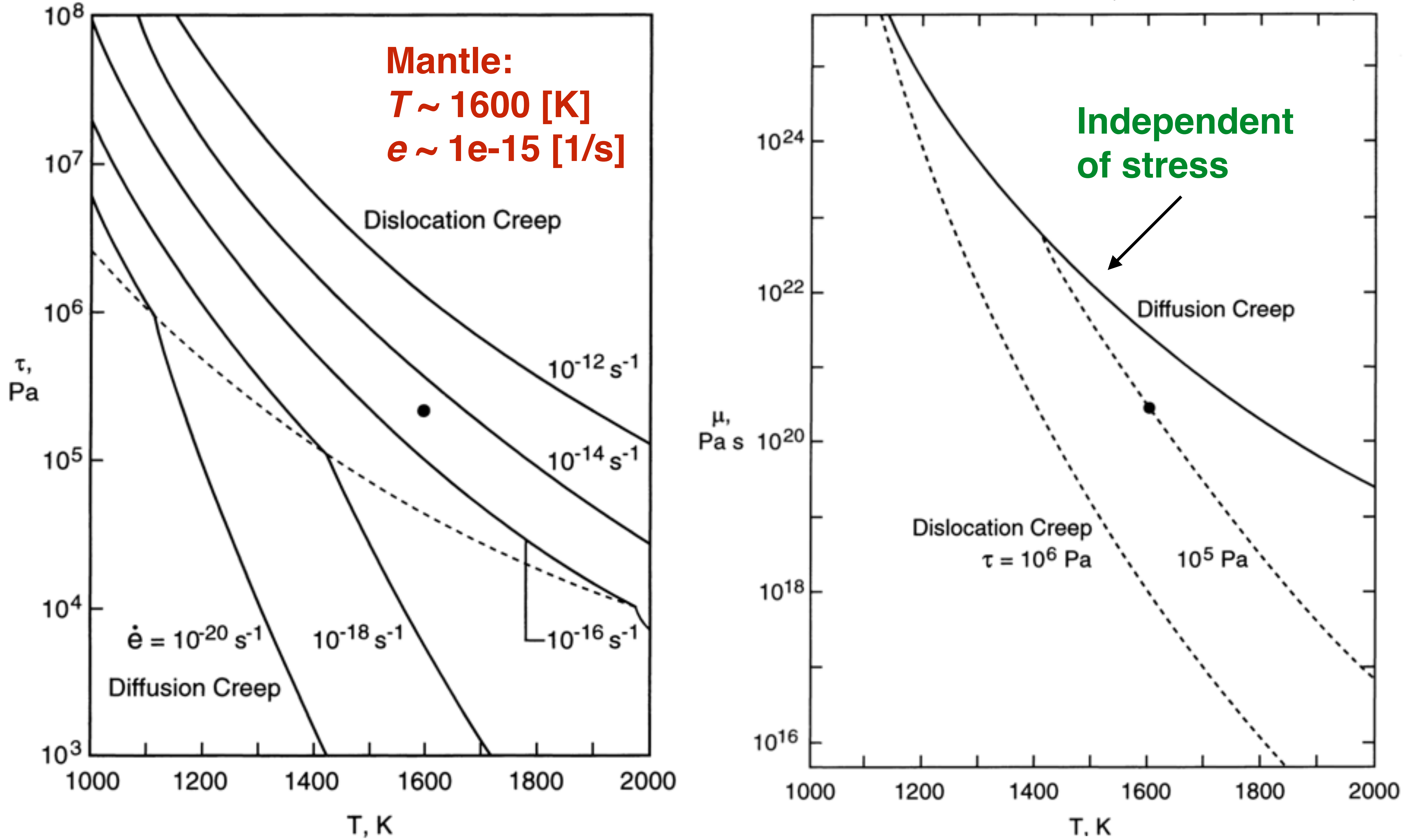
- Considerations:

1. For a given **stress** - the mechanism with the **largest** strain-rate is dominant

2. For a given **strain-rate** - the mechanism with the **lowest** stress is dominant

Mantle deformation maps

(Schubert et al, 2001)



Dry upper mantle, $p = 0$

$$\dot{e} = A\left(\frac{\tau}{\mu}\right)^n\left(\frac{b}{d}\right)^m \exp\left[-\frac{(E^* + pV^*)}{RT}\right]$$

Olivine deformation map

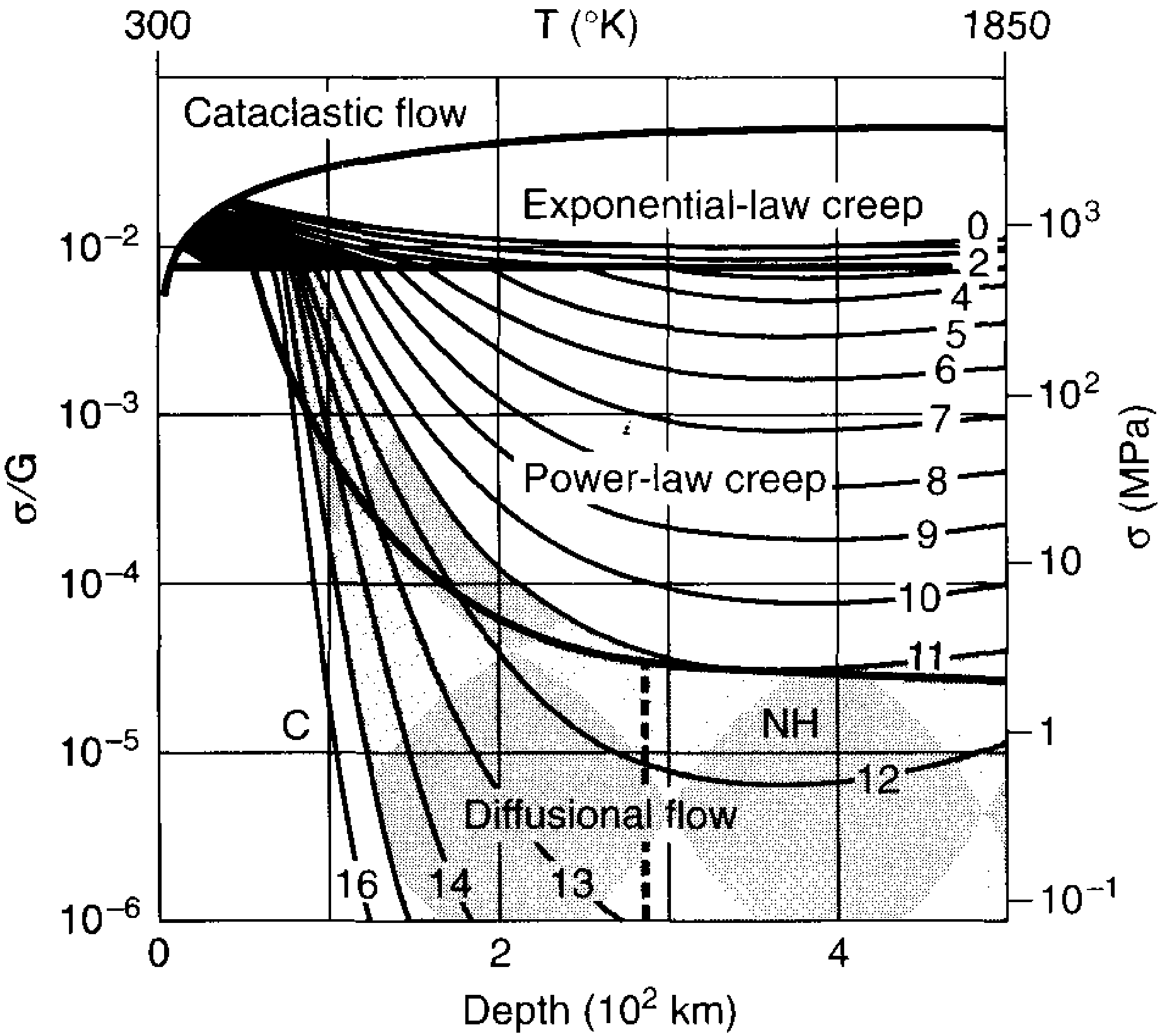


Figure 9.35 Deformation mechanism map for olivine with a grain size of 100 μm. Variables are the same as in Figure 9.32, except that depth is substituted for temperature given an exponentially decreasing geothermal gradient with 300°C at the surface and 1850°C at 500 km depth.

Viscous deformation map

(Kameyama et al, 2001)

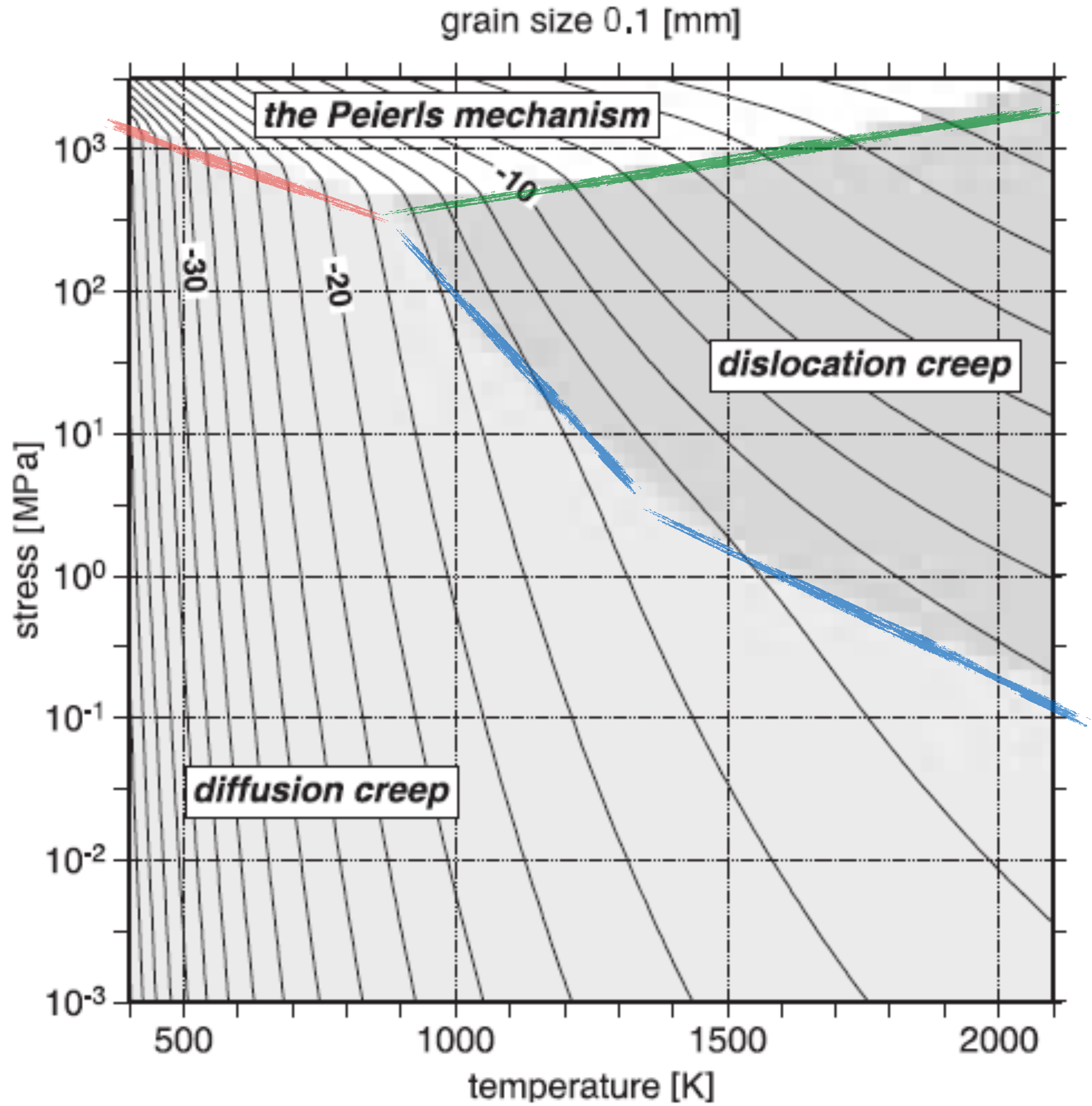
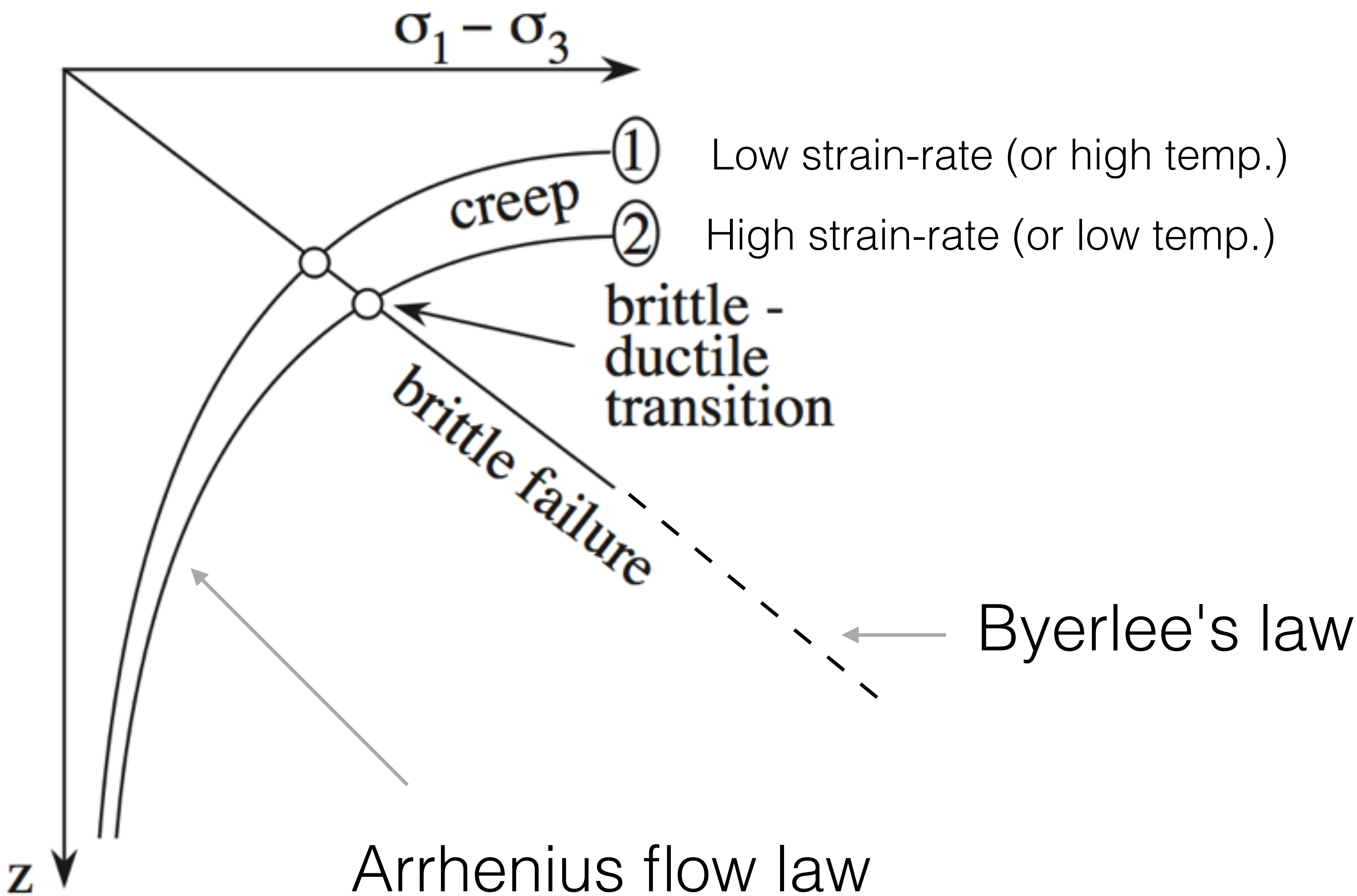


Fig. 1. Deformation mechanism map calculated for grain size $a = 0.1$ mm. The lightly shaded area indicates that deformation mainly occurs by diffusion creep. The densely shaded area indicates that deformation mainly occurs by power-law creep. The white region indicates that deformation mainly occurs by the Peierls mechanism. The solid curves are lines of constant strain rate. The numbers attached to each contour indicate the logarithm of the strain rate in the unit of s^{-1}.

Strength envelopes



Strength of the mantle-lithosphere

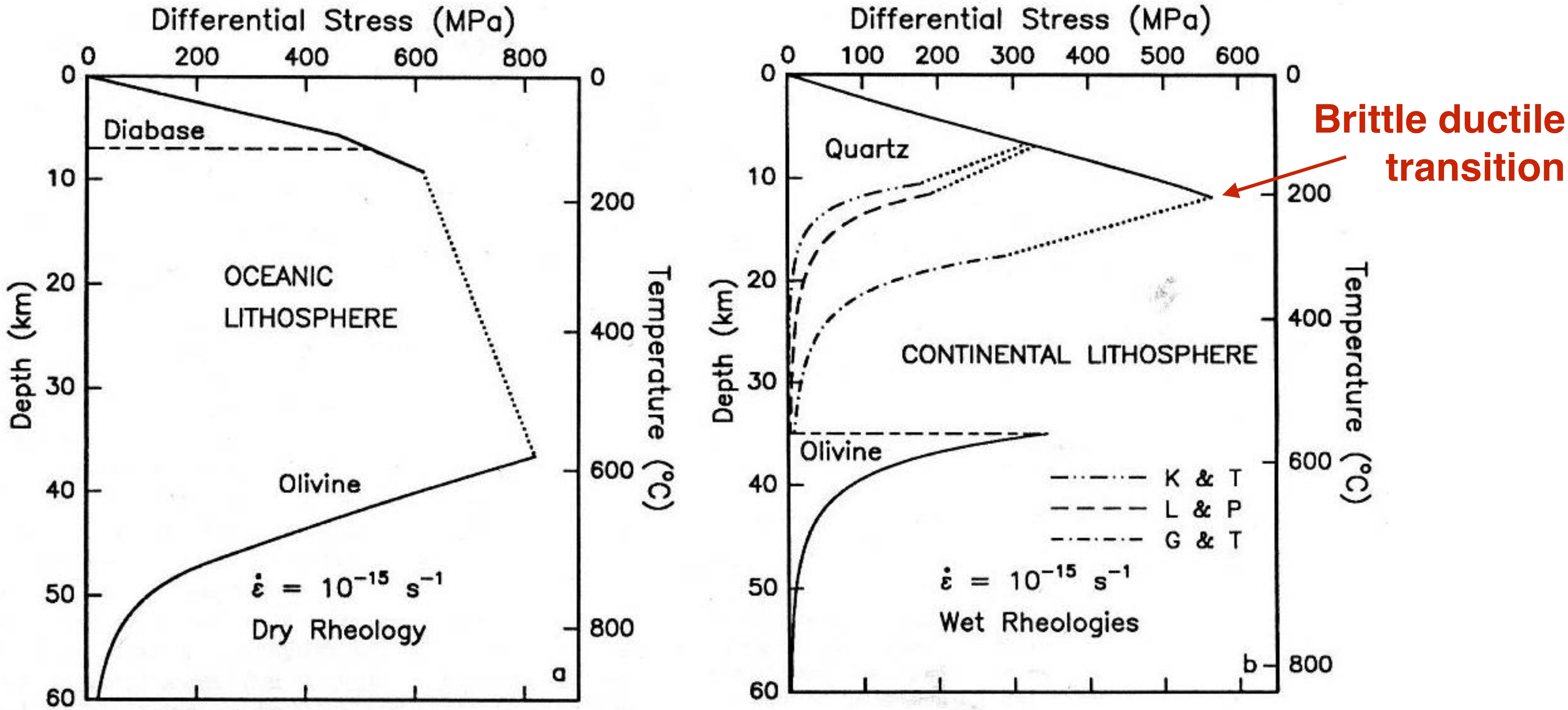
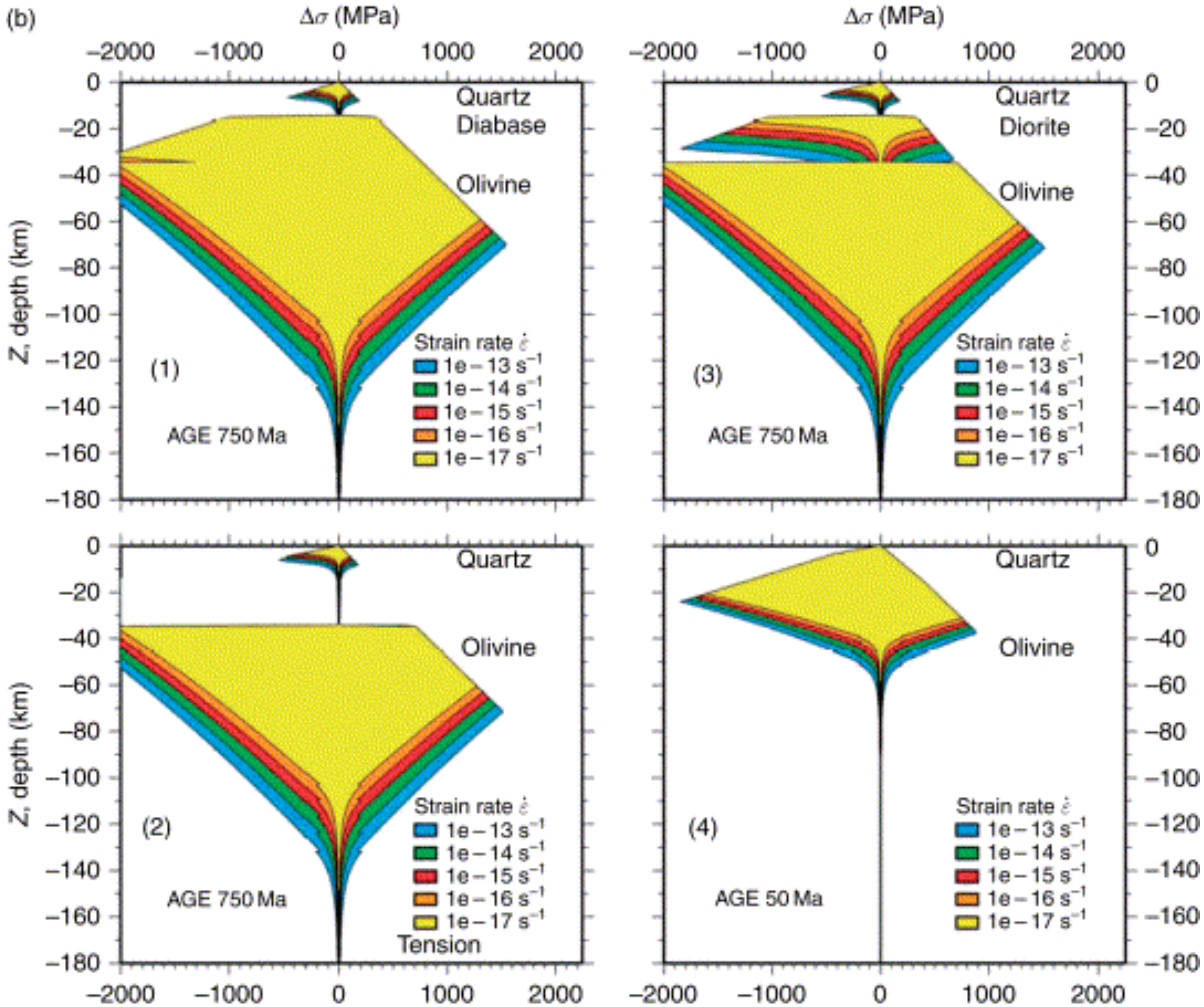
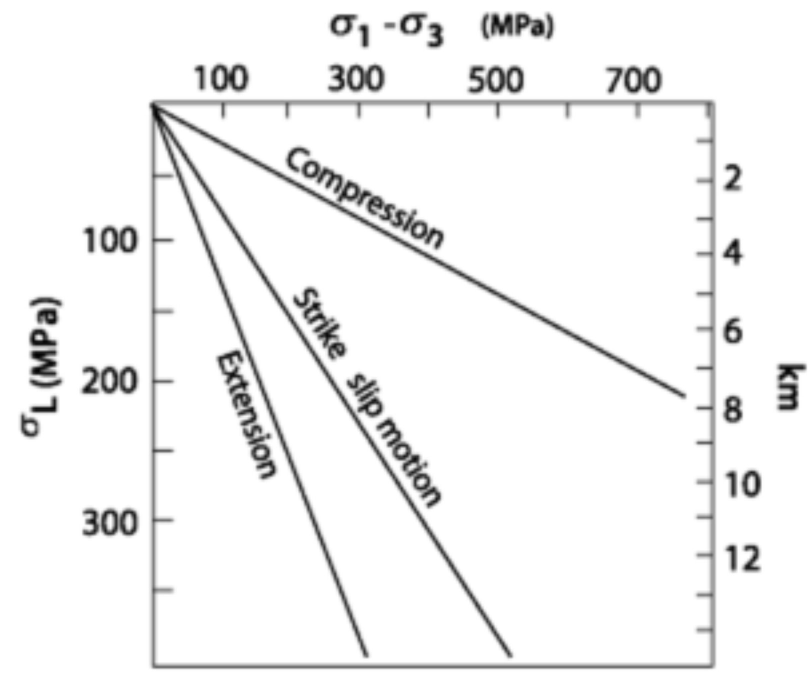


Figure 9. Strength envelopes for oceanic and continental lithosphere. (a) For the oceanic lithosphere, a geotherm for 60-m.y.-old lithosphere was used [e.g., *Turcotte and Schubert*, 1982 pp. 163-167]. A rheology for dry olivine [*Chopra and Paterson*, 1984] was used because water strongly partitions into the melt during partial melting. (b) For the continental lithosphere, a geotherm for a surface heat flow of 60 mW m^{-1} was employed [*Chapman*, 1986]. The rheologies for wet quartzite are those used in Figure 5; the olivine rheology is for wet Anita Bay dunite from *Chopra and Paterson* [1984]. Wet rheologies were used, consistent with high fluid pressures in fault zones. Plastic flow strength was corrected for water fugacity using a water fugacity exponent of unity and assuming lithostatic pore pressure. The BDT and BPT, determined as described in the text, have been connected by a dotted line.

(Kohlstedt et al., Strength curves for different materials: lithosphere, 1995)

Compression versus extension





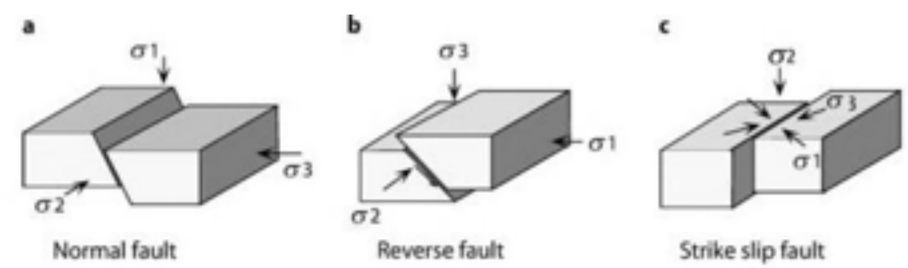
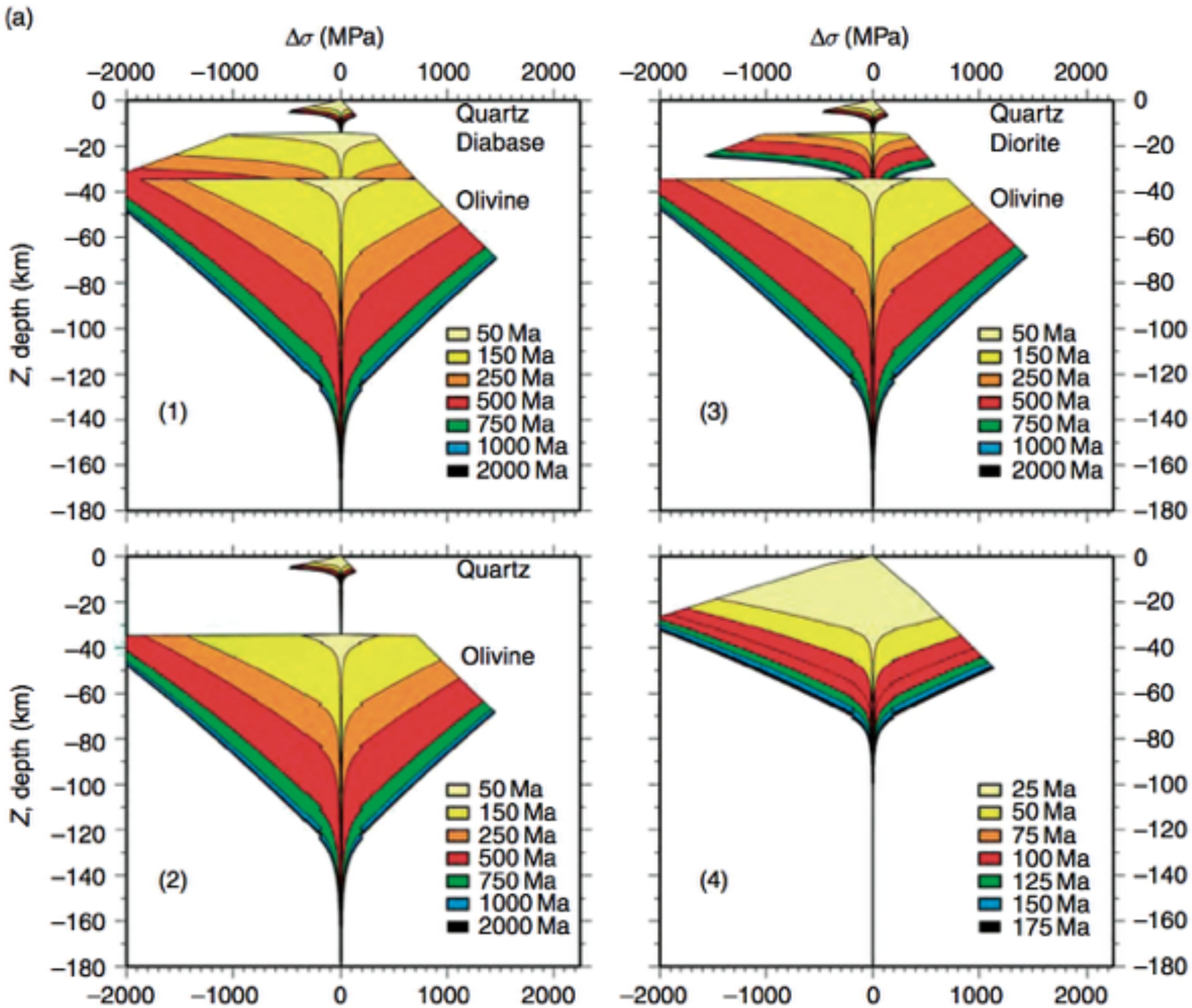


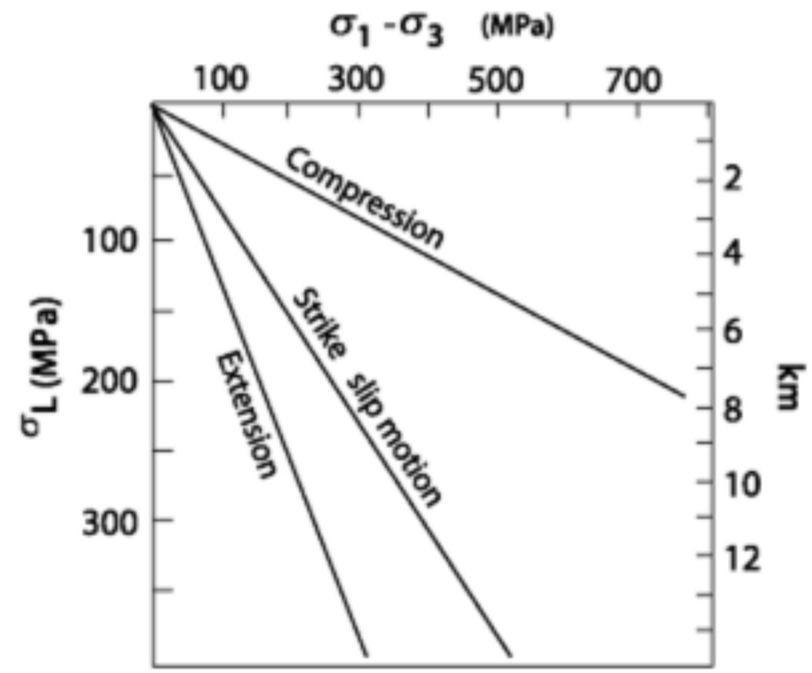
Figure 5.8. Geometry of a brittle fault during: **a** extension, **b** compression and **c** strike slip motion

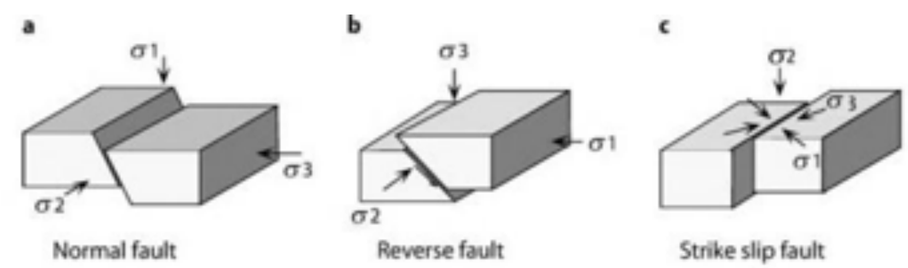
Difference come from the dependence of Byerlee's law on the normal stress
Compression results in large normal stress (tectonic loading)

(Burov E., Treatise on Geophysics V. 6, 2007)

Compression versus extension







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Summary

- Learned the vocabulary of rheology
- Examined three basic classes
  - elastic
  - viscous
  - plastic / brittle
- Deformation maps and strength envelops