Rheology of the Earth

(651-4008-00 G)
Schedule

- Rheology basics
  - Viscous, elastic and plastic
- Creep processes
- Flow laws
- Yielding mechanisms
- Deformation maps
- Yield strength envelopes
- Constraints on the rheology from the laboratory, geology, geophysics and numerical modelling (next time)
Rheology: What is it?

- The branch of science concerned with how material “flows”
- More precisely, “the response of a material to deformation (e.g. applied strain, or strain-rate, or stress)”

- Some examples
Rheology of Fluids
... steel ...
... concrete ...
... visco-elastic fluids ...
Strain

Tensile strain - or - direct strain
\[ \epsilon_{xx}, \epsilon_{yy} \]

Shear strain
\[ \epsilon_{xy} \]

Volumetric strain (dilatation)

original shape
dearformation
\textit{due to displacement}
Strain-rate

• Analogous to the strain tensor, but involves gradients of velocity (and not displacement)

\[ \ddot{\epsilon}_{xx}, \ddot{\epsilon}_{yy}, \ddot{\epsilon}_{xy} \ [1/s] \]

\[ \mathbf{x} = (x, y) \] position

\[ \mathbf{v} = (v_x, v_y) \] velocity

\[ \ddot{\epsilon}_{xx} = \frac{\partial v_x}{\partial x} \]

\[ \ddot{\epsilon}_{yy} = \frac{\partial v_y}{\partial y} \]

\[ \ddot{\epsilon}_{xy} = \frac{1}{2} \left( \frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x} \right) \]

\[ \ddot{\epsilon}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \]

• Strain-rate invariant: a scalar measure of the magnitude of a tensorial quantity

\[ \ddot{\epsilon}_{II} = \sqrt{\frac{1}{2} \ddot{\epsilon}_{ij}^2} \]
World strain-rate map

Kreemer et al. (2003)
GSRM v.1.2

Second Invariant Strain Rate, $1 \times 10^{-9} \text{yr}^{-1}$
Three basic rheologies

• Elastic

• Viscous

• Plastic
0D elastic and viscous media

Elastic

\[ \sigma = E\varepsilon \]

Viscous

\[ \sigma = 2\eta\dot{\varepsilon} \]
### Characteristic values

<table>
<thead>
<tr>
<th>Material</th>
<th>Viscosity (Pa · s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>$10^{-5}$</td>
</tr>
<tr>
<td>Water</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>Olive oil</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>Honey</td>
<td>4</td>
</tr>
<tr>
<td>Glycerin</td>
<td>83</td>
</tr>
<tr>
<td>Lava</td>
<td>$10^{-10^{-4}}$</td>
</tr>
<tr>
<td>Asphalt</td>
<td>$10^{5}$</td>
</tr>
<tr>
<td>Pitch</td>
<td>$10^{9}$</td>
</tr>
<tr>
<td>Ice</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>Rock salt</td>
<td>$10^{17}$</td>
</tr>
<tr>
<td>Sandstone slab</td>
<td>$10^{18}$</td>
</tr>
<tr>
<td>Asthenosphere (upper mantle)</td>
<td>$10^{20}$</td>
</tr>
<tr>
<td>Lower mantle</td>
<td>$10^{21}$</td>
</tr>
</tbody>
</table>

**Sources**: Several sources, including Turcotte and Schubert (1982).
Visco-elasticity

**Visco-elastic**

**Kelvin-Voigt body**

\[ \sigma = E \varepsilon + 2\eta \dot{\varepsilon} \]

**Elastico-viscous**

**Maxwell body**

\[ \dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{2\eta} \]
Visco-elasticity

A) Maxwell model \[ \dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{2\eta} \]

B) Voigt model \[ \sigma = E\varepsilon + 2\eta \dot{\varepsilon} \]

Stress relaxation

<table>
<thead>
<tr>
<th>Stress</th>
<th>Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Graph" /></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
</tbody>
</table>

Creep

<table>
<thead>
<tr>
<th>Stress</th>
<th>Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image3" alt="Graph" /></td>
<td><img src="image4" alt="Graph" /></td>
</tr>
</tbody>
</table>

Time
Maxwell relaxation time

Maxwell relaxation time: \( t_m = \frac{2\eta}{E} \)

Time required for initial stress to reduce by \( \frac{1}{e} \)

Parameters values: \( E \sim 10^{10} - 10^{11} \text{ Pa}, \ \eta = 10^{17} - 10^{27} \text{ Pa s} \)

Implying \( t_M = 11 \text{ days} - 3000 \text{ million years} \)
Material behaviour classification

Stress and strain (strain-rate) relationships

- Linear
- Non-linear
- Ductile
- Plastic
- Brittle

![Stress-strain diagram]

- **A** Yield stress
- **B** Failure stress
- **C** Permanent strain
- **D** Elastic strain
- **E** Work hardening
- **F** Ultimate strength

The diagram illustrates stress-strain curves for different material behaviors, with linear, non-linear, ductile, plastic, and brittle behaviors distinctly marked.
Non-linear (non-Newtonian) behaviour

\[ \dot{\varepsilon} \propto \tau^n \]

- General stress - strain-rate relation
  - \( n = 1 \): Newtonian
  - \( n > 1 \): non-Newtonian (shear thinning)
  - \( n \) infinite: pseudo-brittle
- Local slope: Effective viscosity
- Application: Different viscosities within the lithosphere due to different absolute plate motions
Rocks are not rheologically simple: Loading experiment

- Simple compression experiment
- Constant stress loading
- Many effects observed (viscous, elastic, brittle)
Rocks are not rheologically simple: Unloading experiment
Combined effects

Figure 5.19 Brittle (a) to brittle-ductile (b, c) to ductile (d) deformation, reflecting the general subdivision that is used in the subsequent chapters.
Pressure-temperature effects

Figure 5.3 Effects of pressure (left: Carrara marble) and temperature (right: granite) on the stress–strain behaviour of rocks. Numbers on curves give confining pressure (in MPa) and temperature (in °C), respectively (from Jaeger and Cook 1979).
Temperature dependence

- SHIVA test
- High speed, torsion experiment
- Gabbro test sample
- Axial load of ~ 8 MPa
- Rotational velocity of 5 m/s
Creep

• The slow, continuous deformation of a material over time
• Mechanism occurs under applied stress, due to thermally activated motion of atoms and ions associated with crystal defects
• Thermally activated diffusion process
• Viscous behaviour (strain-rate)
• Flow law

\[ \dot{\varepsilon} = f(\sigma, t, T, ...) \]

• Solid state creep is a major deformation mechanism in the Earth’s crust and mantle
Creep processes

- **Diffusion creep**
- Migration of atoms through the (i) interior of the crystalline lattice (Herring-Nabarro), or (ii) along grain boundaries (Coble)

\[ \dot{\varepsilon} = A_{\text{diff}} \tau \]

Function of:
- crystal grain size \((d)\)
- pressure \((P)\)
- temperature \((T)\)

- Dominant at low stresses
- Linear (Newtonian) flow law
Creep processes

• Dislocation creep

• Migration of defects (dislocations) within the crystalline lattice. Dislocations may assume line or point geometries

\[ \dot{\varepsilon} = A_{\text{disc}} \tau^n \]

Function of:
* crystal grain size \((d)\)
* pressure \((P)\)
* temperature \((T)\)

• Dominant at high stresses

• Non-linear (non-Newtonian) flow law
Viscous creep law

- Experimental data
- The viscosity of rocks is strongly dependent on pressure, temperature, stress (strain-rate), grain size, water content, melt and mineralogy, ...

Arrhenius flow law

\[ \dot{\varepsilon} = A \tau^n \sigma^{-p} C^{s OH} \exp(-\alpha\phi) \exp\left[\frac{-(E + PV)}{RT}\right] \]

Effective viscosity

\[ \eta_{\text{effective}} = \frac{\tau}{2\dot{\varepsilon}} \]
Viscous creep laws typically used

\[
\dot{e}_{ij} = \frac{1}{2\eta_{\text{eff}}} \tau_{ij}
\]

\[
\eta_{\text{eff}} = B (\tau_{II})^{1-n} \exp \left[ \frac{E + pV}{RT} \right], \quad \tau_{II} = \sqrt{\frac{1}{2} \tau_{ij} \tau_{ij}}
\]

- \( B \) depends on grain size (in the linear domain)
- \( n = 1 \rightarrow \text{diffusion creep} \)
- \( n > 1 \rightarrow \text{dislocation creep} \)
- Common simplification \( \rightarrow \) Frank-Kamenetskii approx.

\[
\eta \propto \exp(-\theta T)
\]

Satisfactory for a limited \( p,T \) range
Upper mantle

\[
\dot{\epsilon} = A \left( \frac{\tau}{\mu} \right)^n \left( \frac{b}{d} \right)^m \exp \left[ -\frac{(E^* + pV^*)}{RT} \right]
\]

Table 5.3. Parameter Values for Diffusion Creep and Dislocation Creep in a Dry Upper Mantle

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Diffusion Creep</th>
<th>Dislocation Creep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-exponential factor ( A ) ( (s^{-1}) )</td>
<td>( 8.7 \times 10^{15} )</td>
<td>( 3.5 \times 10^{22} )</td>
</tr>
<tr>
<td>Stress exponent ( n )</td>
<td>1</td>
<td>3.5</td>
</tr>
<tr>
<td>Grain size exponent ( m )</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Activation energy ( E^* ) ( (kJ \ mol^{-1}) )</td>
<td>300</td>
<td>540</td>
</tr>
<tr>
<td>Activation volume ( V^* ) ( (m^3 \ mol^{-1}) )</td>
<td>( 6 \times 10^{-6} )</td>
<td>( 2 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

\(^a\) After Karato and Wu (1993). Other relevant parameter values are \( \mu_{\text{shear}} = 80 \) GPa, modulus \( b = 0.55 \) nm, and \( R = 8.3144 \) J K\(^{-1}\) mol\(^{-1}\).
Values valid for the following form of the flow law

\[ \dot{\varepsilon} = A \tau^n \exp\left[-\frac{E^*}{RT}\right] \]

**Table 5.6** Experimentally Derived Creep Parameters for Some Common Rock Types

<table>
<thead>
<tr>
<th>Rock type</th>
<th>(^{10}\log A) (MPa(\cdot)s(^{-1}))</th>
<th>n</th>
<th>(E^*) (kJ \cdot mol(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albite rock</td>
<td>18</td>
<td>3.9</td>
<td>234</td>
</tr>
<tr>
<td>Anorthosite</td>
<td>16</td>
<td>3.2</td>
<td>238</td>
</tr>
<tr>
<td>Clinopyroxenite</td>
<td>17</td>
<td>2.6</td>
<td>335</td>
</tr>
<tr>
<td>Clinopyroxenite (wet)</td>
<td>5.17</td>
<td>3.3</td>
<td>490</td>
</tr>
<tr>
<td>Diabase</td>
<td>17</td>
<td>3.4</td>
<td>260</td>
</tr>
<tr>
<td>Granite</td>
<td>6.4</td>
<td>3.4</td>
<td>139</td>
</tr>
<tr>
<td>Granite (wet)</td>
<td>7.7</td>
<td>1.9</td>
<td>137</td>
</tr>
<tr>
<td>Marble</td>
<td>33.2</td>
<td>4.2</td>
<td>427</td>
</tr>
<tr>
<td>Olivine rock</td>
<td>4.5</td>
<td>3.6</td>
<td>535</td>
</tr>
<tr>
<td>Olivine rock (wet)</td>
<td>4.0</td>
<td>3.4</td>
<td>444</td>
</tr>
<tr>
<td>Quartz diorite</td>
<td>11.5</td>
<td>2.4</td>
<td>219</td>
</tr>
<tr>
<td>Quartzite</td>
<td>10.4</td>
<td>2.8</td>
<td>184</td>
</tr>
<tr>
<td>Quartzite (wet)</td>
<td>10.8</td>
<td>2.6</td>
<td>134</td>
</tr>
<tr>
<td>Rock salt</td>
<td>-1.59</td>
<td>5.0</td>
<td>82</td>
</tr>
</tbody>
</table>

*Source: Kirby and Kronenberg (1987).*
Rock and mineral aggregates

- Diffusion creep and dislocation creep mechanisms are not independent -
- Both simultaneously occur at a given stress state
- Composite rheology

Effective composite viscosity

\[
\frac{1}{\eta_{\text{eff}}} = \frac{1}{\eta_{\text{diff}}} + \frac{1}{\eta_{\text{disc}}}
\]

under strain rate decomposition assumption (Maxwell like)

\[
\dot{\varepsilon} = \dot{\varepsilon}_{\text{diff}} + \dot{\varepsilon}_{\text{disc}}
\]
Rocks have a finite strength

- The differential stress \((\sigma_1 - \sigma_3)\) is limited in nature
- Caused by micro-defects, breaking bonds between atoms, growth of micro cracks
Differential stresses in the crust

\[ p \propto (\sigma_1 - \sigma_3) \quad \text{(Townend & Zoback Geology 2000)} \]
Plastic vs. viscous deformation

(a) brittle/plastic  (b)  (c) viscous

An undeformed sample
Fracture style as a function of confining pressure

Figure 6.25  (a) A representative composite failure envelope on a Mohr diagram. The different parts of the envelope are labeled, and are discussed in the text. (b) Sketches of the fracture geometry that forms during failure. Note that the geometry depends on the part of the failure envelope that represents failure conditions, because the slope of the envelope is not constant.
Coulomb failure criteria

\[ \tau = C + \sigma \tan(\phi) \]

- friction angle
- normal stress
- cohesion
- max shear stress

\[ \sigma = \sigma_m - \tau_m \sin(\phi) \quad \tau_m = \frac{1}{2} (\sigma_1 - \sigma_3) \]

\[ \tau = \tau_m \sin(\phi) \quad \sigma_m = \frac{1}{2} (\sigma_1 + \sigma_3) \]

\[ \tau_m = C \cos(\phi) + \sigma_m \sin(\phi) \]

\[ \rightarrow \sigma_1 - \sigma_3 = C' + p\mu' \]
Byerlee’s law

- Coulomb plasticity is empirical theory, however seems to work reasonably well for upper crustal rocks.
Byerlee's law

Figure 5.7-10: Relation between shear stress and normal stress for frictional sliding.

- $200 \text{ MPa} \leq \sigma_n \leq \sim 2000 \text{ MPa}$

- $\sigma_n < 200 \text{ MPa}$

- $\tau = 0.85\sigma_n$, $\sigma_n \leq 200 \text{ MPa}$

- $\tau = 50 \text{ MPa} + 0.6\sigma_n$, $1700 \text{ MPa} > \sigma_n > 200 \text{ MPa}$

- $\sigma_1 = 4.7\sigma_3$, $\sigma_3 \leq 114 \text{ MPa}$

- $\sigma_1 = 3.1\sigma_3 + 177 \text{ MPa}$, $1094 \text{ MPa} > \sigma_3 > 114 \text{ MPa}$
Numerical models of (brittle) localisation
Numerical models of (brittle) localisation

- Brittle failure in the upper crust may result in localised zones of deformation due to Mohr-Coulomb plasticity (which mimics Byerlee’s law)
Peierls creep and strength of rocks

(Kameyama et al, 2001)

• Problem:
  • Byerlee is valid for upper crustal rocks. At deeper levels, the mechanism limiting stress is not very clear
  • Low temperature plasticity (Peierls creep) has been suggested based on; experiments, numerical calculations and theoretical considerations

• Consequences
  • This plasticity form does not produce localised faults as easy, requires shear-heating feedbacks to “break” the lithosphere
  • Possible explanation of the small number of earth quakes in the lithospheric mantle compared to upper crustal rocks?
Question: Is diffusion creep or dislocation creep the dominant deformation mechanism in the upper mantle?

Considerations:

1. For a given stress - the mechanism with the largest strain-rate is dominant

2. For a given strain-rate - the mechanism with the lowest stress is dominant
Mantle deformation maps

Mantle:

\[ T \sim 1600 \text{ [K]} \]
\[ \varepsilon \sim 1 \times 10^{-15} \text{ [1/s]} \]

Dislocation Creep

\[ \dot{\varepsilon} = 10^{-20} \text{ s}^{-1} \]
\[ 10^{-18} \text{ s}^{-1} \]
\[ 10^{-16} \text{ s}^{-1} \]

Diffusion Creep

\[ \dot{\varepsilon} \]

Independent of stress

\[ \dot{\varepsilon} = A \left( \frac{\tau}{\mu} \right)^n \left( \frac{b}{d} \right)^m \exp \left[ -\frac{(E^* + pV^*)}{RT} \right] \]

Dry upper mantle, \( p = 0 \)

(Schubert et al, 2001)
Figure 9.35  Deformation mechanism map for olivine with a grain size of 100 μm. Variables are the same as in Figure 9.32, except that depth is substituted for temperature given an exponentially decreasing geothermal gradient with 300°C at the surface and 1850°C at 500 km depth.
Viscous deformation map

(Kameyama et al, 2001)

Fig. 1. Deformation mechanism map calculated for grain size $a = 0.1$ mm. The lightly shaded area indicates that deformation mainly occurs by diffusion creep. The densely shaded area indicates that deformation mainly occurs by power-law creep. The white region indicates that deformation mainly occurs by the Peierls mechanism. The solid curves are lines of constant strain rate. The numbers attached to each contour indicate the logarithm of the strain rate in the unit of $s^{-1}$. 
Strength envelopes

Low strain-rate (or high temp.)
High strain-rate (or low temp.)

Byerlee's law
Arrhenius flow law
Strength of the mantle-lithosphere

Figure 9. Strength envelopes for oceanic and continental lithosphere. (a) For the oceanic lithosphere, a geotherm for 60-m.y.-old lithosphere was used [e.g., Turcotte and Schubert, 1982 pp. 163-167]. A rheology for dry olivine [Chopra and Paterson, 1984] was used because water strongly partitions into the melt during partial melting. (b) For the continental lithosphere, a geotherm for a surface heat flow of 60 mW m$^{-2}$ was employed [Chapman, 1986]. The rheologies for wet quartzite are those used in Figure 5; the olivine rheology is for wet Anita Bay dunite from Chopra and Paterson [1984]. Wet rheologies were used, consistent with high fluid pressures in fault zones. Plastic flow strength was corrected for water fugacity using a water fugacity exponent of unity and assuming lithostatic pore pressure. The BDT and BPT, determined as described in the text, have been connected by a dotted line.
Difference come from the dependence of Byerlee’s law on the normal stress. Compression results in large normal stress (tectonic loading)

(Burov E., Treatise on Geophysics V. 6, 2007)
Compression versus extension

Difference come from the dependence of Byerlee’s law on the normal stress. Compression results in large normal stress (tectonic loading).

(Burov E., Treatise on Geophysics V. 6, 2007)
Summary

• Learned the vocabulary of rheology
• Examined three basic classes
  • elastic
  • viscous
  • plastic / brittle
• Deformation maps and strength envelops