Dynamics of the mantle and lithosphere (2016)
Practical: Thermal structure

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Q1 Thermal flux

(a) Using the values in Table 4.1 (see below), compute the heat flow through each layer.

(b) Compute the mean heat flow over the depth range, 380-515 m.

(c) For each layer, indicate the direction of the heat flow (i.e. towards the surface, or away from the surface).

TABLE 4–1 **Temperatures Between Layers of Rock Types**

Depth (m)	**Temp. (°C)**	**Rock Type**	**k (Wm^{-1} K^{-1})**
380	18.362		
		Sandstone	3.2
402	18.871		
		Shale	1.7
412	19.330		
		Sandstone	5.3
465	20.446		
		Salt	6.1
475	20.580		
		Sandstone	3.4
510	21.331		
		Shale	1.9
515	21.510		

Q2 Conservation of energy

(a) Write down the equation describing the conservation of energy in 1D. Identify all variables introduced and state their units.

(b) Derive an expression for the *steady state temperature profile* across the continental crust, considering only a radiogenic heat source, $H_r = 1 \times 10^{-6}$ W/m^3. The surface temperature T_0 and temperature at the base of the crust T_1 are given by 300 K and 700 K respectively. Assume that crustal thickness is given by L and the thermal conductivity by $k = 2$ W/(m K). Include in your answer a diagram indicating the coordinate

system and define all quantities. Indicate the direction of heat flow.
Note: Derive your expression for the temperature profile in terms of the variables defined above - only substitute numbers into the expression at the end.

(c) Plot your temperature solution $T(y)$. What is the value of the heat flux at the surface? What would the temperature field look like if you only considered radiogenic heating in only the *upper half* of the continental crust?

Q3 Lithospheric thickness

A solution of the conservation of energy equation

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} \tag{1}$$

with the boundary conditions

$$T = T_1 \quad \text{at} \quad t = 0,$$
$$T = T_0 \quad \text{at} \quad y = 0,$$
$$T \to T_1 \quad \text{as} \quad y \to \infty,$$

is given by

$$\frac{T - T_0}{T_0 - T_1} = \text{erf}\left(\frac{y}{2\sqrt{\kappa t}}\right). \tag{2}$$

This solution represents an idealisation of mid-ocean ridge (spreading centre). The geometry and sketch of the solution is shown in Fig. 1

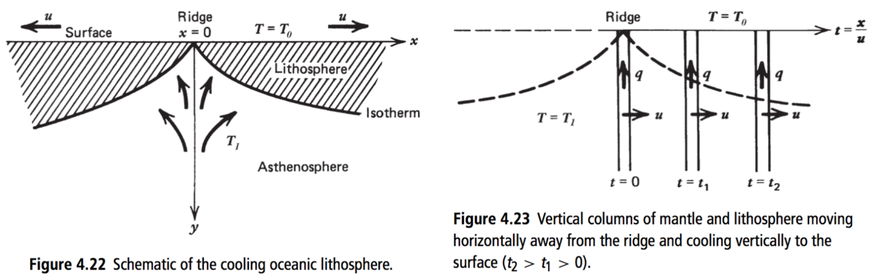

Figure 1: Taken from Turcotte & Schubert, *Geodynamics* (2014).

Using this simple analytic solution, we will determine an approximate thickness of the oceanic lithosphere. In this context, we will assume that the mantle temperature $T_1 = 1773$ K and that the surface temperature $T_0 = 273$ K. We will also assume the following parameter values are valid for the oceanic lithosphere: $C_p = 1000$ J/kg/K and $k = 1.67$ W/(m K).

(a) Compute the thermal diffusivity κ of the lithosphere. State any assumptions made. The answer should be given in units of mm^2 s^{-1}

(b) Using Eqn.(2), plot $T - T_0$ as function depth y and time t. Consider depths ranging from 0 to 200 km and for a time up to 150 Myr.

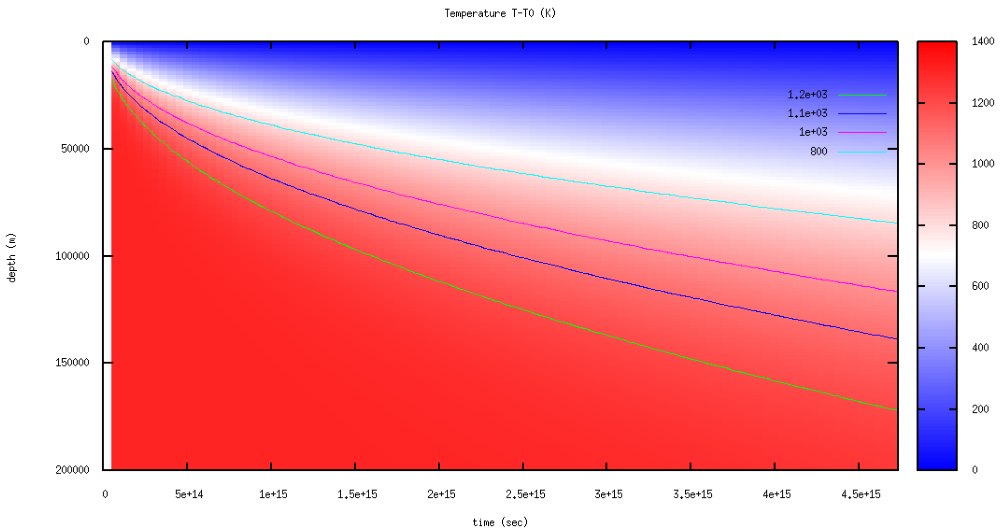

Figure 2: Example relative temperature field $T(t, y) - T_0$, when $\kappa = 1$ mm^2 s^{-1} and $T_1 - T_0 = 1300$ K.

(c) The oceanic lithosphere is the surface plate that moves rigidly over the deeper viscous mantle. These plates are created from the hot mantle rock which is flowing upward beneath the ridge. The rising (buoyant) rock is cooled by the seawater and forms the rigid plates which then moves away from the ridge. Consequently, the oceanic lithosphere can be identified as the part of the upper mantle whose temperature is less than some value below which mantle rocks do not readily deform over geologic time.

Assuming that the contour $T - T_0 = 1300$ K represents the mantle-lithosphere boundary, compute (or estimate) the thickness of the lithosphere at $t = 50$ Myr and $t = 100$ Myr.

Table 4.5 The Error Function and the Complementary Error Function

η	erf η	erfc η
0	0	1.0
0.02	0.022565	0.977435
0.04	0.045111	0.954889
0.06	0.067622	0.932378
0.08	0.090078	0.909922
0.10	0.112463	0.887537
0.15	0.167996	0.832004
0.20	0.222703	0.777297
0.25	0.276326	0.723674
0.30	0.328627	0.671373
0.35	0.379382	0.620618
0.40	0.428392	0.571608
0.45	0.475482	0.524518
0.50	0.520500	0.479500
0.55	0.563323	0.436677
0.60	0.603856	0.396144
0.65	0.642029	0.357971
0.70	0.677801	0.322199
0.75	0.711156	0.288844
0.80	0.742101	0.257899
0.85	0.770668	0.229332
0.90	0.796908	0.203092
0.95	0.820891	0.179109
1.0	0.842701	0.157299
1.1	0.880205	0.119795
1.2	0.910314	0.089686
1.3	0.934008	0.065992
1.4	0.952285	0.047715
1.5	0.966105	0.033895
1.6	0.976348	0.023652
1.7	0.983790	0.016210
1.8	0.989091	0.010909
1.9	0.992790	0.007210
2.0	0.995322	0.004678
2.2	0.998137	0.001863
2.4	0.999311	0.000689
2.6	0.999764	0.000236
2.8	0.999925	0.000075
3.0	0.999978	0.000022

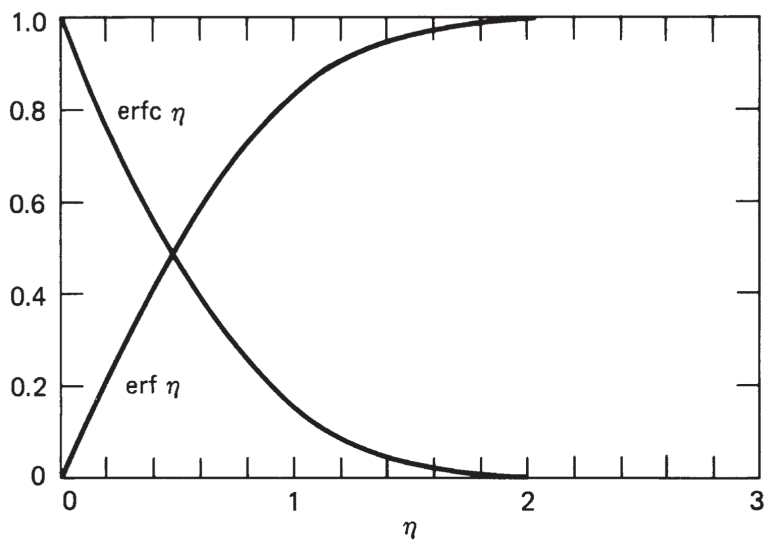

Figure 3: Tabulated and plots of the the error function (erf) and the complementary error function (erfc). Taken from Turcotte & Schubert, *Geodynamics*, (2014).