Recent Geodynamic Research: *Subduction Dynamics*

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Paper discussions

- *[Paper 1] “Plate tectonics and arcuate plate boundaries”*
  A free plate surface and weak oceanic crust produce single sided subduction on Earth, Crameri, et al, Geophysical Research Letters, Vol. 39, L03306, (2012)
- *[Paper 2] “Subduction dynamics: An analytic perspective”*
  A simple analytic solution for slab detachment, Schmalholz, Earth and Planetary Science Letters, 304 (2011), pp. 45–54
- *[Paper 3] “Potential problems with numerical subduction models”*
  A benchmark comparison of spontaneous subduction models —Towards a free surface, Schmeling et al, Physics of the Earth and Planetary Interiors, Vol. 17, (2008), pp. 198—223

Paper 1: “Plate tectonics and arcuate plate boundaries”

- **A free plate surface and weak oceanic crust produce single sided subduction on Earth**, Crameri, et al, Geophysical Research Letters, Vol. 39, L03306, (2012)

Objectives

- Continuing the quest for self consistent plate tectonics…
- How to make models of subduction zones which evolve in a manner like those observed on Earth
- What are the necessary “ingredients”? (minimal physics)

GEOPHYSICAL RESEARCH LETTERS, VOL. 39, L03306, doi:10.1029/2011GL050046, 2012

A free plate surface and weak oceanic crust produce single-sided subduction on Earth

F. Crameri,[1] P. J. Tackley,[1] I. Meilick,[1] T. V. Gerya,[1] and B. J. P. Kaus[1,2]

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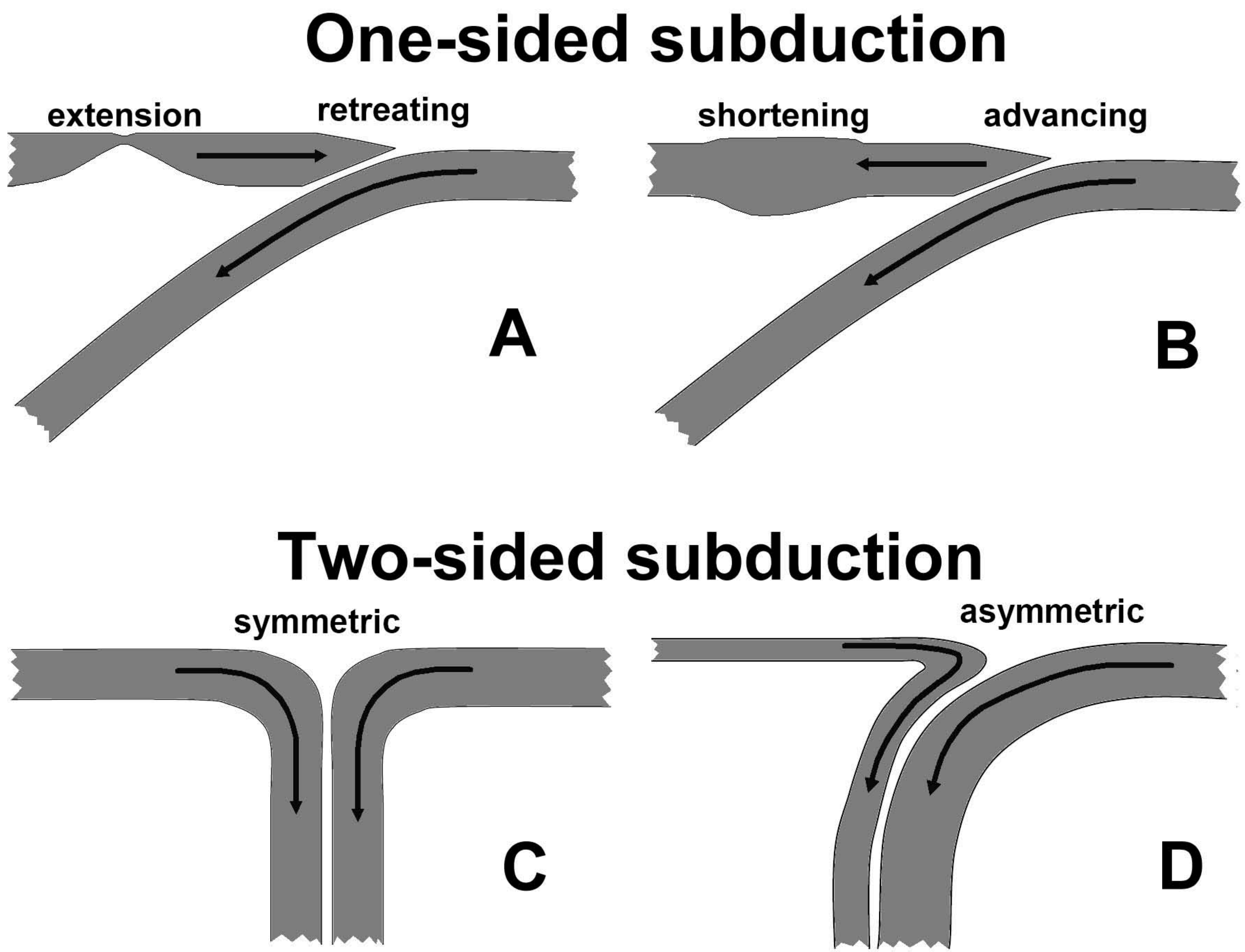
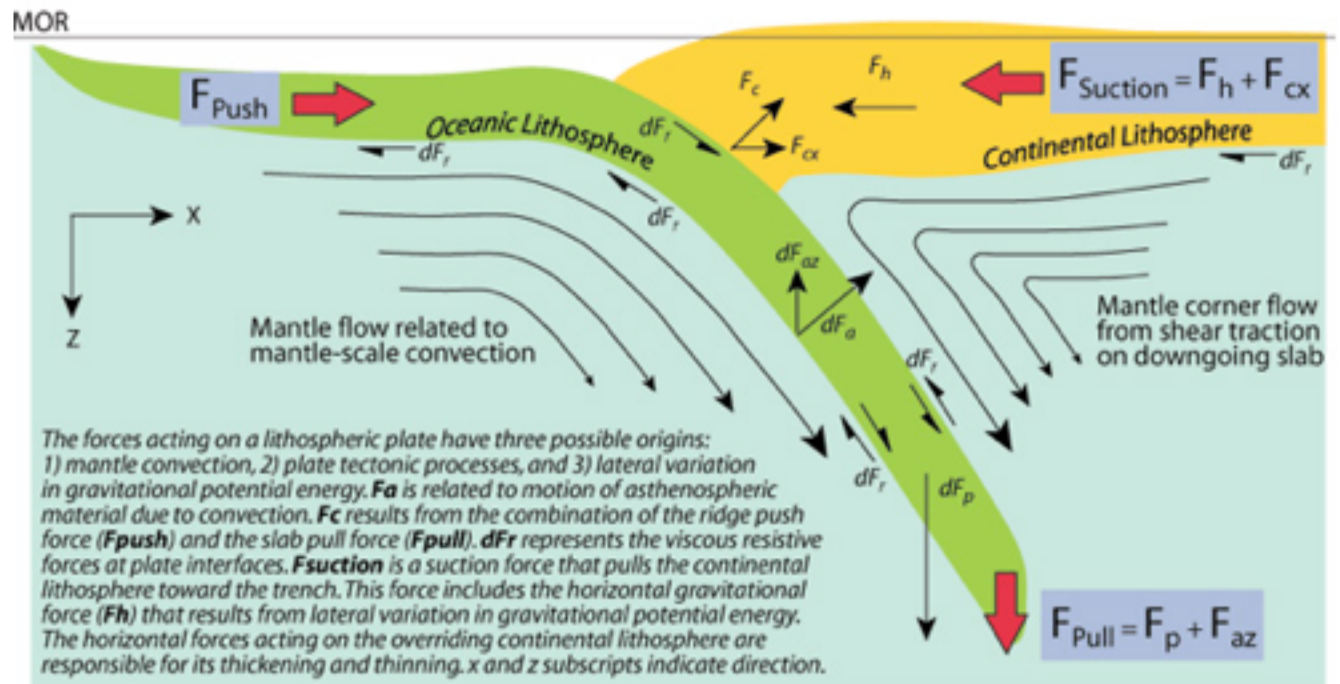
[1] Earth's lithosphere is characterized by the relative movement of almost rigid plates as part of global mantle convection. Subduction zones on present-day Earth are strongly asymmetric features composed of an overriding plate above a subducting plate that sinks into the mantle. While global self-consistent numerical models of mantle convection have reproduced some aspects of plate tectonics, the assumptions behind these models do not allow for realistic single-sided subduction. Here we demonstrate that the asymmetry of subduction results from two major features of terrestrial plates: (1) the presence of a free deformable upper surface and (2) the presence of weak hydrated crust atop subducting slabs. We show that assuming a free surface, rather than the conventional free-slip surface, allows the dynamical behavior at convergent plate boundaries to change from double-sided to single-sided. A weak crustal layer further improves the behavior towards steady single-sided subduction by acting as lubricating layer between the sinking and the overriding plate. This is a first order finding of the causes of single-sided subduction, which by its own produces important features like the arcuate curvature of subduction trenches. **Citation:** Crameri, F., P. J. Tackley, I. Meilick, T. V. Gerya, and B. J. P. Kaus (2012), A free plate surface and weak oceanic crust produce single-sided subduction on Earth, *Geophys. Res. Lett.*, *39*, L03306, doi:10.1029/2011GL050046.

Problem: “two-sidedness”



Gerya et al, Geology, (2007)

What are "natural" slabs?



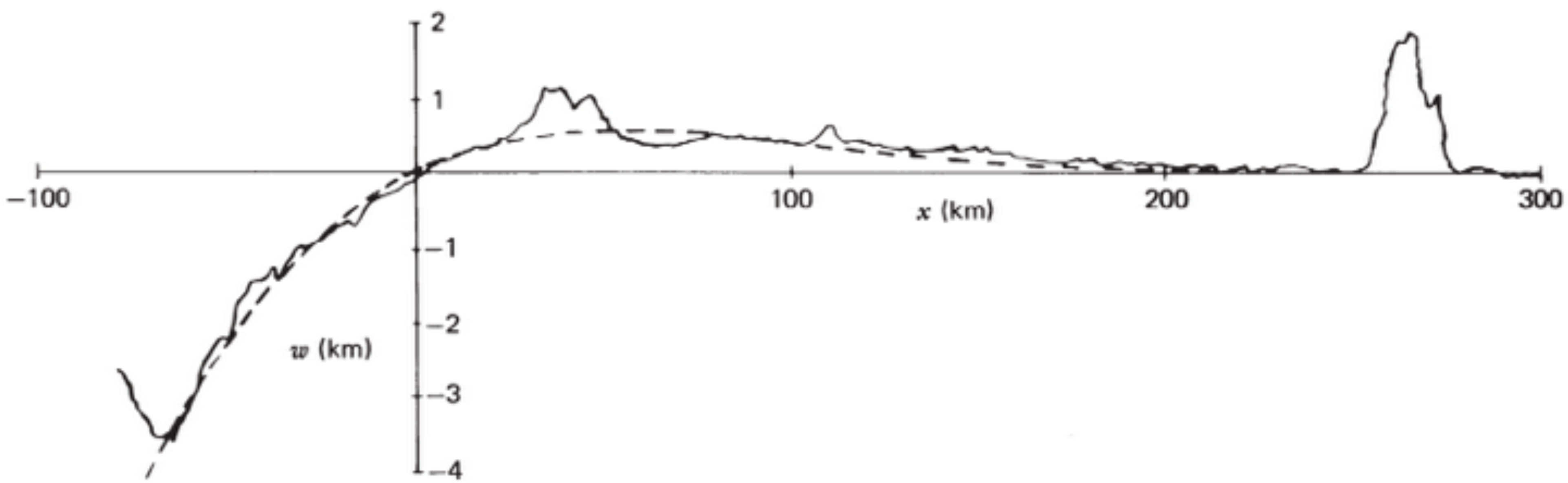


Figure 3.35 Comparison of a bathymetric profile across the Mariana trench (solid line) with the universal lithospheric deflection profile given by Equation (3.159) (dashed line); $x_b = 55$ km and $w_b = 0.5$ km.

Turcotte & Schubert (2014)

What are "numerical" slabs?

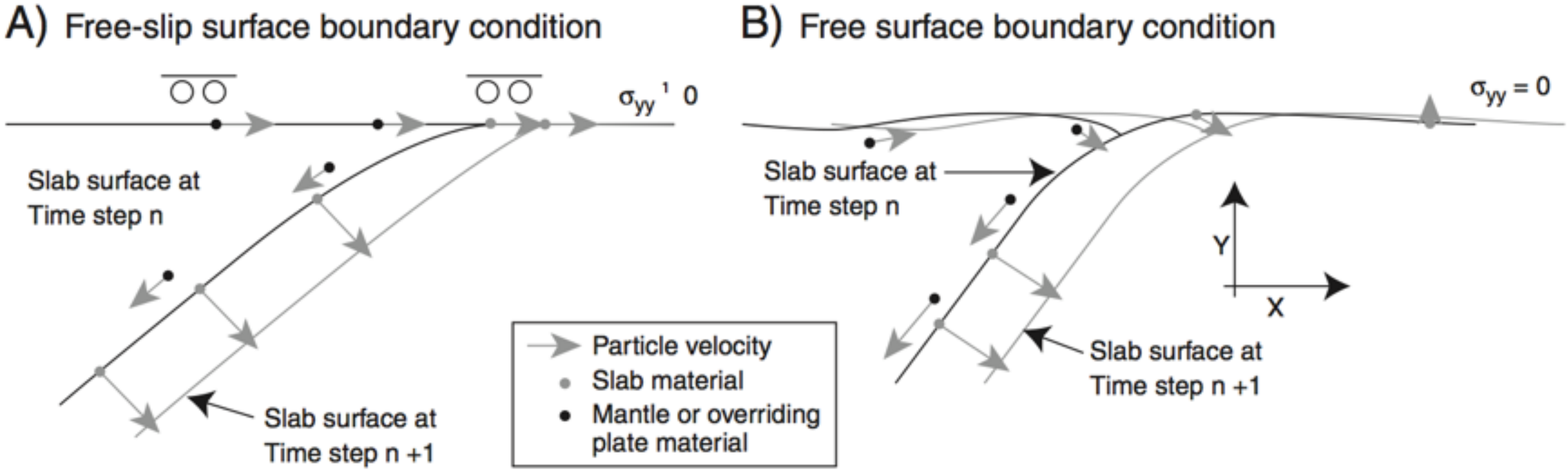


Fig. 12. Simple schematics of the effects of A) free-slip and B) free surface boundary conditions on the particles located at the trench (see text).

Quinquis et al, Tectonophysics, (2011)

$$\sum_i u_i n_i = 0$$

$$\sum_{i,j} n_i \tau_{ij} t_j = 0$$

$$\sum_j \sigma_{ij} n_j = 0$$

Method

- Incompressible, variable viscosity Stokes equations (conservation of mass & momentum)
- Conservation of energy (with internal heating)
- Viscosity is temperature dependent (Arrhenius)
- Byerlee law is used to limit strength of rocks
- Cohesion (*C*) = 0.6 MPa

$$\eta(T,p) = \eta_0 \cdot \exp\left[\frac{E_{act} + pV_{act}}{RT}\right] \qquad \sigma_{yield} = C + p\mu$$

- *Friction coefficient and activation volume varied*

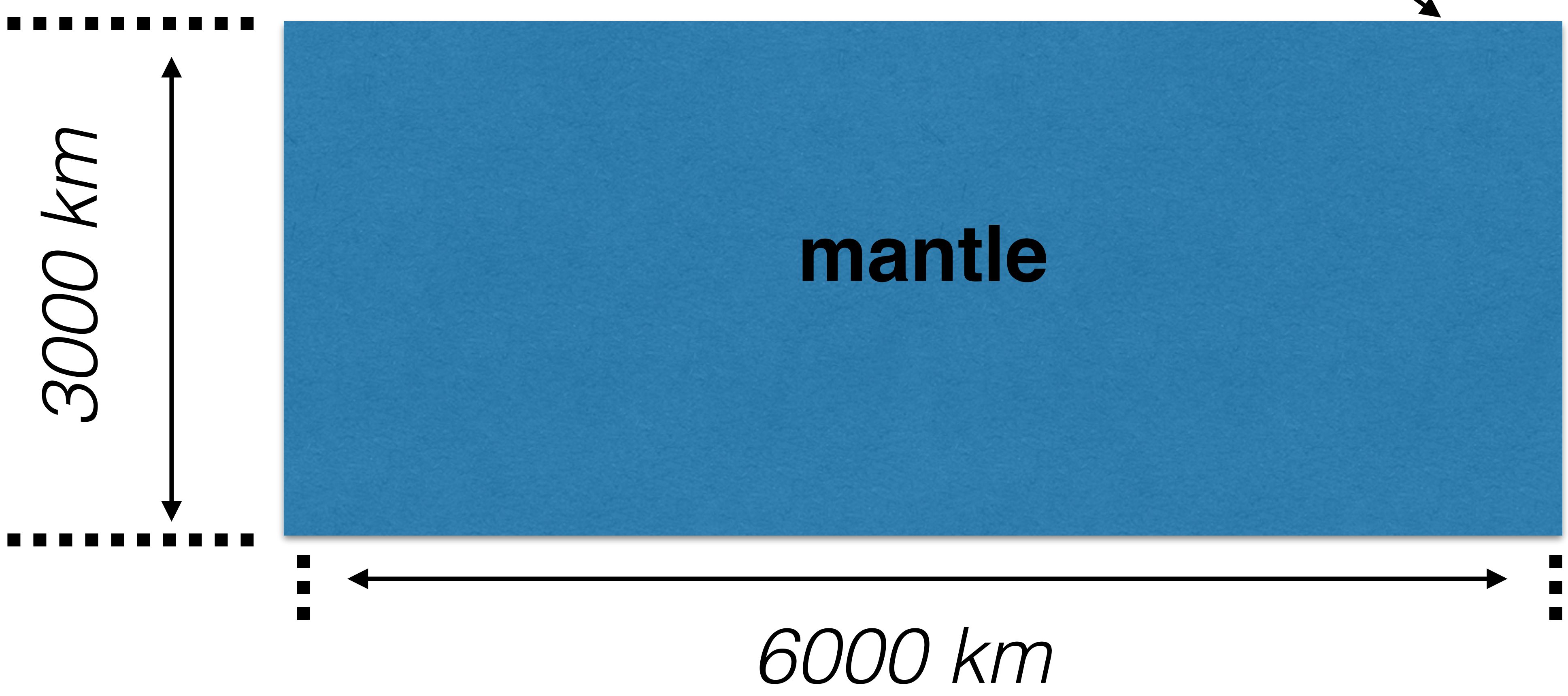
Weak crustal layer

- Defined to have two orders of magnitude lower viscosity and yield stress compared to mantle
- Material within d_{crust} (~6 km) distance of the surface is assumed to be weak crustal material
- Any material at > 900 km depth identified as “crustal” is converted into regular mantle material

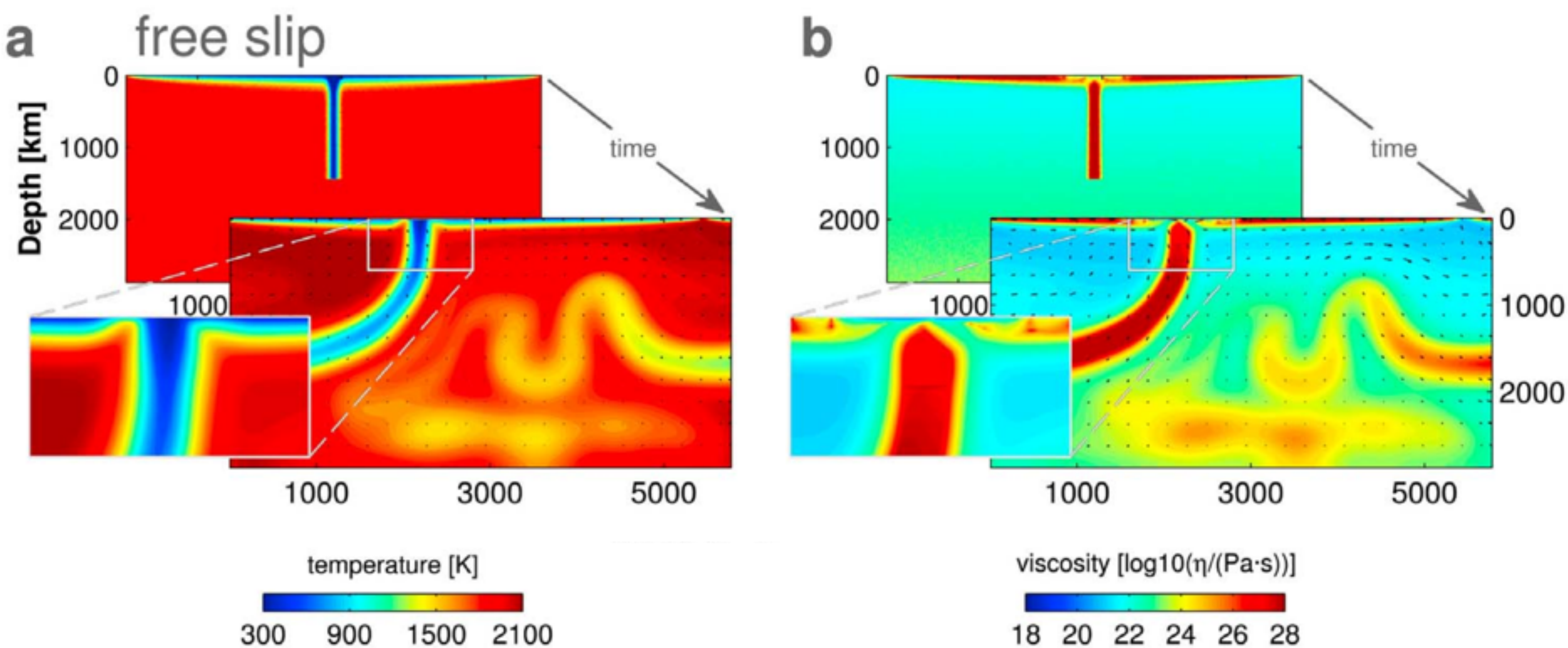
Free-slip

$$\sum_i u_i n_i = 0$$

$$\sum_{i,j} n_i \tau_{ij} t_j = 0$$



Findings



a
free slip
0
1000
2000
Depth [km]
time
1000
1000
3000
5000
temperature [K]
300
900
1500
2100
b
0
1000
2000
time
0
1000
1000
2000
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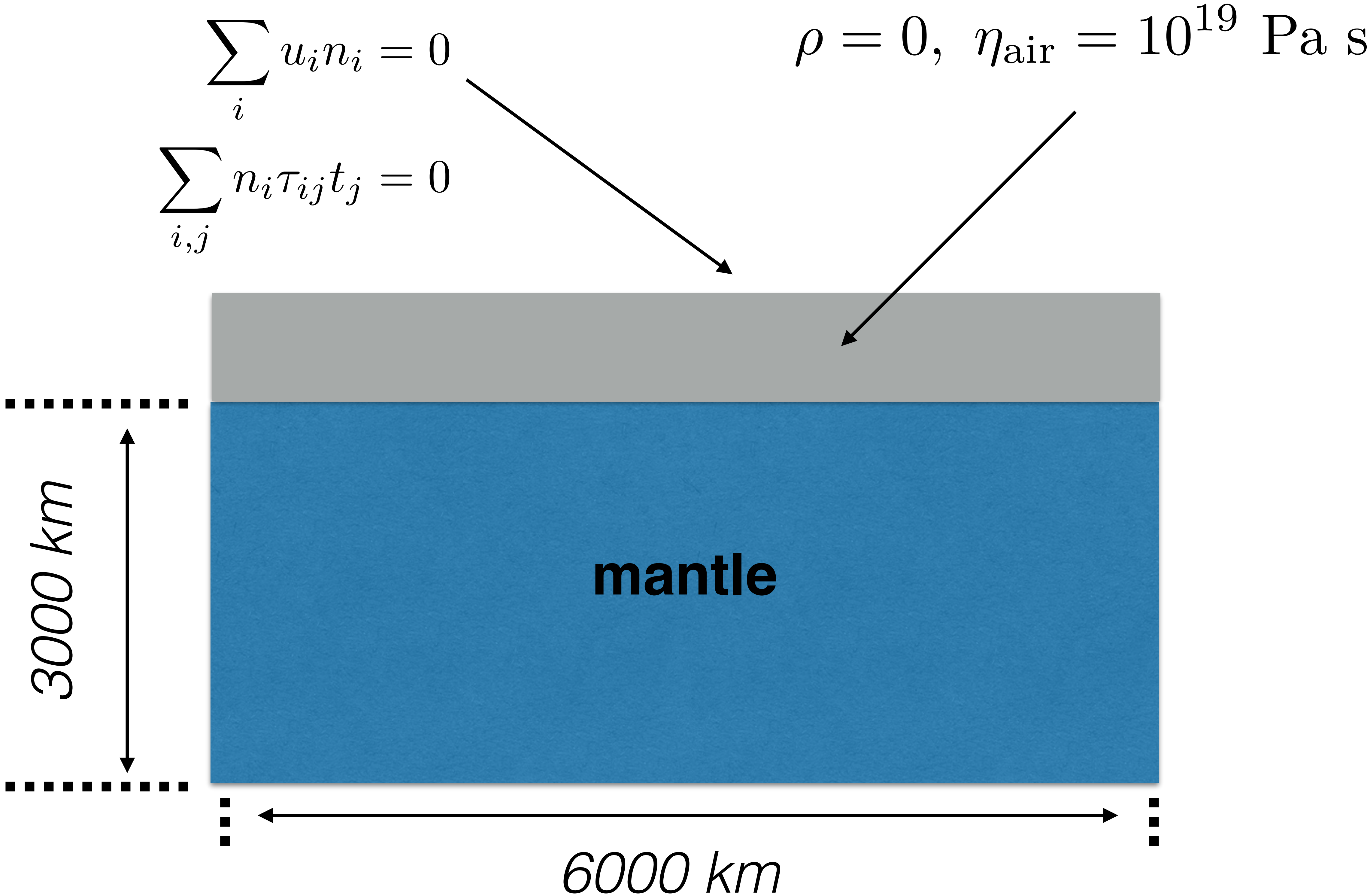
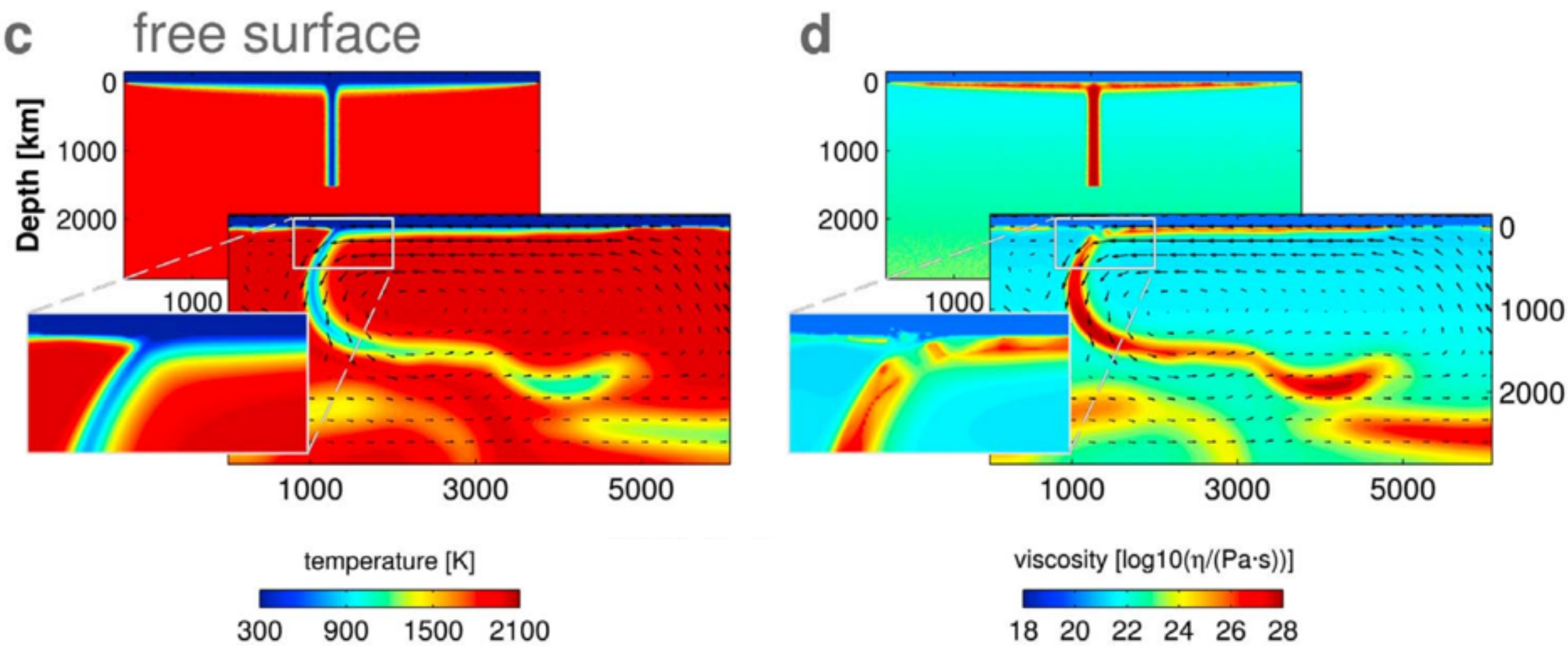
3000
5000
viscosity [log10(η/(Pa·s))]
18
20
22
24
26
28

“Sticky-air” model



Findings


c
free surface
Depth [km]
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2000
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1000
3000
5000
temperature [K]
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900
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2100
d
0
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2000
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viscosity [log10(η/(Pa·s))]
18
20
22
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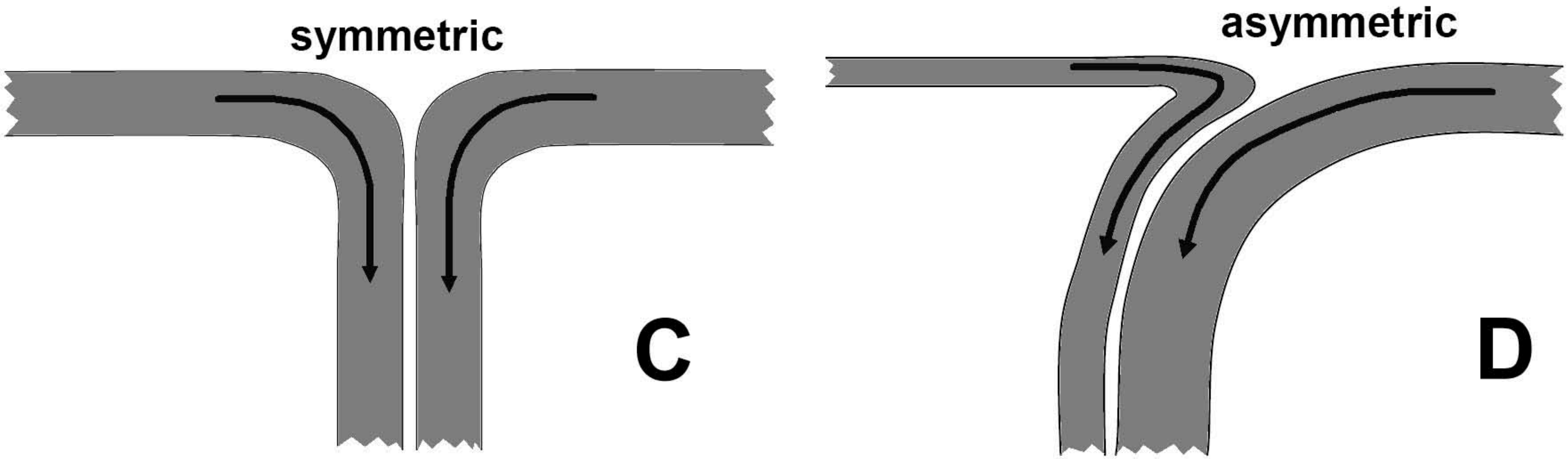
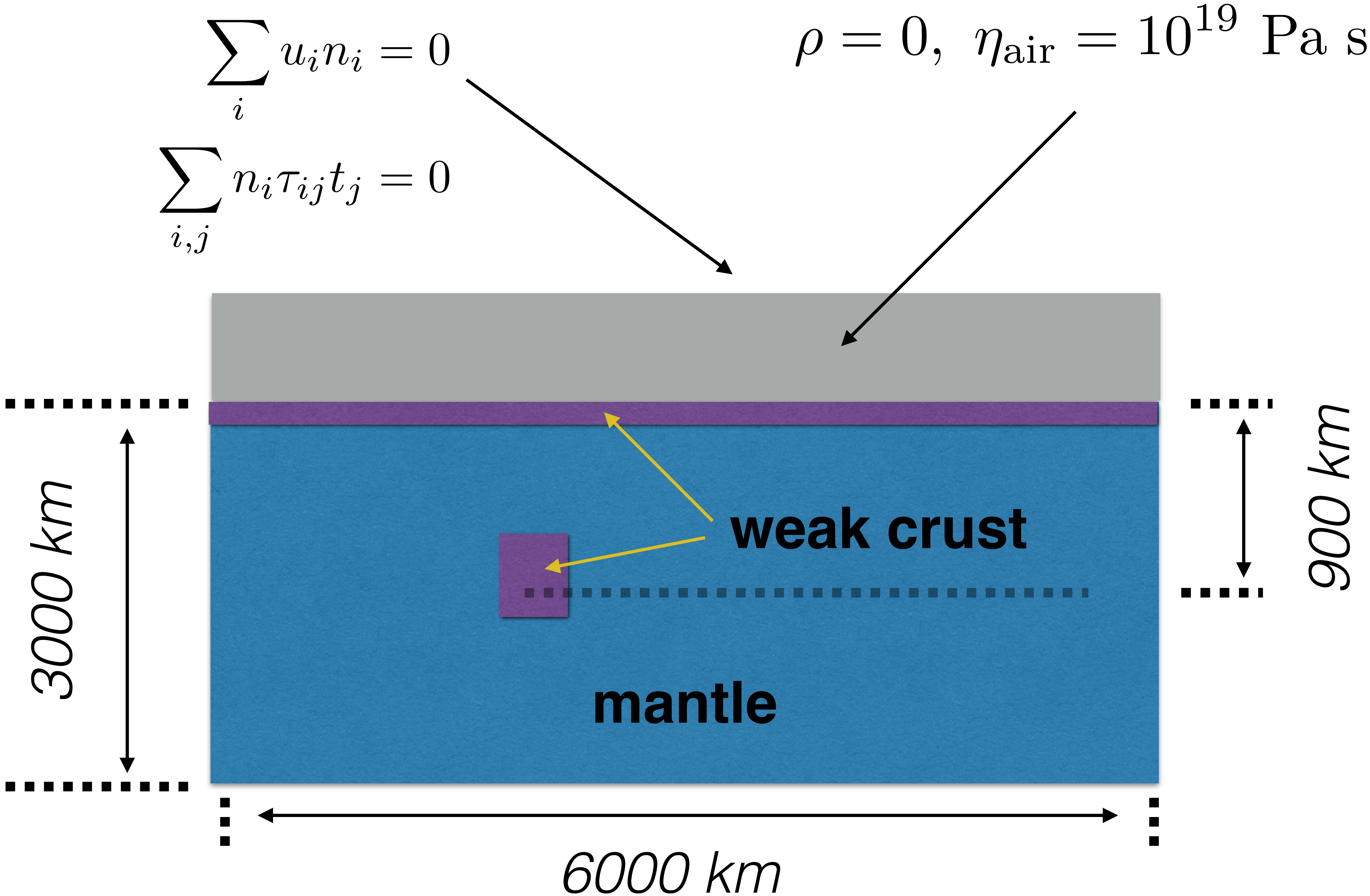
26
28

Weakening at the trench

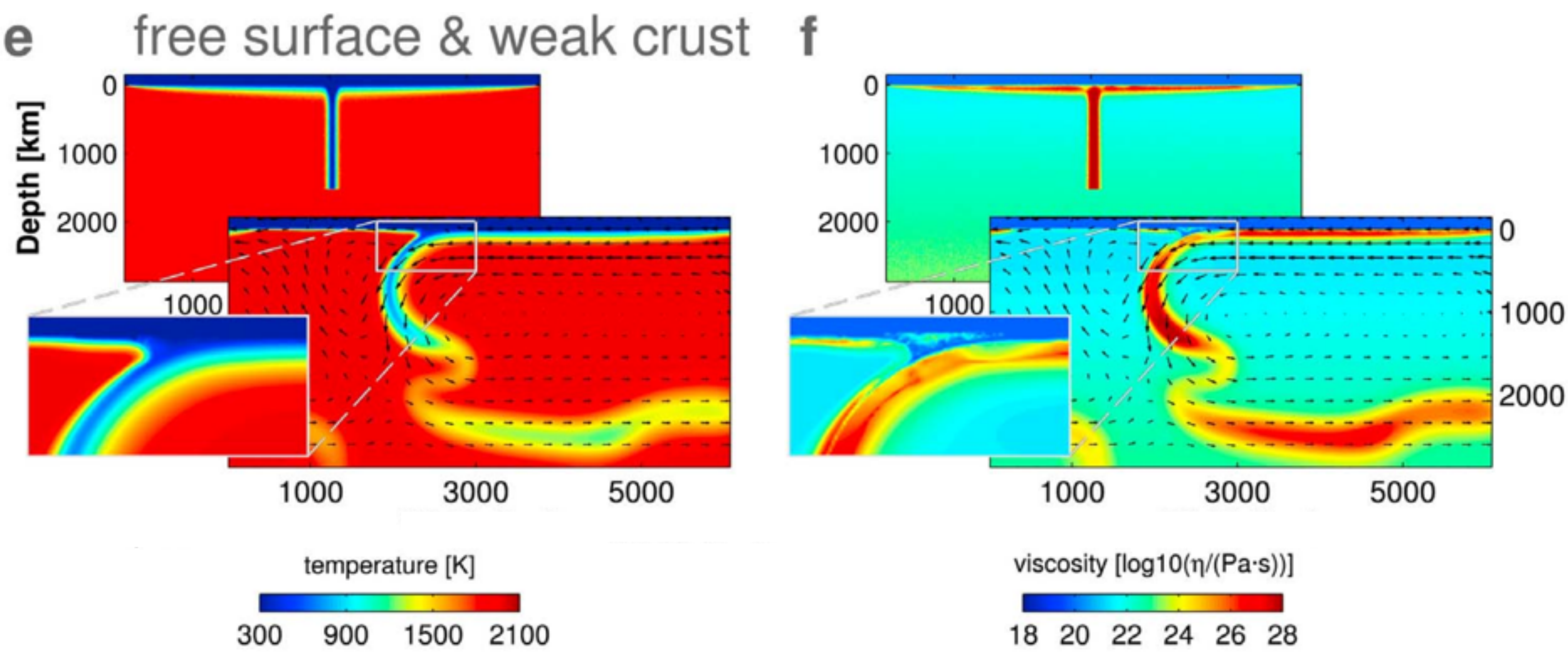


$$\dot{\epsilon} \propto \frac{\partial v_x}{\partial y}$$

Sticky-air + weak crust model



Findings



e
free surface & weak crust
f
Depth [km]
0
1000
2000
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5000
temperature [K]
300
900
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2100
viscosity [log10(η/(Pa·s))]
18
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Regimes

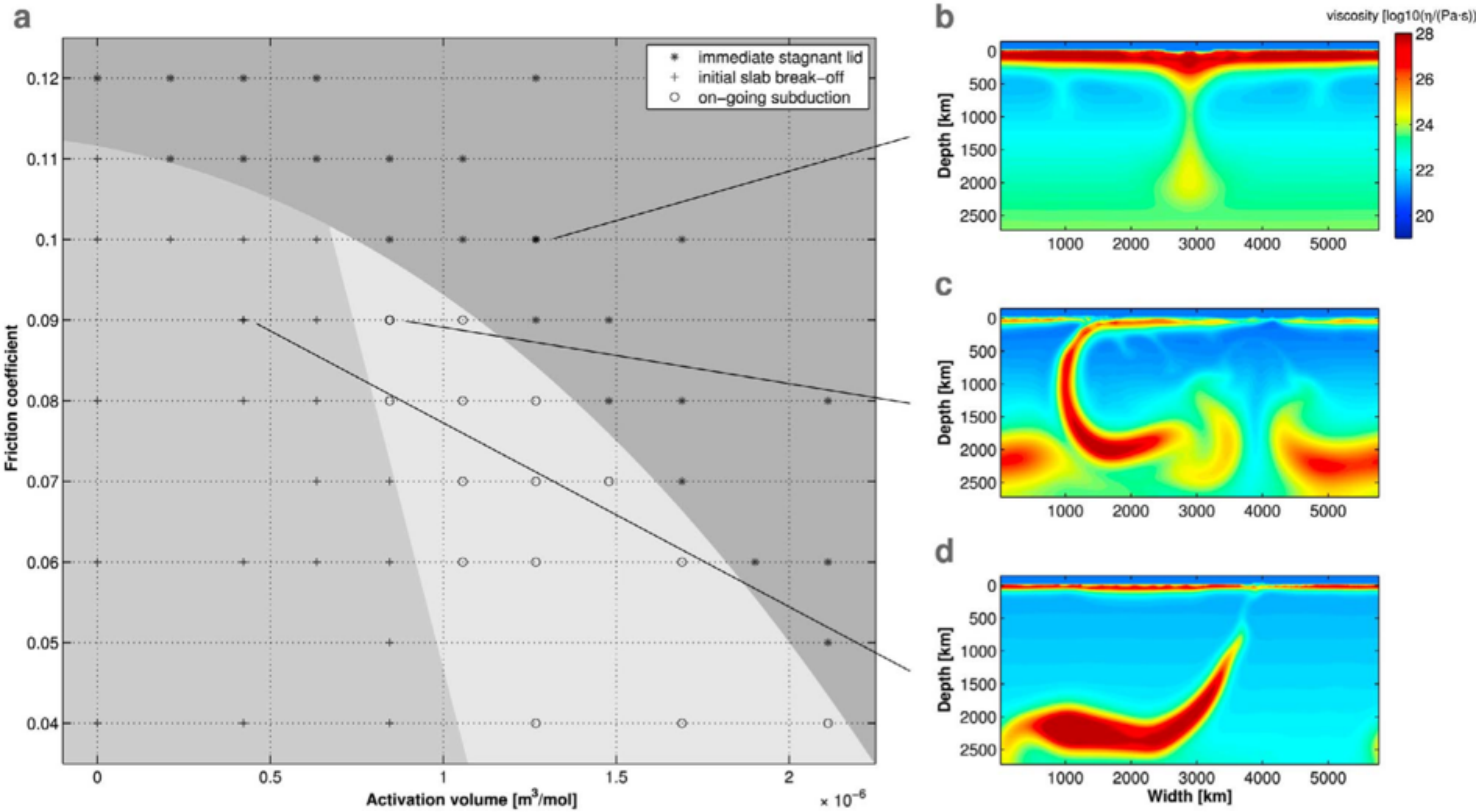
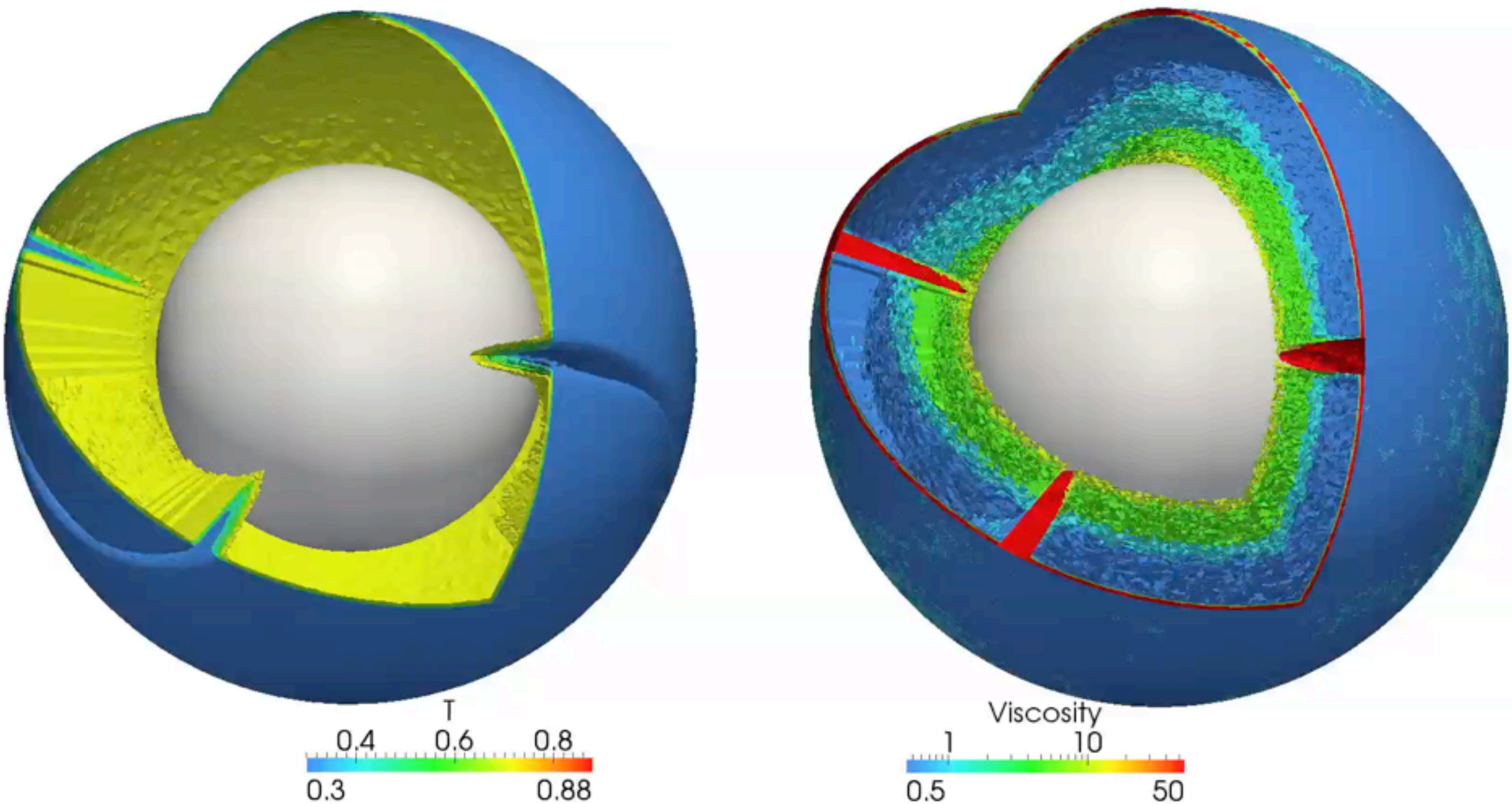


Figure 2. Regime diagram. Tectonic modes of mantle convection derived from 2-D Cartesian simulations in a 2:1 aspect ratio (case 4). (a) Distribution of tectonic modes as function of friction coefficient and activation volume, which are (b) immediate occurrence of a stagnant lid, (c) on-going subduction and (d) initial slab break-off.

Three-dimensionality



Three-dimensionality

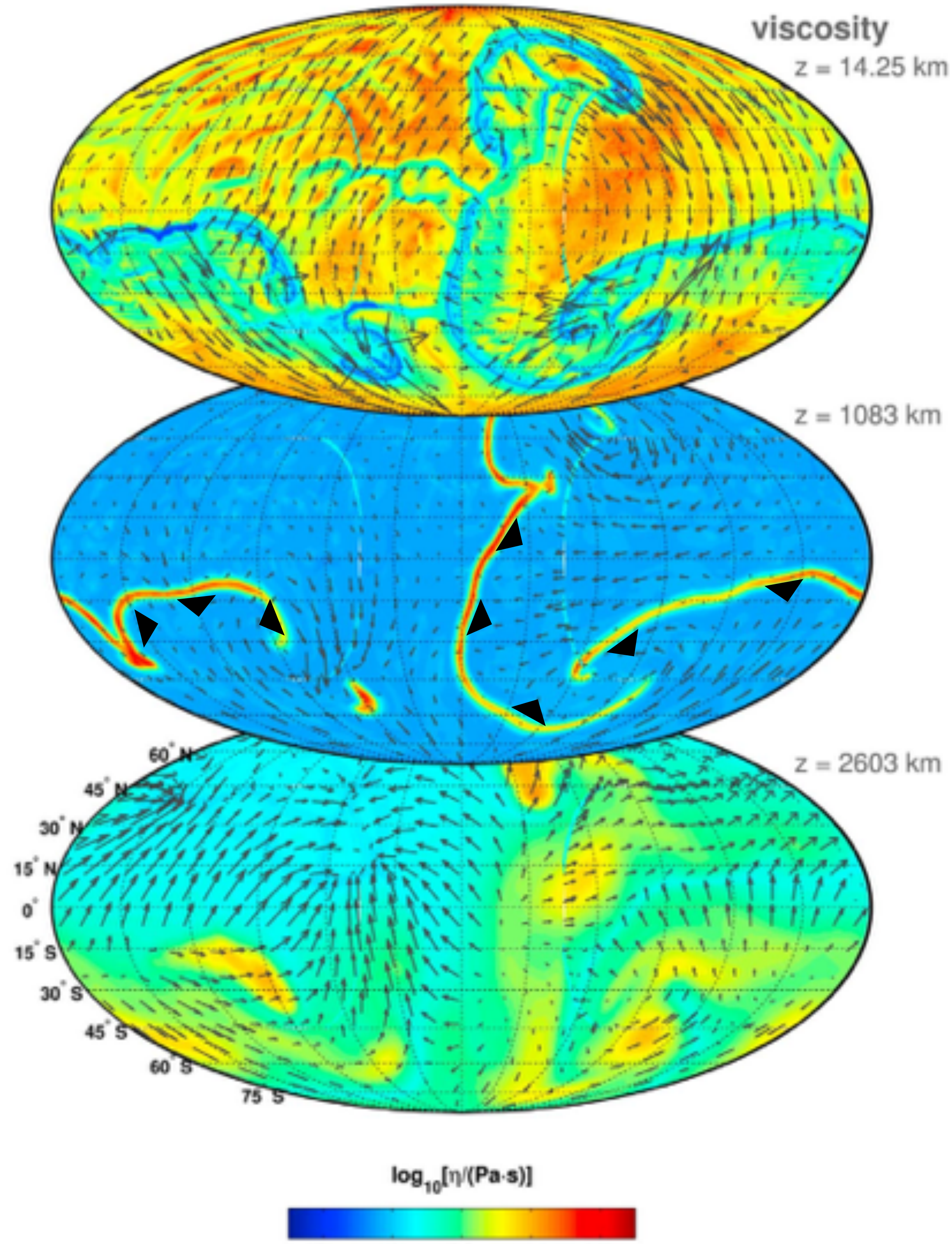


Figure 4. Mollweide projection of 3-D model. Viscosity fields at different depths showing characteristic arcuate trench and slab curvature induced by single-sided subduction (case 5).

Summary

- Independent of geometry, single sided subduction is possible
- Lithospheric strength is crucial in determining subduction style
- Depth dependence of the viscosity is required to prevent slab from immediately breaking off (e.g. through providing resistance to sinking/bending)
- Single-sided subduction promotes curved trenches in 3D
- Ultimate state of nearly all numerical experiments performed in this study is the stagnant lid mode

Paper 1: Some open questions

- What style of collision (subduction) is being modelled in this paper? How realistic a model is this for the Earth?
- Discuss the modelling choices related to the inclusion of the weak crustal material. What are the consequences of removing the crust at depth?
- What ingredients are missing (or would you add) to make the model more Earth like?
- The authors report nearly all models end up in a stagnant lid regime. Is this the expected fate of our Earth?
- Discuss the choice of initial thermal structure (Figure 1a). Why was this used? Is it realistic? Can you propose an alternative?
- Why does single-sided subduction promote curved trenches in 3D geometries?

Further reading

- Tackley, P. J. (2000). Self-consistent generation of tectonic plates in time-dependent, three-dimensional mantle convection simulations. *Geochemistry, Geophysics, Geosystems*, 1 (8).
- Bellahsen, N., Faccenna, C., & Funiciello, F. (2005). Dynamics of subduction and plate motion in laboratory experiments: insights into the “plate tectonics” behavior of the Earth. *Journal of Geophysical Research: Solid Earth* (1978–2012), 110 (B1).
- Van Heck, H. J., & Tackley, P. J. (2008). Planforms of self-consistently generated plates in 3D spherical geometry. *Geophysical Research Letters*, 35 (19).
- Rolf, T., & Tackley, P. J. (2011). Focussing of stress by continents in 3D spherical mantle convection with self-consistent plate tectonics. *Geophysical Research Letters*, 38 (18).
- Crameri, F., & Tackley, P. J. (2014). Spontaneous development of arcuate single-sided subduction in global 3-D mantle convection models with a free surface. *Journal of Geophysical Research: Solid Earth*, 119 (7), pp. 5921-5942.

Paper 2: “The physics of subduction”

- **A simple analytic solution for slab detachment**, Schmalholz, Earth and Planetary Science Letters, 304 (2011), pp. 45–54

Objectives

- Develop understanding of subduction using an analytic model
- Use of simple analytic models versus complex thermo-mechanical models
- Understand the physical controls governing slab break-off, tearing

Earth and Planetary Science Letters 304 (2011) 45–54

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A simple analytical solution for slab detachment

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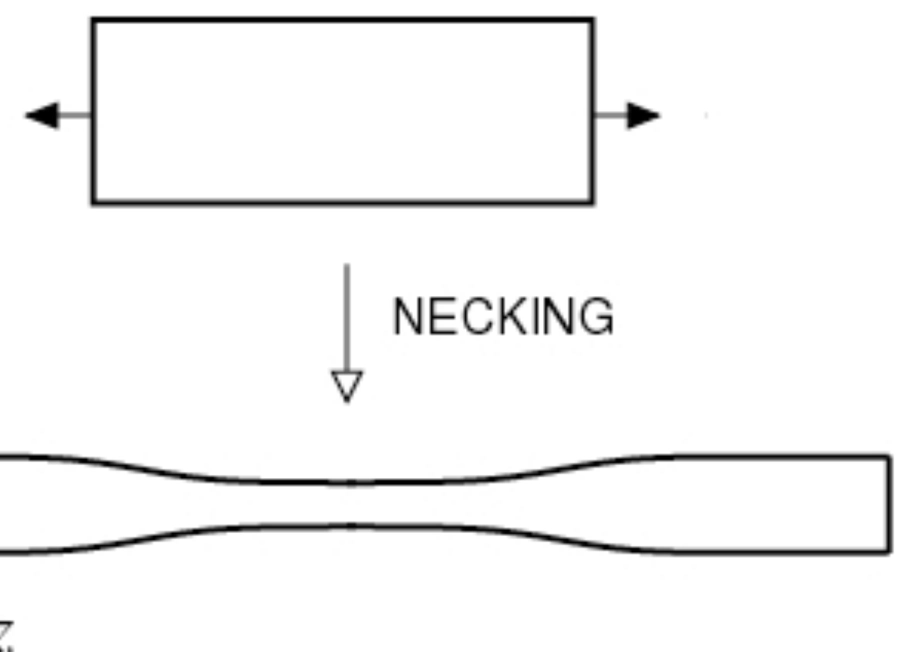
ABSTRACT

An analytical solution is presented for the nonlinear dynamics of high amplitude necking in a free layer of power-law fluid extended in layer-parallel direction due to buoyancy stress. The solution is one-dimensional (1-D) and contains three dimensionless parameters: the thinning factor (i.e. ratio of current to initial layer thickness), the power-law stress exponent, n, and the ratio of time to the characteristic deformation time of a viscous layer under buoyancy stress, t/t_c. t_c is the ratio of the layer's effective viscosity to the applied buoyancy stress. The value of t_c/n specifies the time for detachment, i.e. the time it takes until the layer thickness has thinned to zero. The first-order accuracy of the 1-D solution is confirmed with 2-D finite element simulations of buoyancy-driven necking in a layer of power-law fluid embedded in a linear or power-law viscous medium. The analytical solution is accurate within a factor about 2 if the effective viscosity ratio between the layer and the medium is larger than about 100 and if the medium is a power-law fluid. The analytical solution is applied to slab detachment using dislocation creep laws for dry and wet olivine. Results show that one of the most important parameters controlling the dynamics of slab detachment is the strength of the slab which strongly depends on temperature and rheological parameters. The fundamental conclusions concerning slab detachment resulting from both the analytical solution and from earlier published thermo-mechanical numerical simulations agree well, indicating the usefulness of the highly simplified analytical solution for better understanding slab detachment. Slab detachment resulting from viscous necking is a combination of inhomogeneous thinning due to varying buoyancy stress within the slab and a necking instability due to the power-law viscous rheology ($n>1$). Application of the analytical solution to the Hindu Kush slab provides no "order-of-magnitude argument" against slab detachment and, therefore, supports existing studies suggesting a currently ongoing slab detachment in the Hindu Kush slab.

Necking instability

- Occurs in plastic (non-Newtonian) materials in extension
- Material instability associated with localisation



Slab detachment as a necking instability

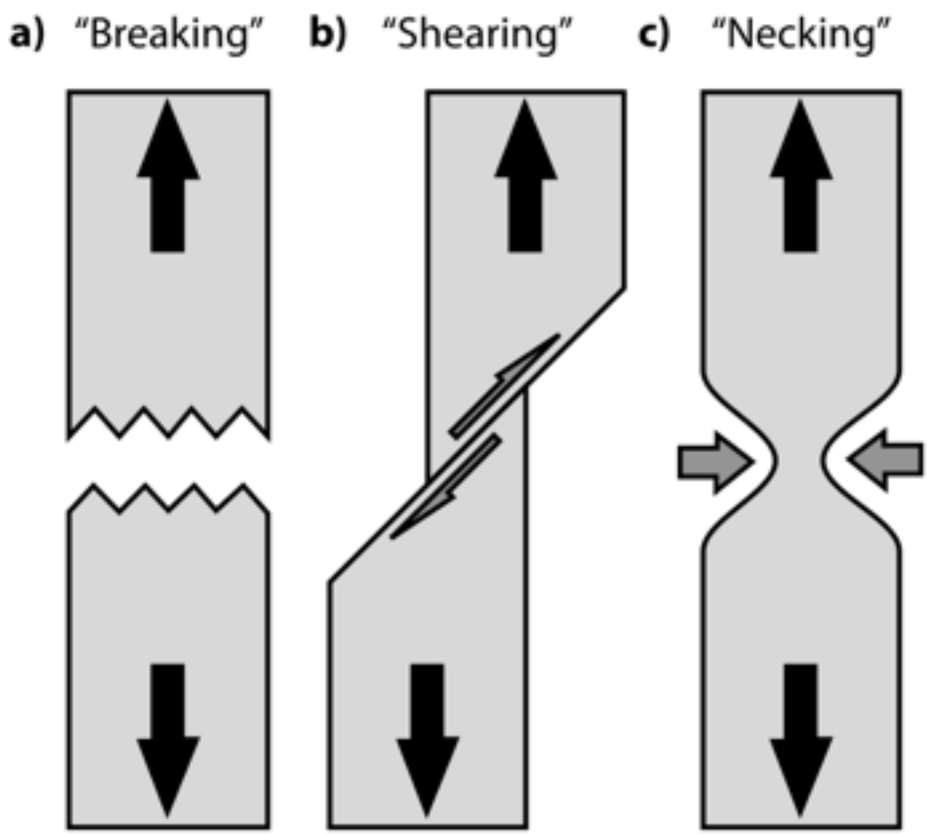


Figure 1. Typical conceptual illustrations of slab detachment. (a) Slab fracture (break-off) as the result of tensile failure (corresponding to a mode I fracture). (b) Simple shear model including the contributions of either plastic or viscous shear zones. (c) Necking model resulting from the extension of a (power law) viscous layer.

Duretz, Schmalholz & Gerya, Geochem. Geophys. Geosys, (2012)

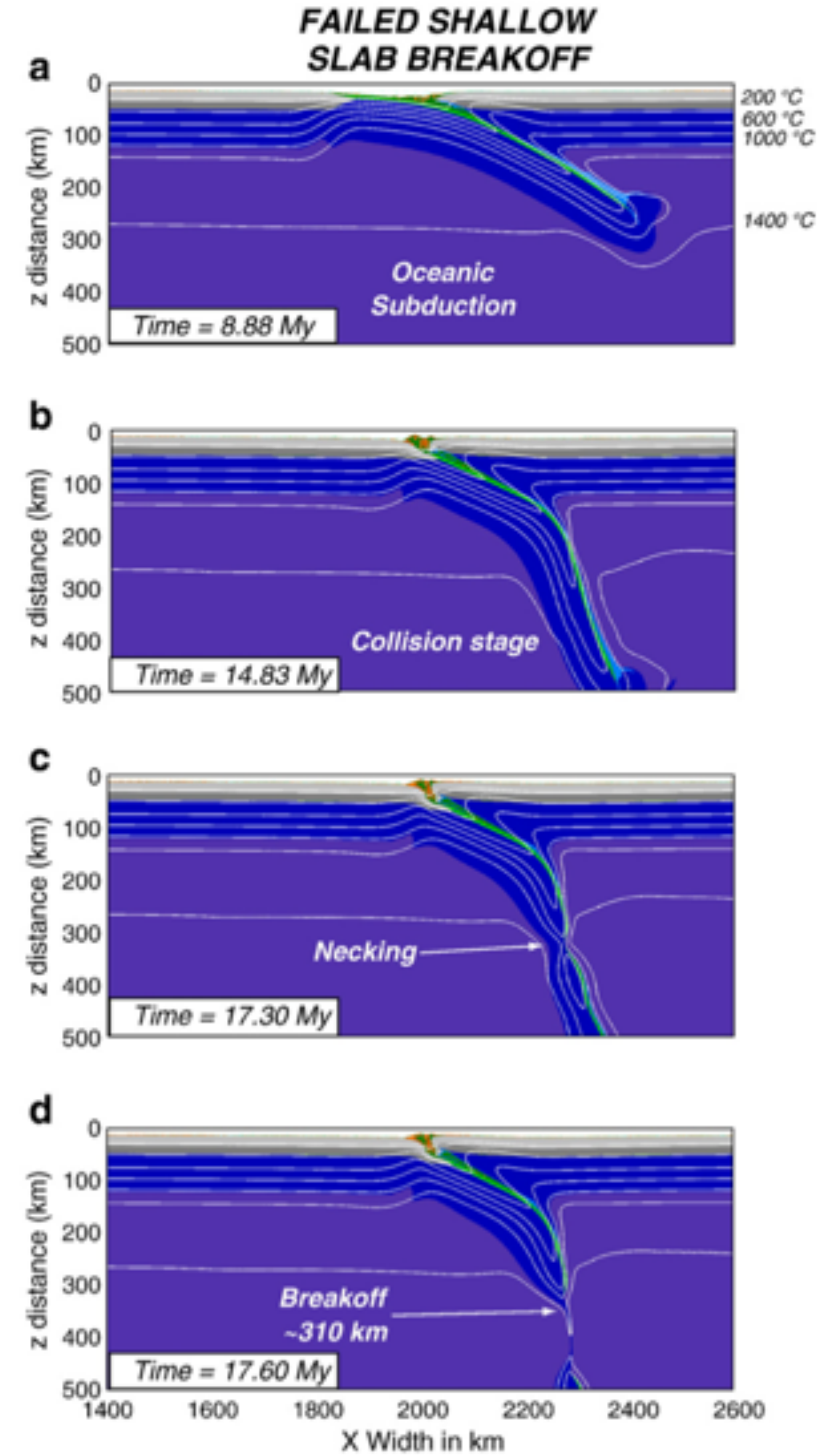
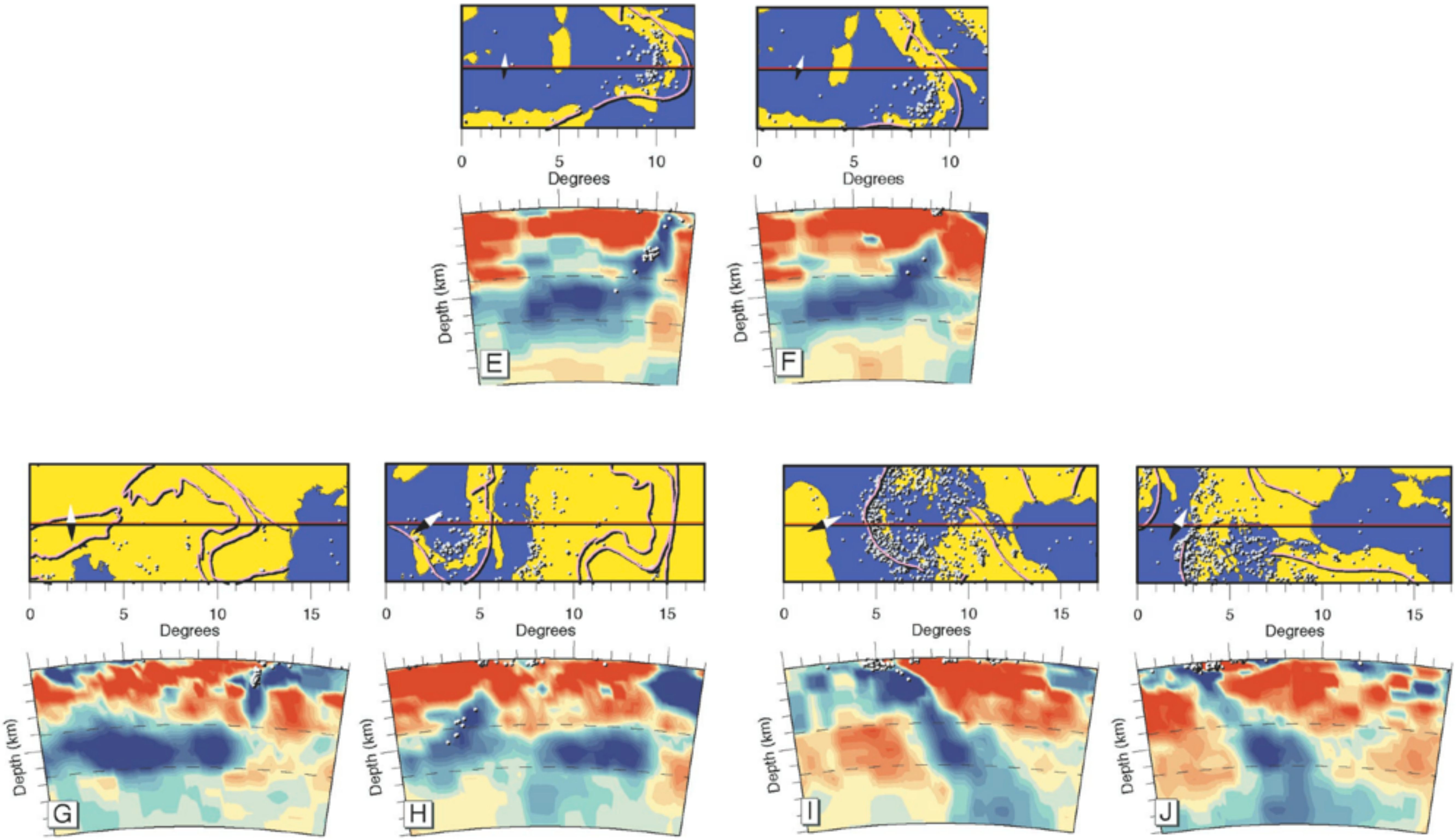


Fig. 5. Evolution of the failed shallow slab breakoff end-member. a) Oceanic subduction. b) Continental collision. c) Necking of the slab d) Breakoff and rebound. Time is incremented from the start of the experiment. Origin of the z distance axis is the top of the box (including the 10 km thick air layer).

Duretz, Gerya & May, Tectonophysics, (2011)

Seismic evidence of detachment



Wortel, Spackman, Science, (2000)

Conceptual model of detachment

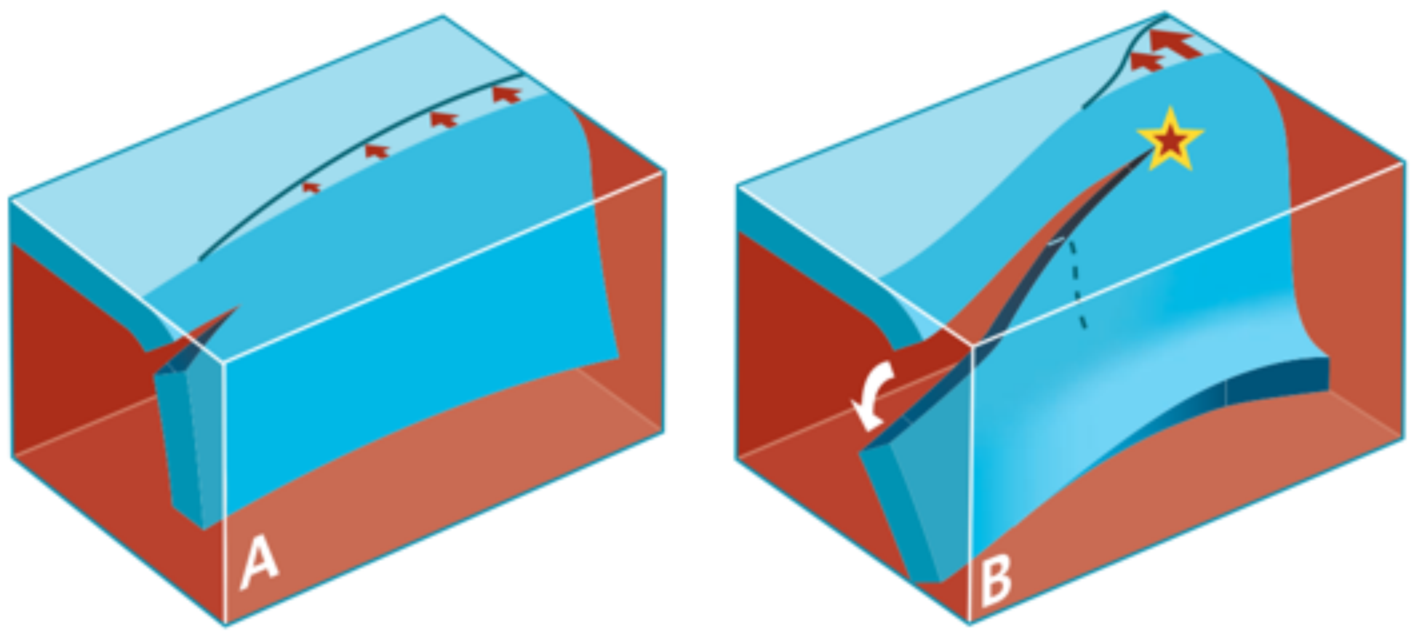


Fig. 3. Lateral migration of slab detachment: a schematic representation [after (*31*)]. An initially small tear in the slab (**A**) propagates approximately horizontally and (**B**) develops into a large tear (*54*). The tear propagation is not expected to take place at a uniform rate; slab detachment most likely occurs episodically, in segments. Eventually the entire slab may break off. The slab pull—the gravitational force associated with the cold, and hence, dense subducted lithosphere—is concentrated in the still continuous part of the slab, leading to pronounced arc curvature. The star indicates seismic activity in the stress concentration region. The initial small tear may develop at one side end of a slab (as indicated here), but also somewhere in an intermediate segment of the subduction zone. The right-hand side of the boxes may, depending on the subduction zone involved, represent the actual side end of a slab, as well as an approximate plane of symmetry. The detached part of the slab does not necessarily remain coherent. The evolving stress distribution may lead to breaking up into separate parts of the detached slab, schematically indicated by the dashed line.

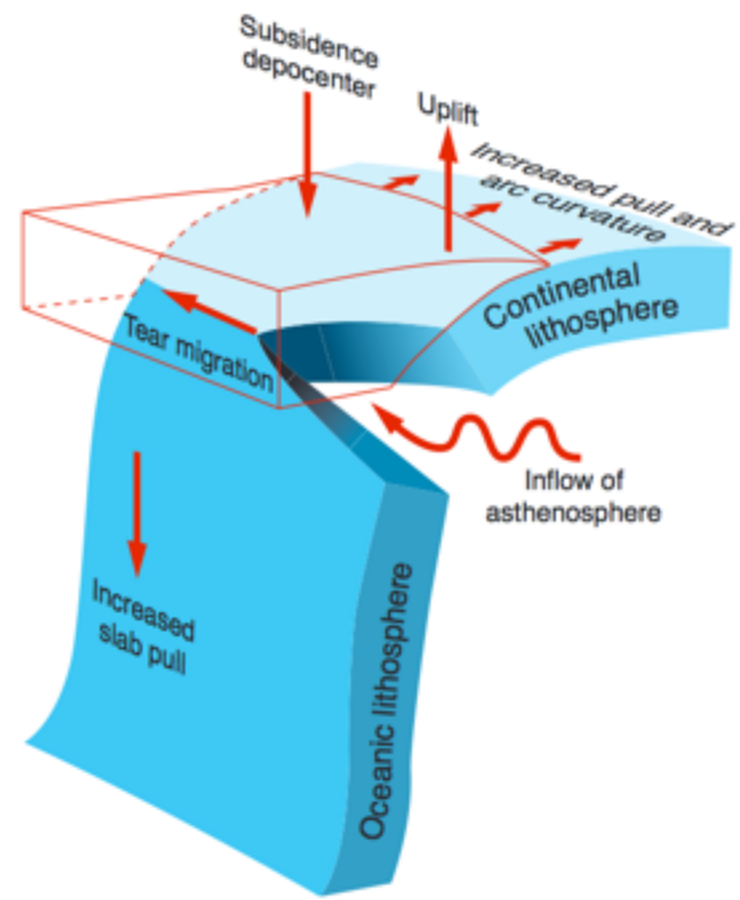


Fig. 4. Plate boundary processes predicted to accompany lateral migration of slab detachment. The concentration of slab pull forces causes a pattern of subsidence (depocenter development) and uplift migrating along strike. It also enhances arc migration (roll-back). Asthenospheric material flows into the gap resulting from slab detachment and causes a specific type of variable composition magmatism, of finite duration, and possibly mineralization.

Wortel, Spackman, Science, (2000)

Objectives

- Complex models have not provided clear understanding of how buoyancy and rheological parameters relat to thinning rate, timing of break-off, etc. More theoretical work required
- Derive analytic solution providing dimensionless parameters defining dynamics of viscous necking driven by buoyancy
- Why? - Simple analytics provides more insight, and more widely applicable to natural examples than complex geodynamic models (e.g. those with all the physics)

Idea

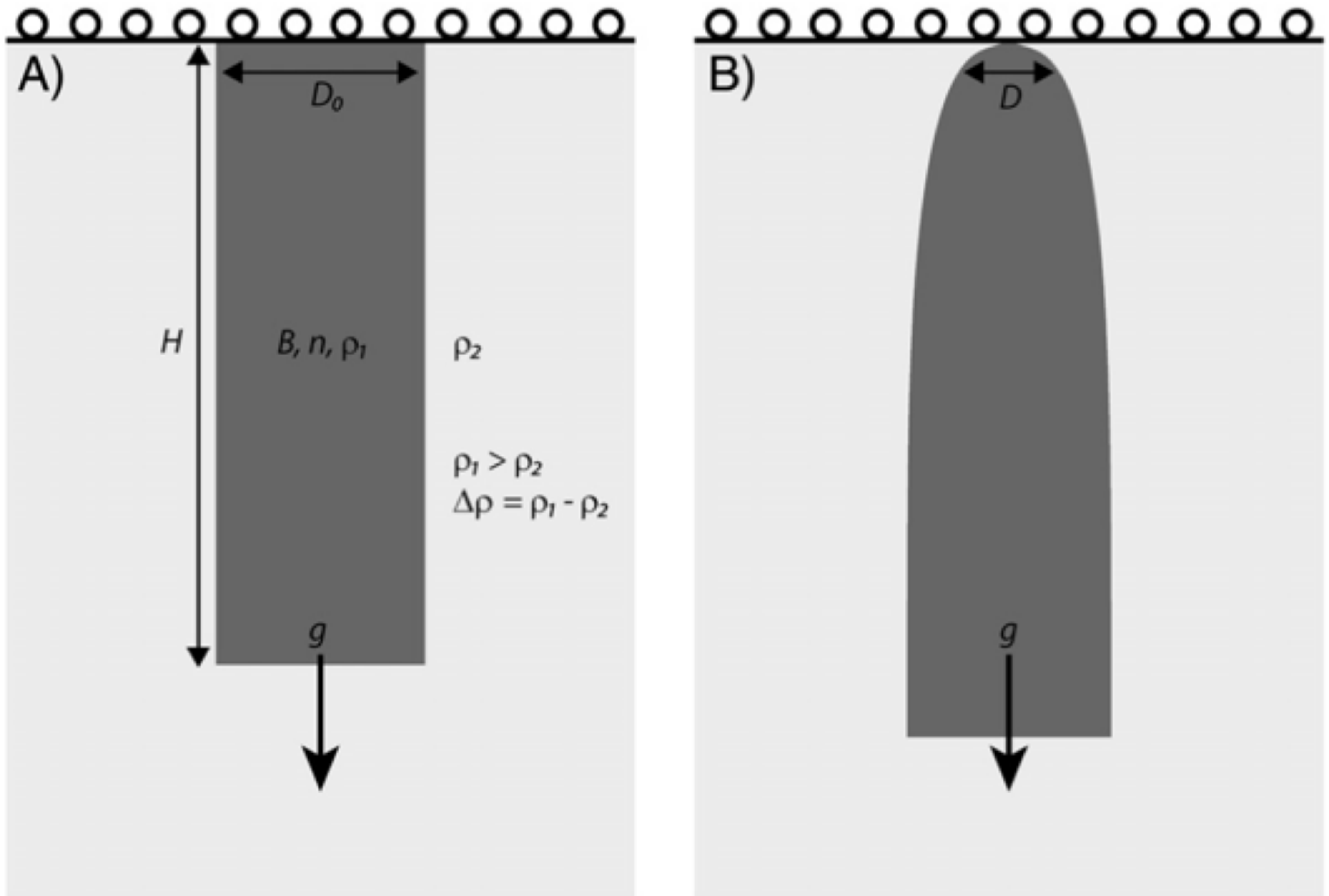


Fig. 1. A) Sketch showing the parameters used for the analytical solution. A layer with length, H, and thickness, D_0, of power-law fluid with stress exponent, n, and coefficient, B, has a larger density, ρ_1, than the inviscid medium with density, ρ_2, surrounding the layer. The layer is attached at its top and gravity acceleration, g, acts downward in a direction parallel to the layer boundaries. B) Typical "cusp" shape of the necked layer with reduced thickness, D, as predicted by the analytical solution.

Model

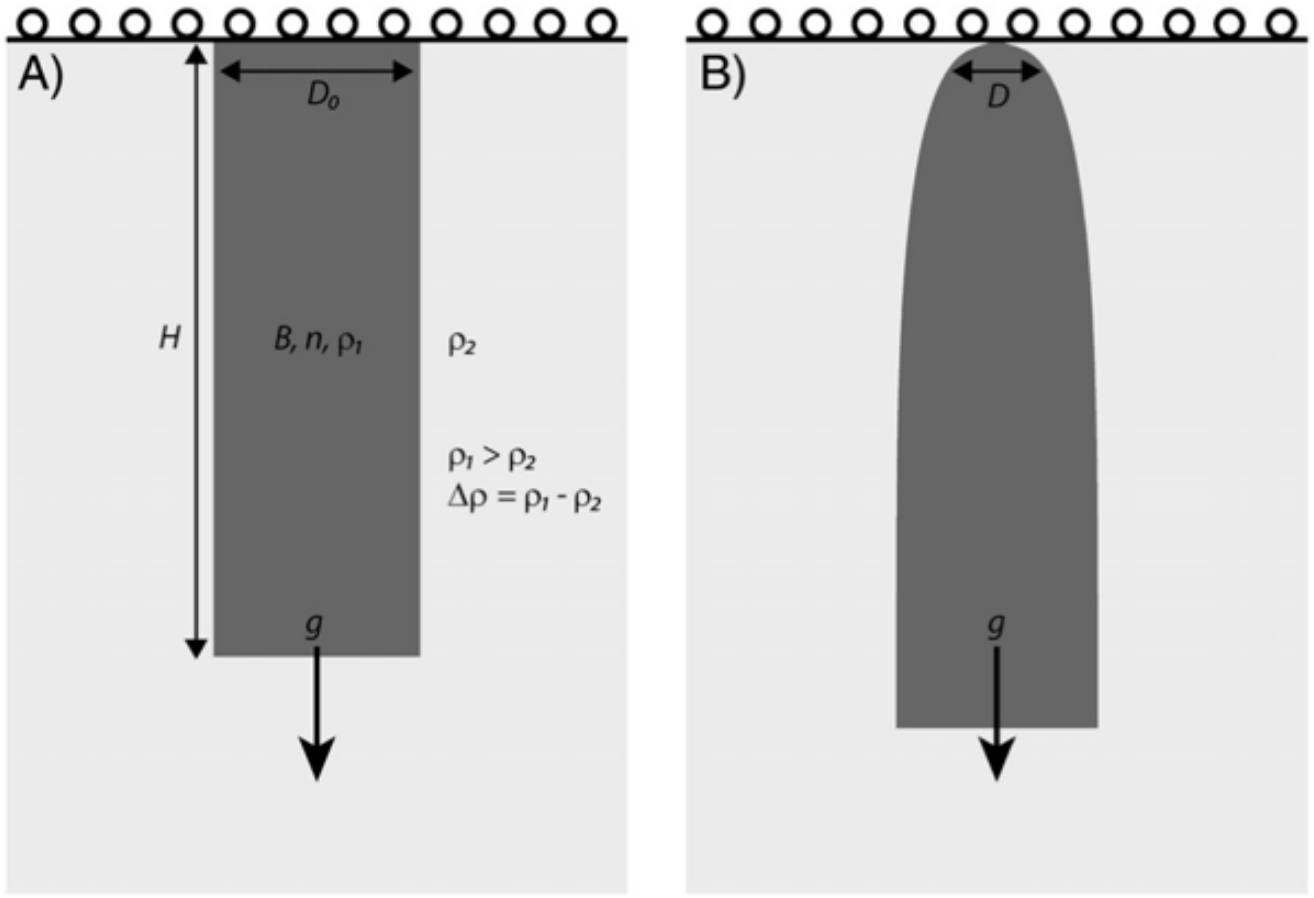
$$\dot{\varepsilon} = -\frac{1}{D}\frac{dD}{dt}$$

$$\tau = \frac{1}{2}\frac{F}{D}$$

$$\dot{\varepsilon} = B\tau^n$$

$$-\frac{1}{D}\frac{dD}{dt} = B\left(\frac{1}{2}\frac{F}{D}\right)^n.$$

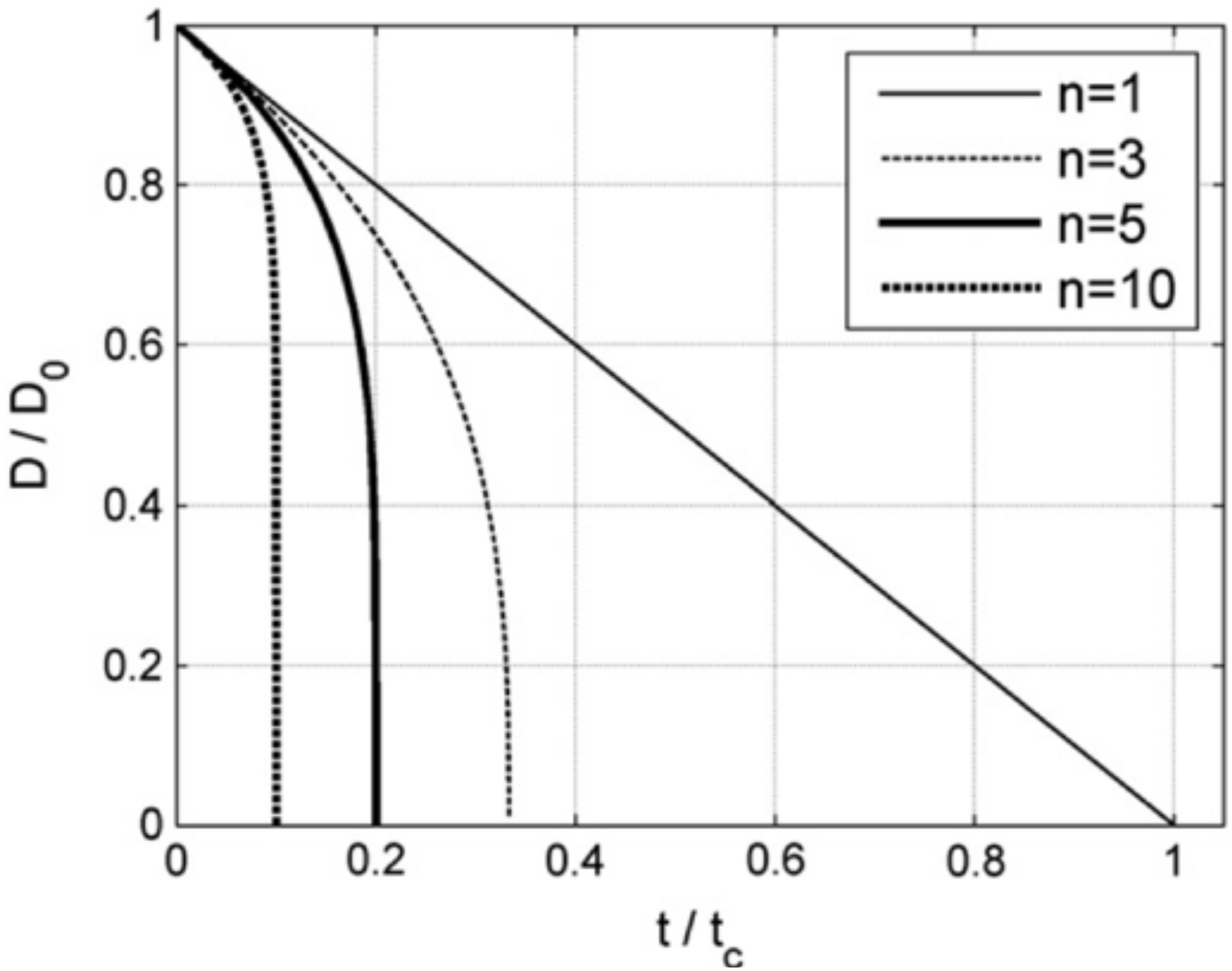
$$F = \Delta\rho g H D_0.$$



- Necking instability only occurs in power-law viscous models (n > 1)

Results

$$\mu_c = \frac{\tau_c}{\dot{\varepsilon}} = \frac{\tau_c^{1-n}}{B}, \quad \tau_c = \frac{1}{2}\Delta\rho g H \quad \text{and} \quad t_c = \frac{\mu_c}{\tau_c} = \frac{1}{B\left(\frac{1}{2}\Delta\rho g H\right)^n}$$



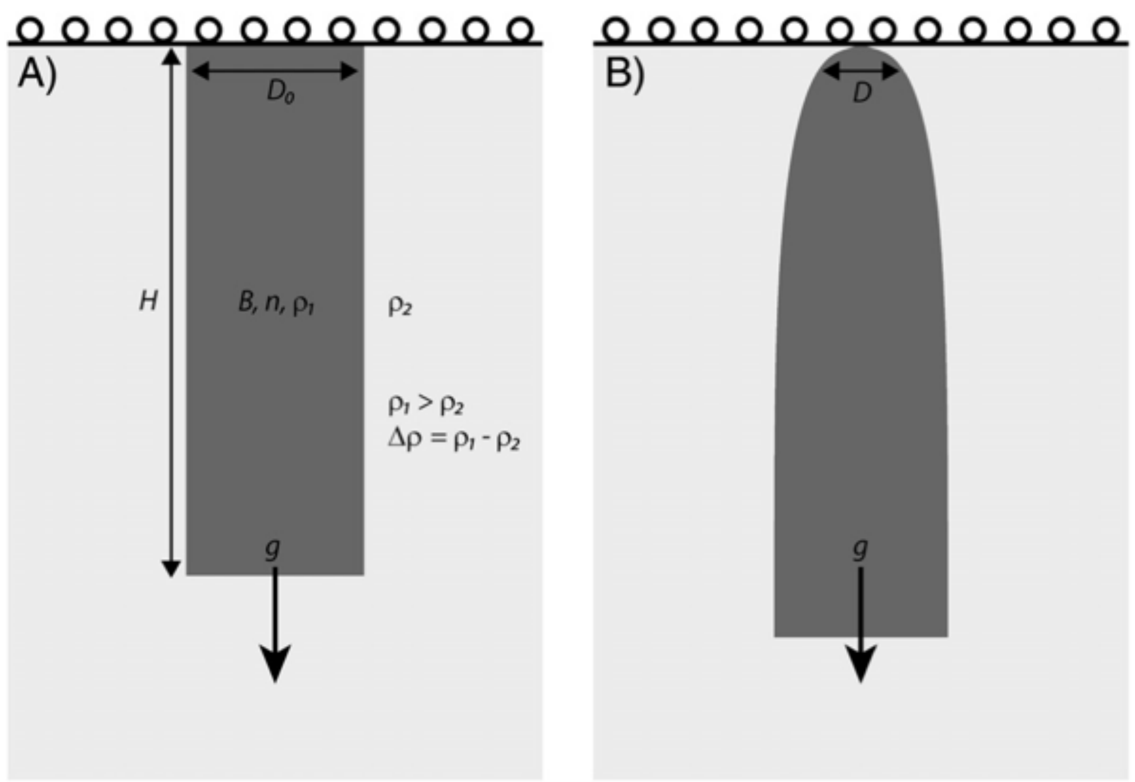
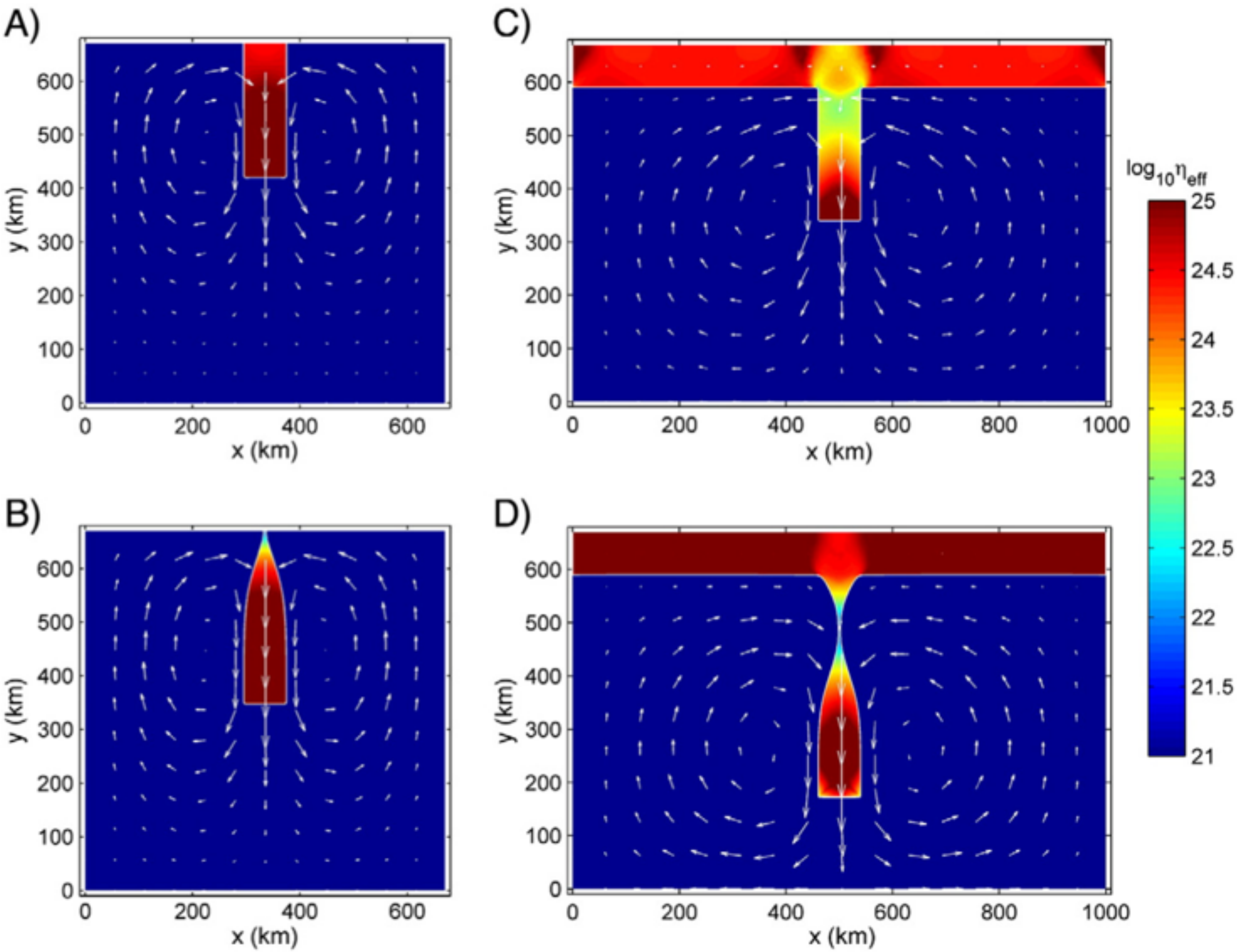
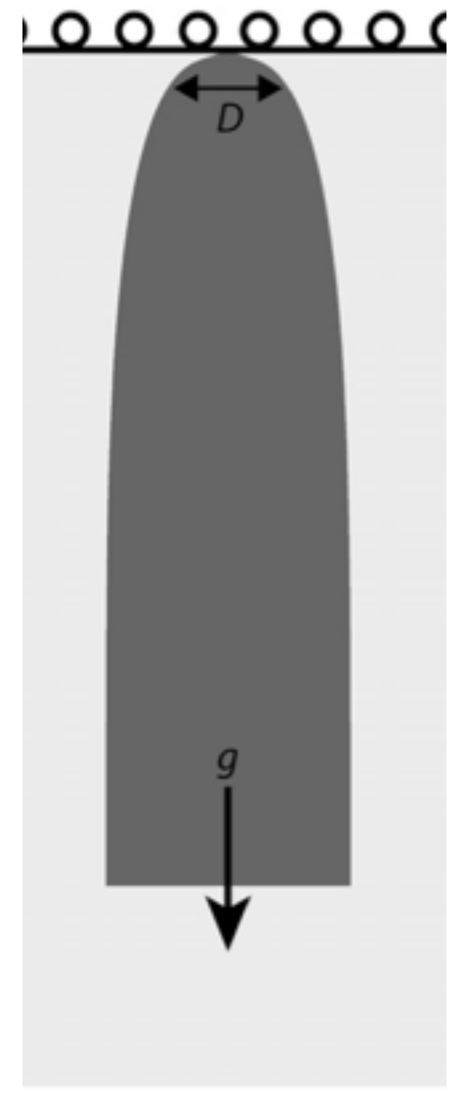


Fig. 2. Thinning factor of the layer, D/D_0, versus dimensionless time, t/t_c, as function of stress exponent, n. D/D_0 is the ratio of current to initial layer thickness, t is time and t_c is the characteristic time defined in Eq. (9). The ratio t_c/n quantifies the time, t, it takes until the layer thickness, D, has thinned to zero.

Comparison with 2D numerics





Comparison with 2D numerics

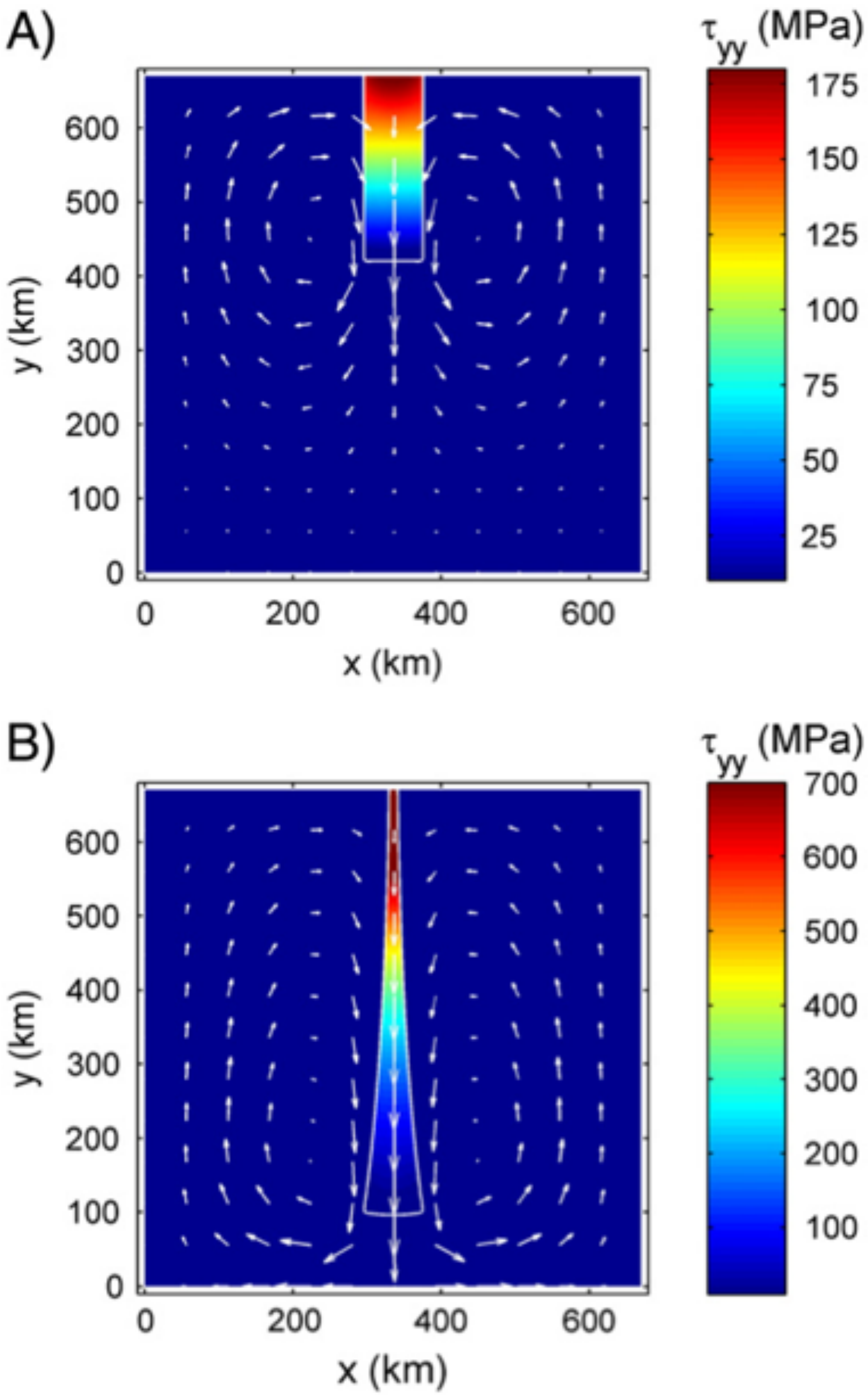


Fig. 4. Simulation results for a linear viscous ($n = 1$, Newtonian) slab. A) Set-up for model 1 with free slip condition at the top of the layer. $\Delta\rho = 150\ \mathrm{kg\ m^{-3}}$, $\eta_0 = 5 \times 10^{23}$ Pa s for the layer, $\eta_0 = 10^{21}$ Pa s for the matrix, and the slab is initially 250 km long and 80 km wide. The vertical deviatoric stress, τ_{yy} (in units of MPa), is shown for the first time step. White arrows indicate the calculated velocity field. B) The layer is more thinned at the top than at the bottom due to the higher deviatoric stress at the top. However, no necking instability and localized thinning developed and, therefore, no actual slab detachment.

Comparison with 2D numerics

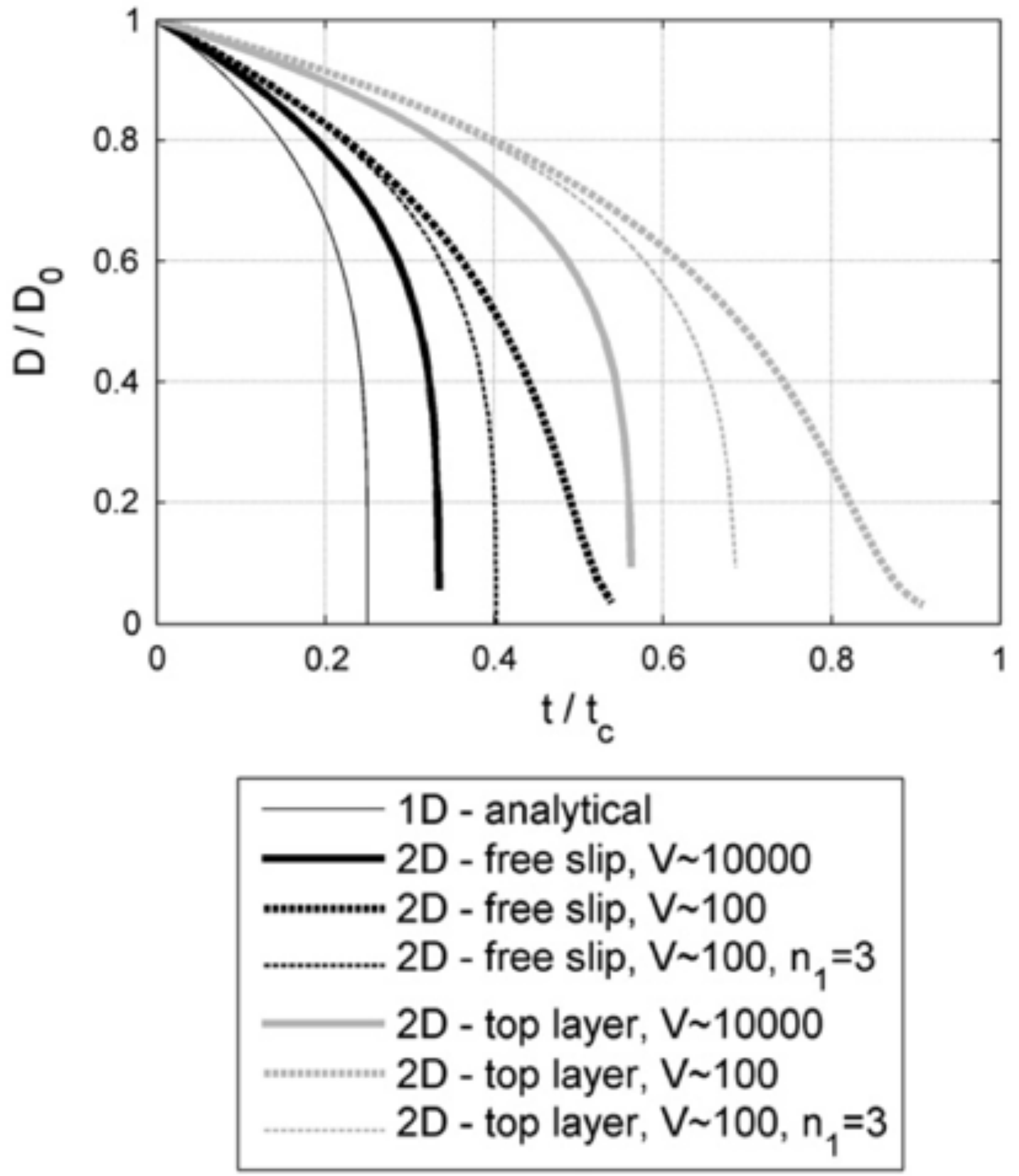


Fig. 5. Evolution of the thinning factor, D/D_0, at the neck versus the dimensionless time, t/t_c, for the 1-D analytical and 2-D numerical simulations. Power-law exponent, n, of the layer is always 4. For the numerical simulations the power-law exponent of the embedding medium $n_1 = 1$ (i.e. linear viscous) if not indicated differently ($n_1 = 3$).

- Quality of fit depends on
  - viscosity contrast between background and slab
  - rheology of background material

Slab detachment model

$$\dot{\varepsilon} = B\tau^n$$

$$B = C\, exp(-E\,/\,RT)$$

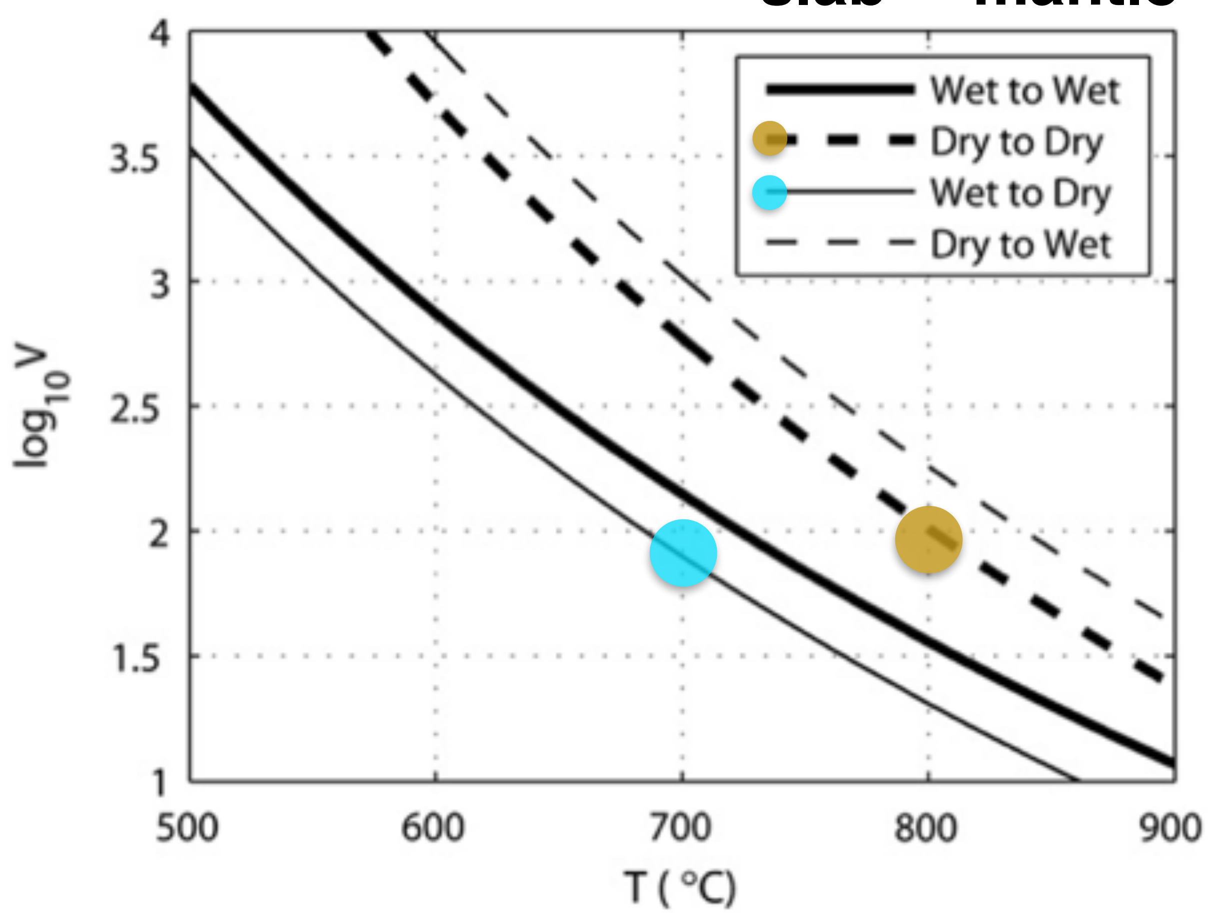
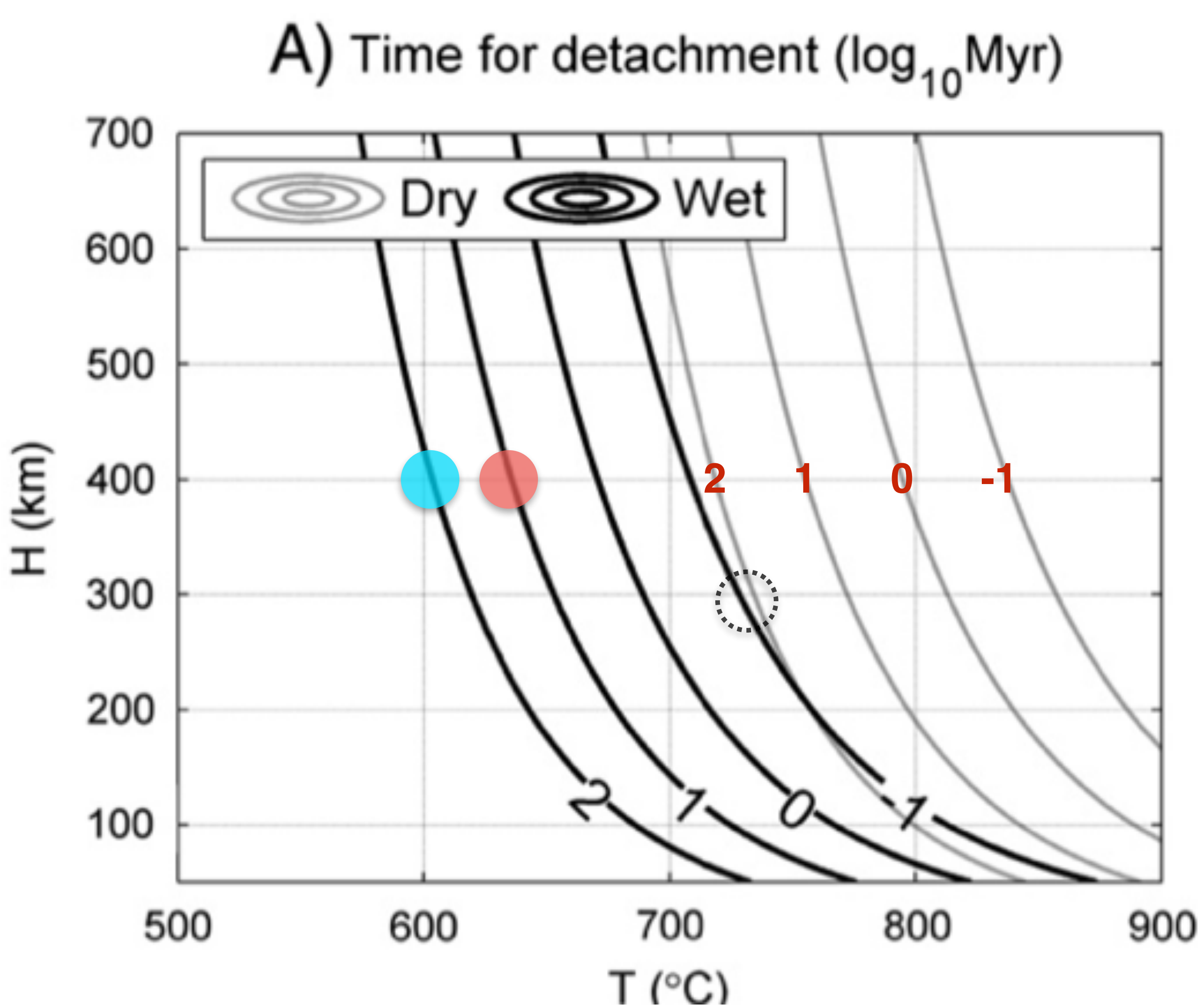


Fig. 6. Effective viscosity ratio, V (shown is $\log_{10}$ of it), between slab and surrounding mantle as function of slab temperature. The mantle temperature is 1200 °C. Both slab and surrounding mantle are assumed to be composed of either wet or dry olivine. The strain rate is $10^{-15}\,s^{-1}$ but strain rate dependence of V is negligible because power-law exponents of wet and dry olivine are similar (3.5 for dry and 4 for wet olivine).

- Assume Arrhenius type flow law
- Assume olivine parameters (dry or wet)
- n = 3.5 (wet)
- n = 4.0 (dry)
- Assume strain-rate of 1e-15 1/s

Slab detachment model



slab temperature

- Wet slab versus dry slab
- Detachment time is a strong function of temperature and rheology
- "Wet" slab has a lower viscosity, thus detachment time is reduced (for a fixed slab temperature)

Slab heating

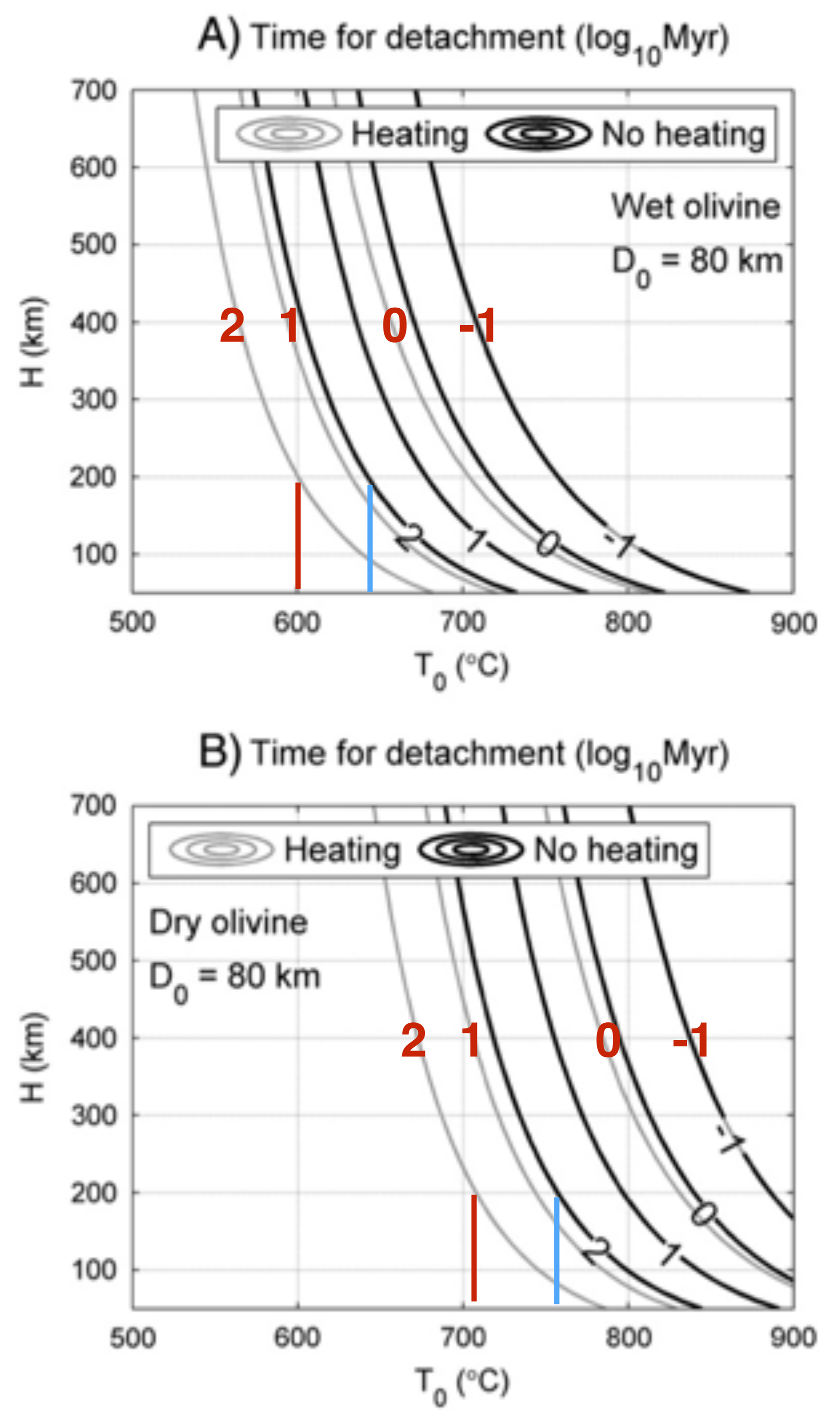
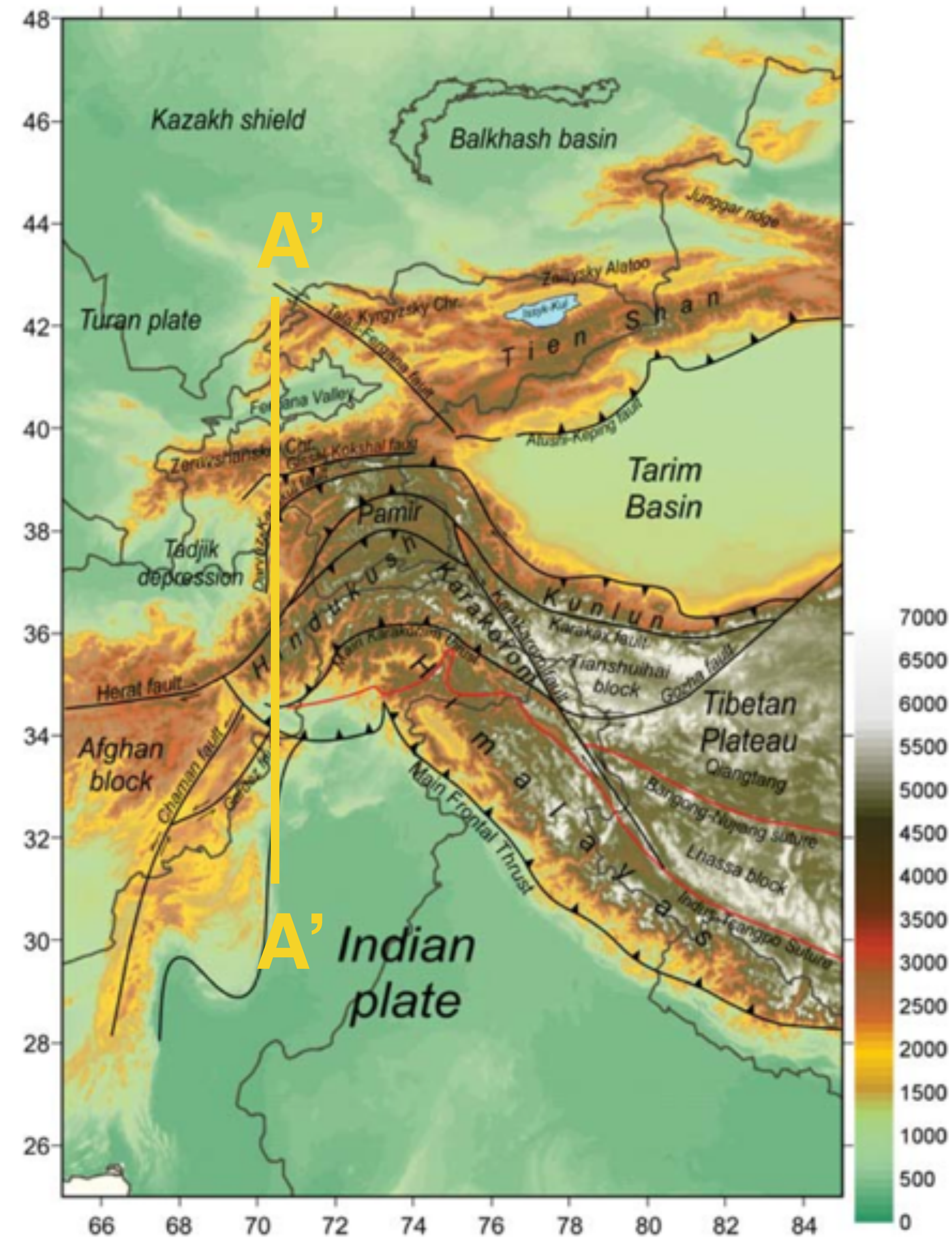


Fig. 8. The time for detachment, t_c/n, for $\Delta\rho = 75\ \mathrm{kg\ m^{-3}}$ is contoured in the space initial slab temperature, T_0 (°C), versus slab length, H (km). The heating due to the transient temperature increase inside the slab has been considered in the calculations using Eq. (15). Results are displayed for wet olivine for an initial slab thickness, D_0, of 80 km (A) and for dry olivine for an initial slab thickness, D_0, of 80 km (B). The gray contour lines are for the same values and in the same order as the labeled black contour lines (labels are values of $\log_{10}$ of time in Myr).

$$T \approx T_m - (T_m - T_0)\exp\left(-\frac{\pi^2 \kappa t}{D^2}\right)$$

- T_m is the surrounding mantle temperature
- T_0 is the initial slab temperature
- Diffusivity = 1e-6 m^2/s

Application

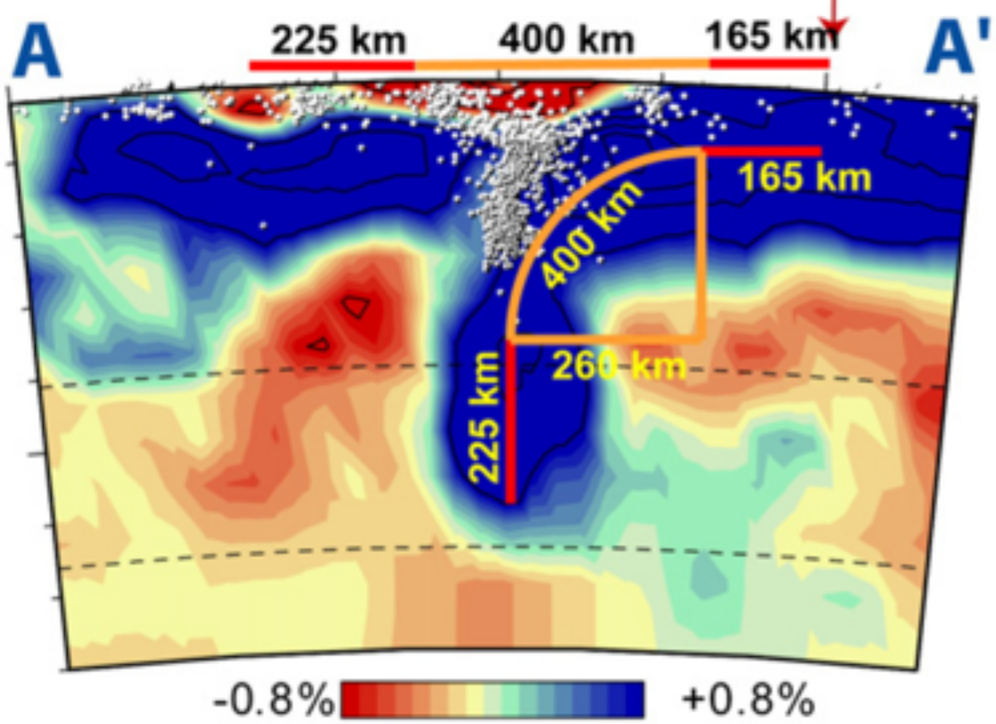


Koulakov & Sobolev, GJI, (2006)

- Hindu Kush slab
- Seismic studies and kinematic studies indicate on-going detachment
- **Assumed that “on-going” implies that necking occurs over 10 Myr period (or less)**
- Test hypothesis

Application

Negredo et al, EPSL, (2007)



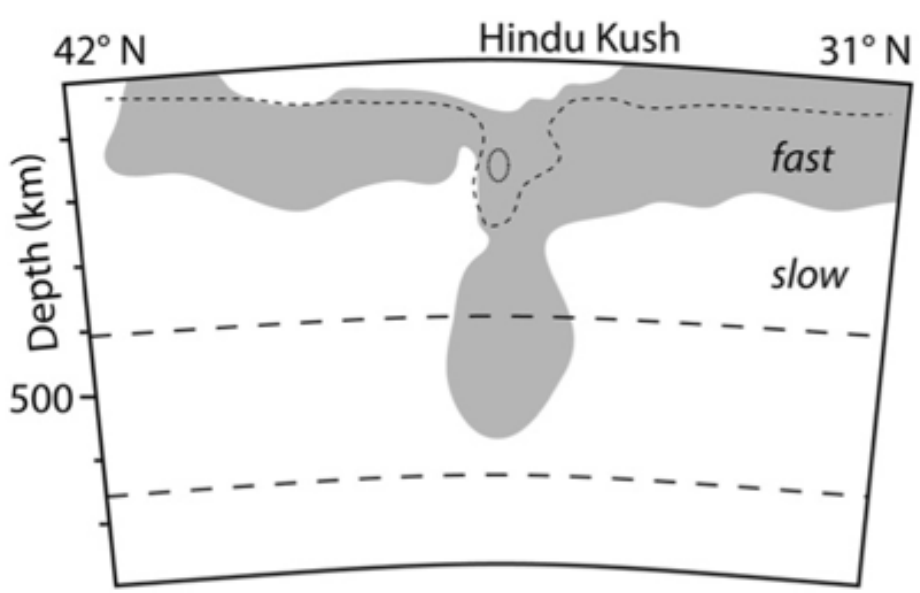


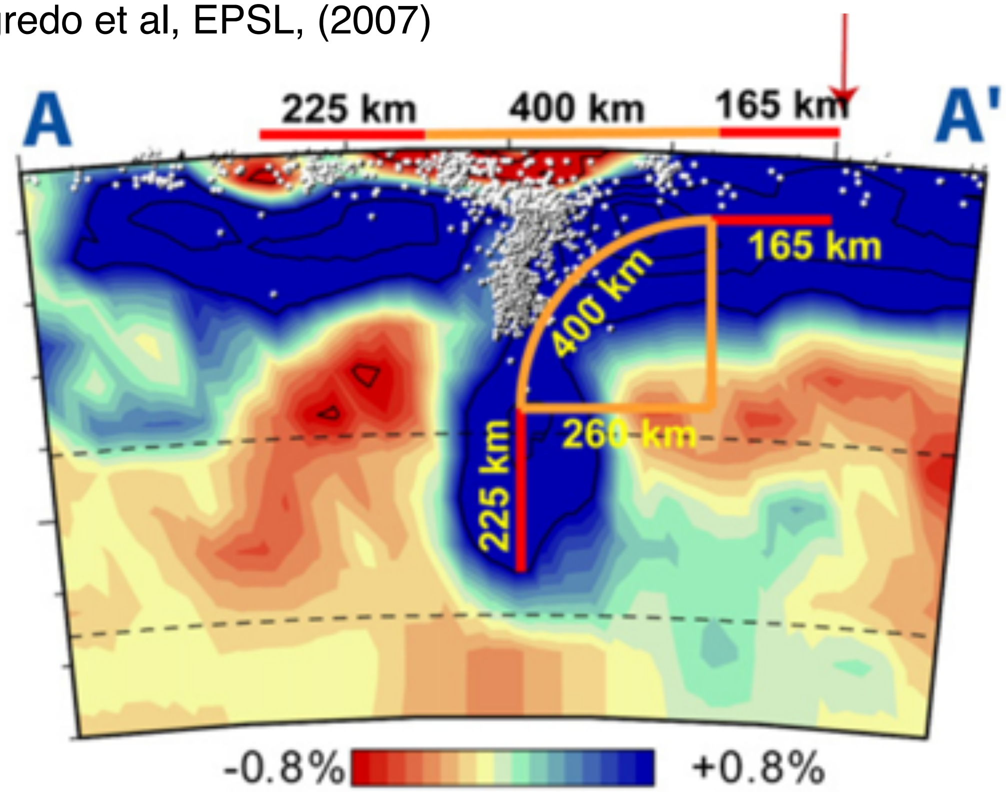
Fig. 9. Sketch of the basic features of the global P-wave tomography model for the Hindu Kush slab after Negredo et al. (2007). The slab dips nearly vertically. The thin dashed line within the gray area of fast P-wave velocities indicates the approximate lower boundary of the occurrence of earthquakes. The dashed ellipse indicates the potential zone of active necking in a depth of 150–200 km according to Lister et al. (2008).

- P-wave tomography
- Interpreted and then slab geometry estimated

- Given H (450 km), require T > 630 degC and strain-rates > 1e-15 1/s.
- Including slab heating would imply T > 600 degC

Application

Negredo et al, EPSL, (2007)



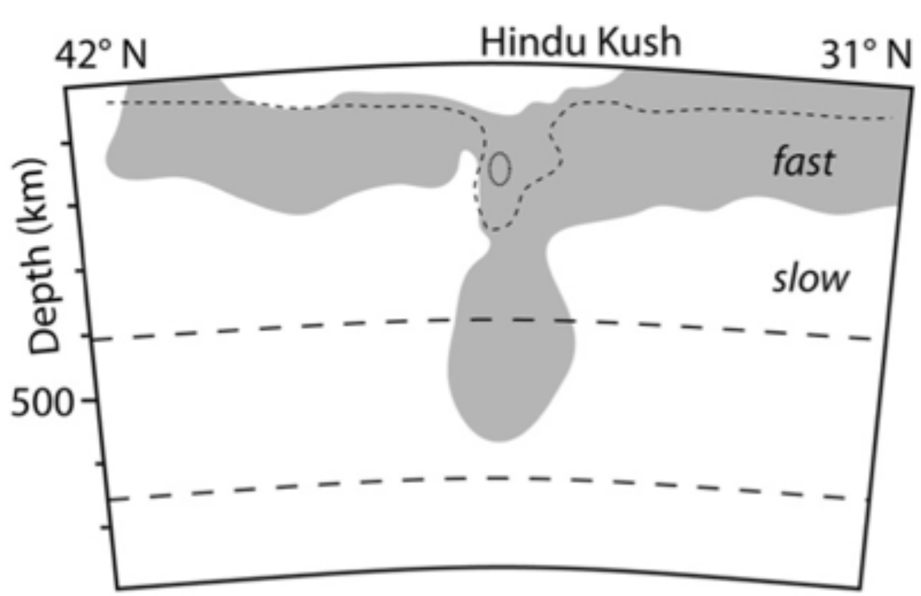


Fig. 9. Sketch of the basic features of the global P-wave tomography model for the Hindu Kush slab after Negredo et al. (2007). The slab dips nearly vertically. The thin dashed line within the gray area of fast P-wave velocities indicates the approximate lower boundary of the occurrence of earthquakes. The dashed ellipse indicates the potential zone of active necking in a depth of 150–200 km according to Lister et al. (2008).

$$t = \frac{t_c}{n} = \frac{\mu_c}{n\tau_c} < 10(\text{Myr}) \Rightarrow \mu_c < 10(\text{Myr})\, n\tau_c.$$

- Consider wet olivine, T=630 C
- Characteristic viscosity ~2e23 Pa s
- If viscosity < 2e23 Pa s, necking last less than 10 Myr
- Characteristic slab strengths at 150-200 km depth imply viscosity ~ 8e22 Pa s and this *“supports the hypothesis of currently ongoing slab detachment in the Hindu Kush slab”*

Paper 2: Some open questions

- What are the major weaknesses / shortcomings of this approach? Discuss in terms of the assumptions made within the definition of the model.
- If the slab is assumed to be a viscous material, can the quantity D/D_0 ever equal zero? Is this a suitable definition of when a slab is detached? If yes, explain your reasoning, if no, discuss an alternative quantitative definition of what slab-detachment could be.
- What are your thoughts about this comment (pg 50)??: *"…in general one can also argue that its use is justified because the error introduced by simplification is likely equal or even smaller than the errors arising from the uncertainty in the input and model parameters (i.e. temperature, rheological para- meters, amount of melt, etc.) needed for the thermo-mechanical numerical simulations."*
- The method is applied the Hindu Kush slab. Why do you think this slab was selected to apply the analytic solution too? How widely applicable do you think this analytic solution is to subduction zones on Earth?
- What conditions should be met in order to confidently apply this analytic solution to understand slab dynamics? How could you constrain such conditions (e.g. ensure they were satisfied)?

Further reading

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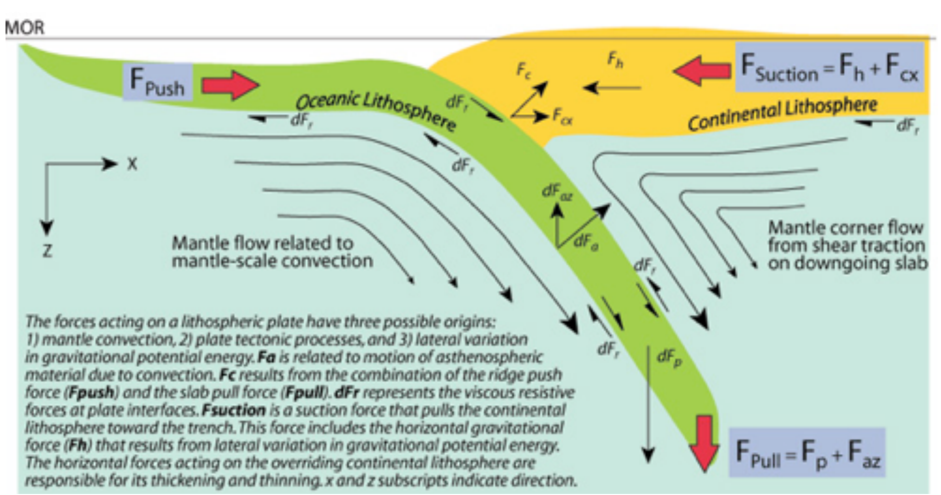
- van Hunen, J., & Allen, M. B. (2011). Continental collision and slab break-off: a comparison of 3-D numerical models with observations. *Earth and Planetary Science Letters*, 302 (1), pp. 27-37.
- Duretz, T., & Gerya, T. V. (2013). Slab detachment during continental collision: Influence of crustal rheology and interaction with lithospheric delamination. *Tectonophysics*, 602, pp. 124-140.
- Duretz, T., Gerya, T. V., & Spakman, W. (2014). Slab detachment in laterally varying subduction zones: 3-D numerical modelling. *Geophysical Research Letters*, 41 (6), pp. 1951-1956.

Paper 3: Numerical modelling of subduction

- **A benchmark comparison of spontaneous subduction models—Towards a free surface**, Schmeling et al, Physics of the Earth and Planetary Interiors, Vol. 17, (2008), pp. 198—223



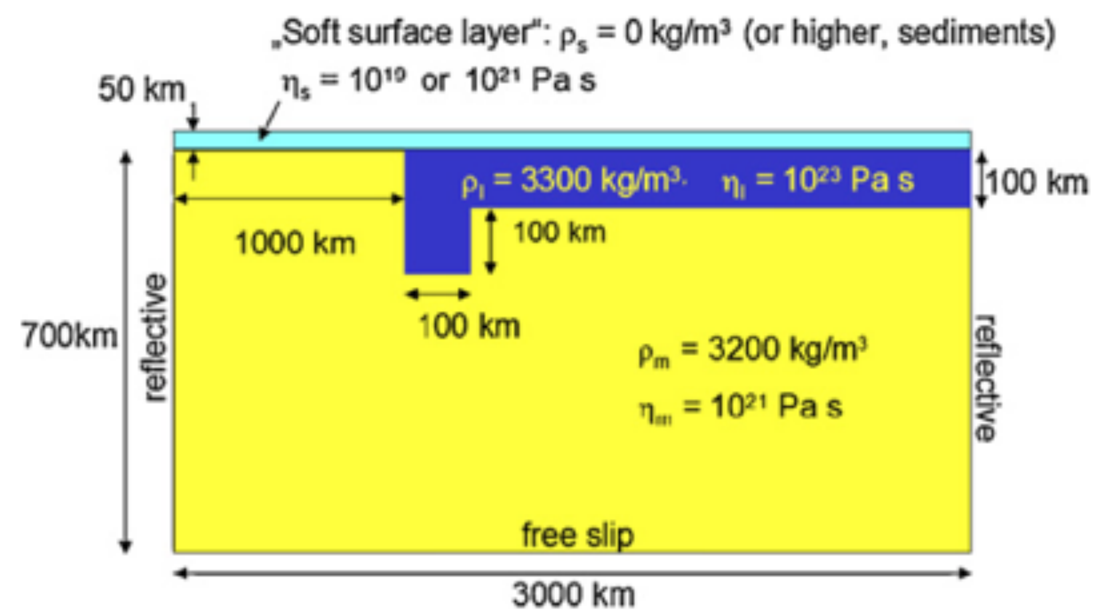


Fig. 1. Model setup and initial condition of the benchmark cases 1 and 2.

Objectives

- Many geodynamic concepts associated with subduction are explained via numerical models.
- To understand how these methods work and their limitations.
- Specifically, to understand how following factors may influence results
  - how discretisation error affects solution
  - how the choice of boundary condition influences solution
  - how the representation of material properties affects solution

Physics of the Earth and Planetary Interiors 171 (2008) 198–223

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journal homepage: www.elsevier.com/locate/pepi

A benchmark comparison of spontaneous subduction models—Towards a free surface

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ABSTRACT

Numerically modelling the dynamics of a self-consistently subducting lithosphere is a challenging task because of the decoupling problems of the slab from the free surface. We address this problem with a benchmark comparison between various numerical codes (Eulerian and Lagrangian, Finite Element and Finite Difference, with and without markers) as well as a laboratory experiment. The benchmark test consists of three prescribed setups of viscous flow, driven by compositional buoyancy, and with a low viscosity, zero-density top layer to approximate a free surface. Alternatively, a fully free surface is assumed. Our results with a weak top layer indicate that the convergence of the subduction behaviour with increasing resolution strongly depends on the averaging scheme for viscosity near moving rheological boundaries. Harmonic means result in fastest subduction, arithmetic means produces slow subduction and geometric mean results in intermediate behaviour. A few cases with the infinite norm scheme have been tested and result in convergence behaviour between that of arithmetic and geometric averaging. Satisfactory convergence of results is only reached in one case with a very strong slab, while for the other cases complete convergence appears mostly beyond presently feasible grid resolution. Analysing the behaviour of the weak zero-density top layer reveals that this problem is caused by the entrainment of the weak material into a lubrication layer on top of the subducting slab whose thickness turns out to be smaller than even the finest grid resolution. Agreement between the free surface runs and the weak top layer models is satisfactory only if both approaches use high resolution. Comparison of numerical models with a free surface laboratory experiment shows that (1) Lagrangian-based free surface numerical models can closely reproduce the laboratory experiments provided that sufficient numerical resolution is employed and (2) Eulerian-based codes with a weak surface layer reproduce the experiment if harmonic or geometric averaging of viscosity is used. The harmonic mean is also preferred if circular high viscosity bodies with or without a lubrication layer are considered. We conclude that modelling the free surface of subduction by a weak zero-density layer gives good results for highest resolutions, but otherwise care has to be taken in (1) handling the associated entrainment and formation of a lubrication layer and (2) choosing the appropriate averaging scheme for viscosity at rheological boundaries.

Why do we use numerics?

- Laboratory models suffer from a number of limitations
  - Restricted range of relevant materials (rheology)
  - Effective boundary conditions are not well understood
  - 3D effects can be suppressed via finite size of model domain
  - Thermal effects are hard to control
  - Extracting quantitative measurements can be difficult (velocity, pressure, strain-rate, …)

What is a benchmark?

- Do the “results” from numerical method A “agree” with numerical method B?
- “results” —> a given model output, e.g. velocity at a point in space, average topography, evolution of a point in space as a function of time
- “agree” —> hopefully a quantitative comparison of the “results” — often geodynamists resort to a visual comparison
- If results from method A and method B do agree - what do we learn and what can we confidently conclude…
- For example, can we say:
    - (i) we solved the system of equations correctly?
    - (ii) the system of equations describes the phenomena of interest?

Reference model

$$-\vec{\nabla}P + \frac{\partial}{\partial x_j}\left[\eta_k\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] - \rho_k g \vec{e}_3 = 0$$

$$\vec{\nabla} \cdot \vec{v} = 0$$

$$\frac{\partial C_k}{\partial t} + \vec{v} \cdot \vec{\nabla} C_k = 0$$

- *No temperature dependence*
    - Constant viscosity
    - Constant density

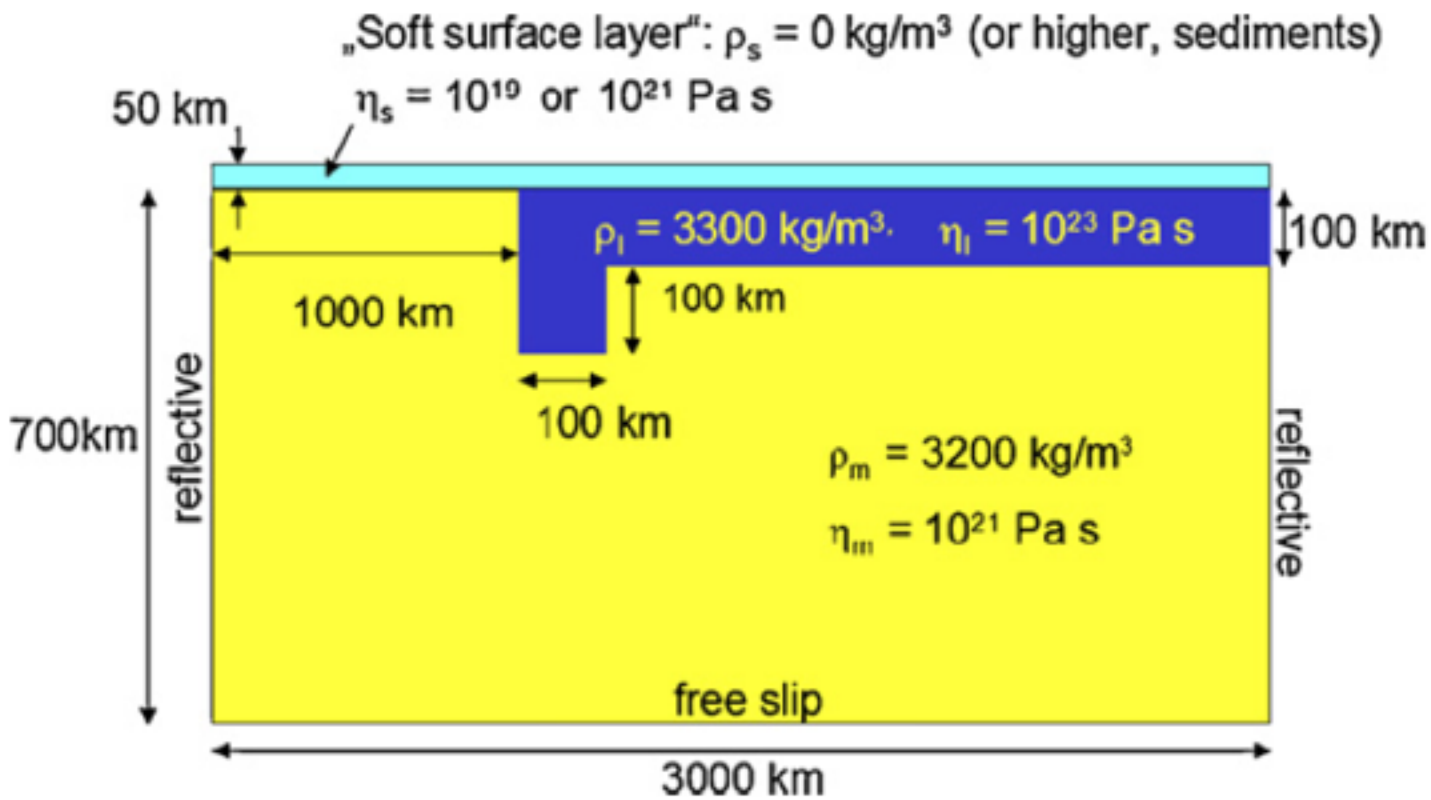


Fig. 1. Model setup and initial condition of the benchmark cases 1 and 2.

Participating methods

Flow solver

$$-\vec{\nabla}P + \frac{\partial}{\partial x_j}\left[\eta_k\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)\right] - \rho_k g \vec{e}_3 = 0$$

$$\vec{\nabla}\cdot\vec{v} = 0$$

FDCON
I2VIS

Finite difference methods

LAPEX-2D
CITCOM
ABAQUS
LAMEM
FEMS-2D

Finite element methods

Participating methods

Transport solver

$$\frac{\partial C_k}{\partial t} + \vec{v} \cdot \vec{\nabla} C_k = 0$$

FDCON
I2VIS
LAPEX-2D
LAMEM

Lagrangian markers

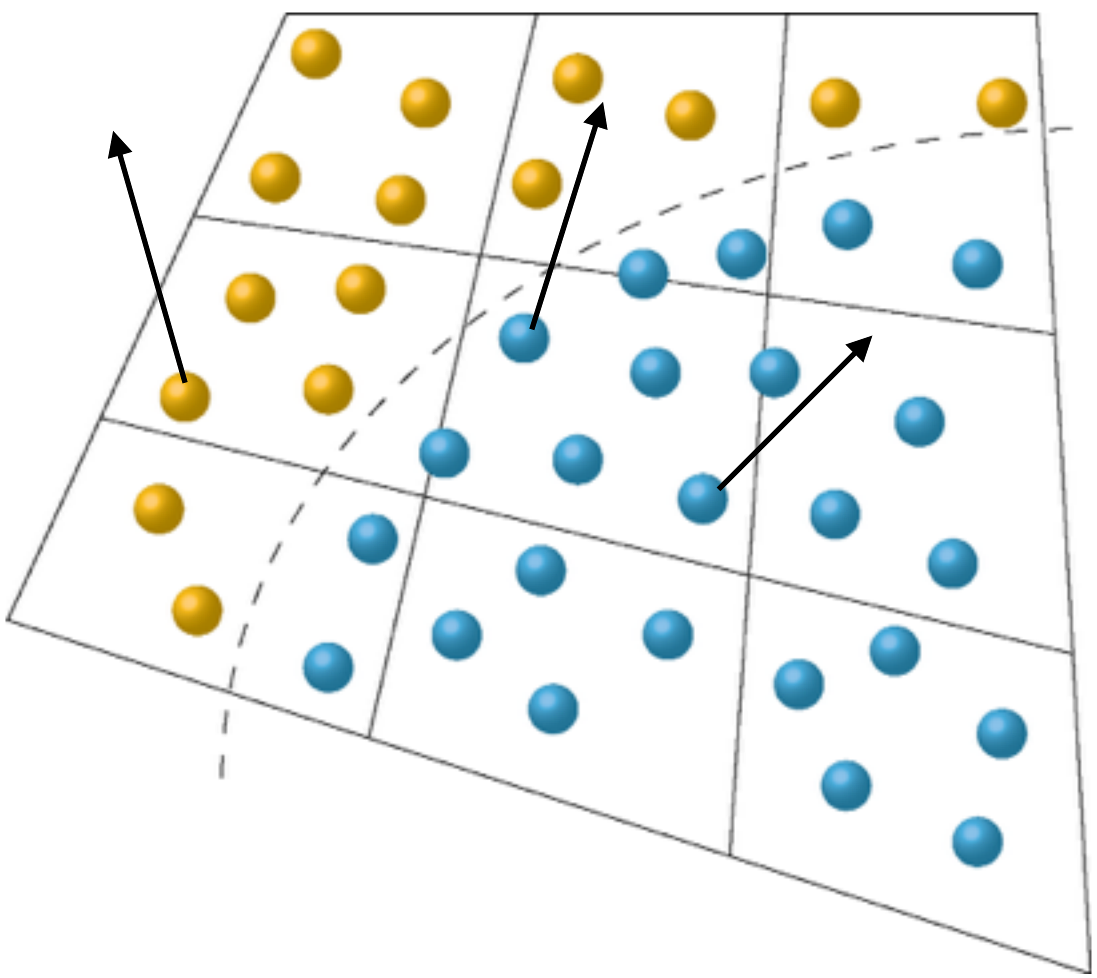
CITCOM
ABAQUS

Mesh based field approach

FEMS-2D

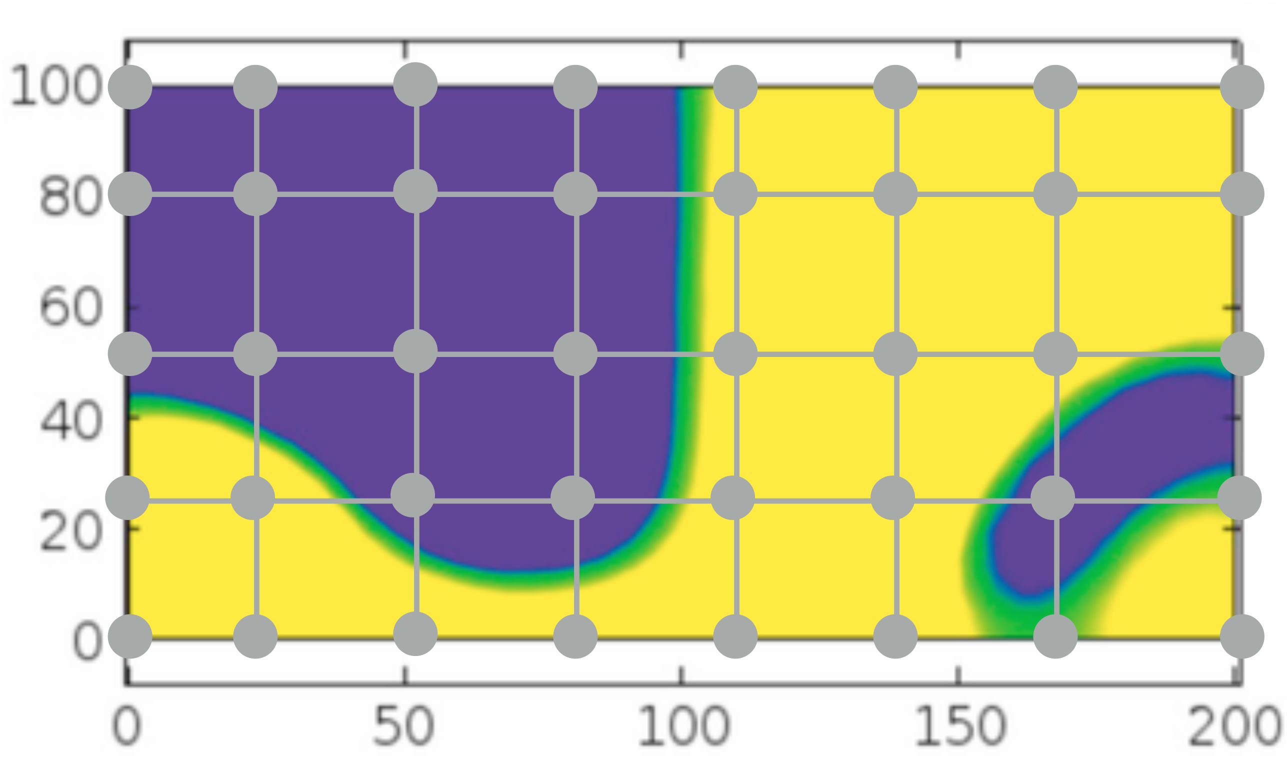
Interface tracking

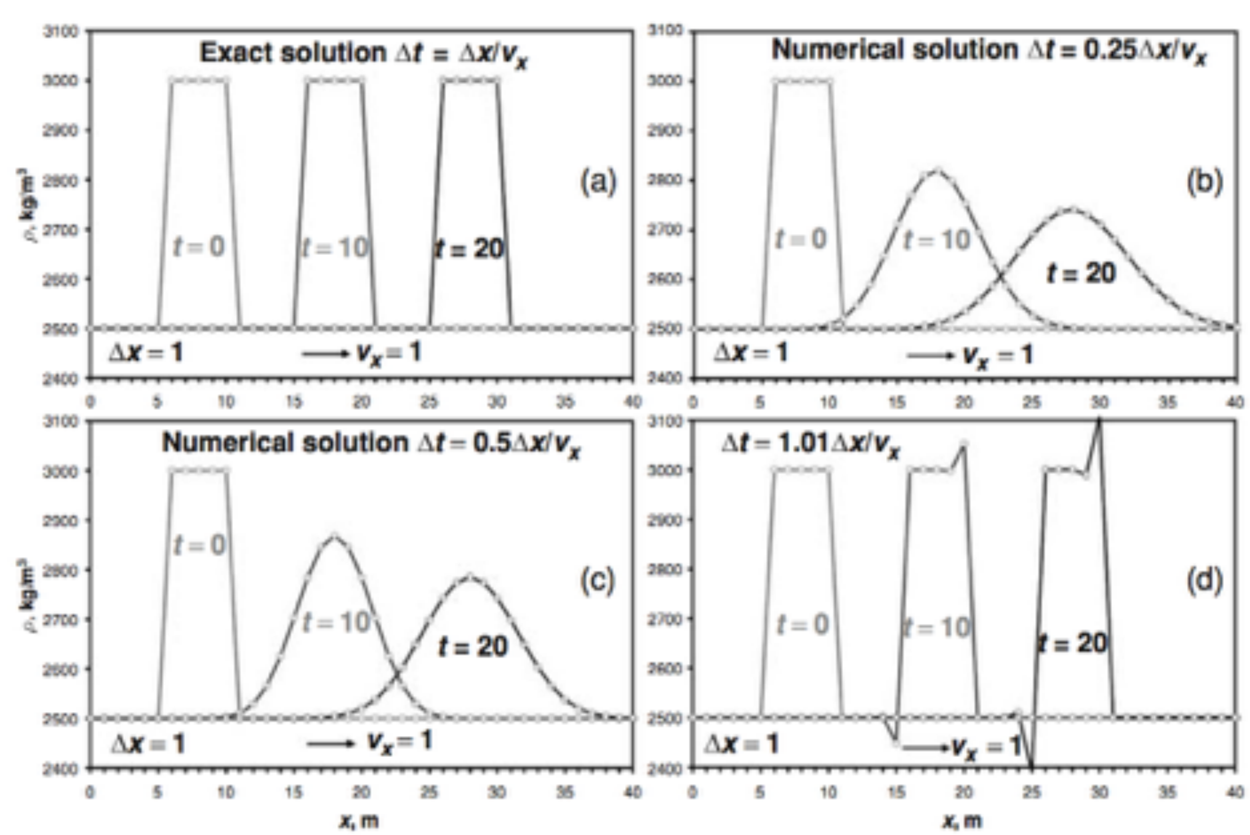
Lagrangian markers



- Volumetric representation of a material
- Particles track composition (rheology)
- Particles move through the mesh
- Material properties on particles are are “averaged” onto the mesh
- Large strain is easy to accommodate - particles naturally mix
- You never actually know where the interface is (as you don’t track it)
- Elements will contain mixtures of multiple materials

Field based approach

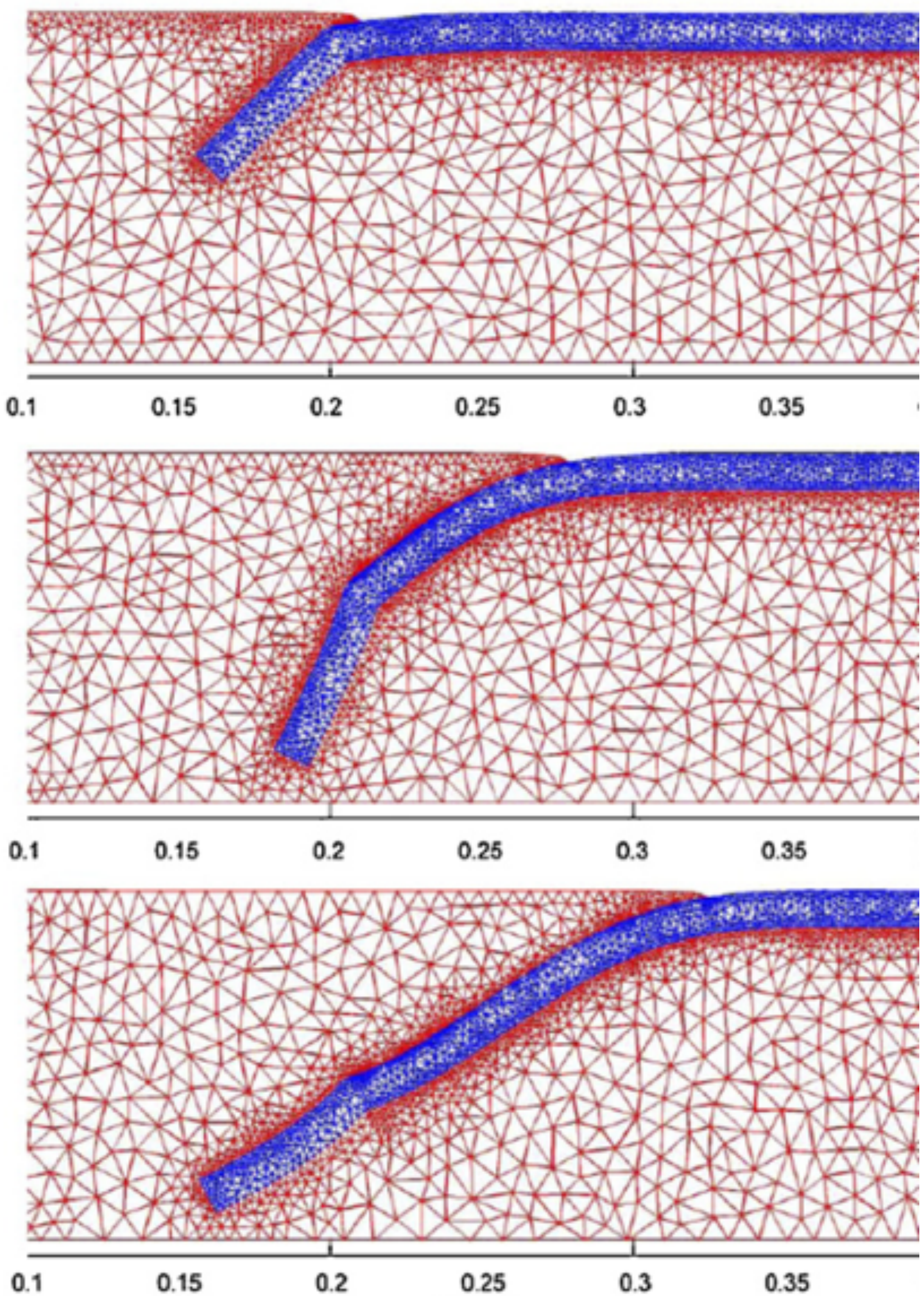




- Define scalar field (s) on a mesh.
- s = 0 —> purple material
- s = 1 —> yellow material
- s = 1/2 green)—> interface between purple and yellow material
- Works with structured meshes
- Care must be taken when advecting quantity to avoid numerical problems

Gerya (2010)

Interface tracking



- Interfaces between materials are explicitly tracked and updated with time
- Interior and exterior meshed
- Best to use an unstructured triangular mesh for geometric flexibility
- Large deformation requires special treatment on interface to ensure interfaces do not overlap

A numerical subducting slab

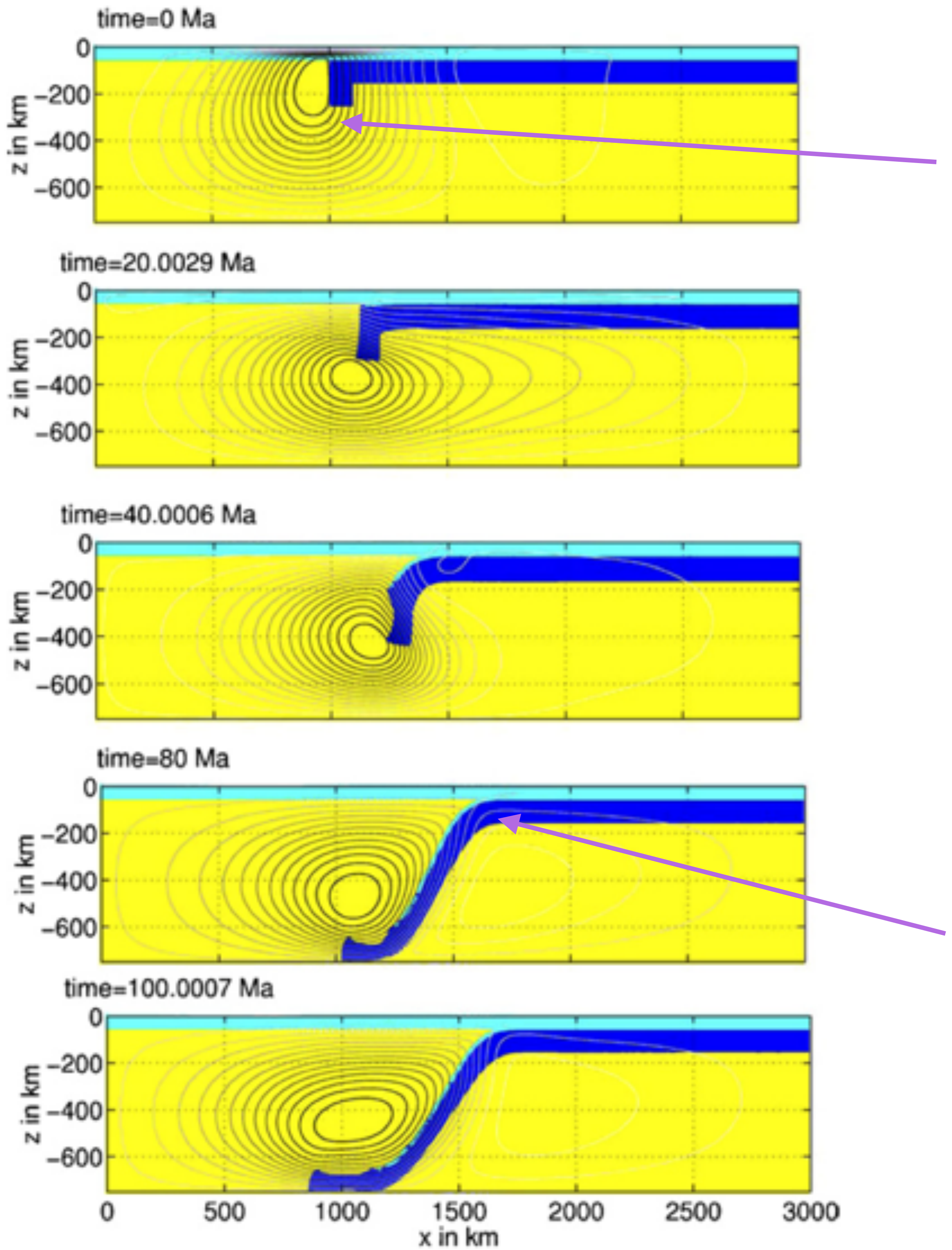


Fig. 3. Typical behaviour of a case 1 model (here FDCON-4 is shown). Streamlines are also shown.

Coherent slab structure (no viscous dripping observed)

Slab tip geometry

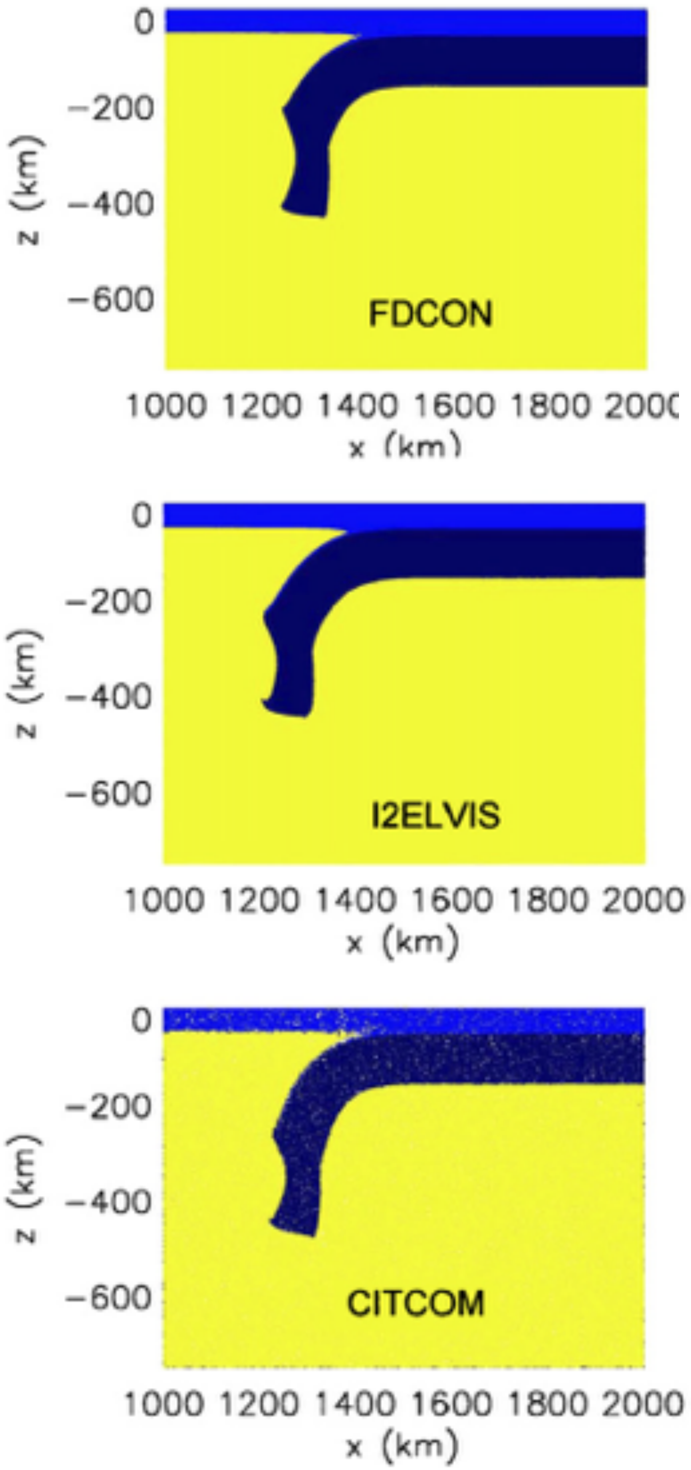


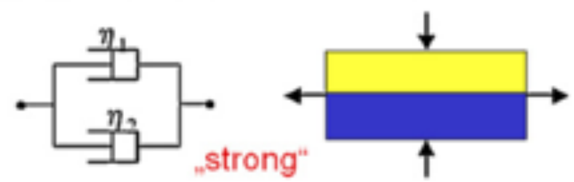
Fig. 4. Shapes of different case 1 models at similar stages: FDCON: 40 Myears, I2ELVIS: 34.7 Myears, CITCOM: 38.1 Myears. Viscosity averaging: geometric mean in all cases.

Viscosity averaging at compositional interfaces

Arithmetic mean

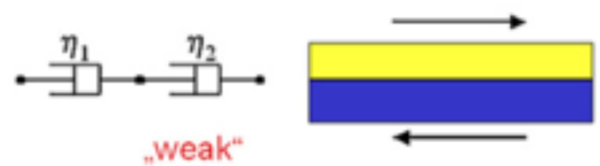
$$\eta_{ave} = c_1\eta_1 + c_2\eta_2$$

c_1, c_2 = weights



Harmonic mean

$$\frac{1}{\eta_{ave}} = \frac{c_1}{\eta_1} + \frac{c_2}{\eta_2}$$



Geometric mean

$$\eta_{ave} = \eta_1^{c_1}\eta_2^{c_2}, \quad (\log \eta_{ave} = c_1 \log \eta_1 + c_2 \log \eta_2)$$

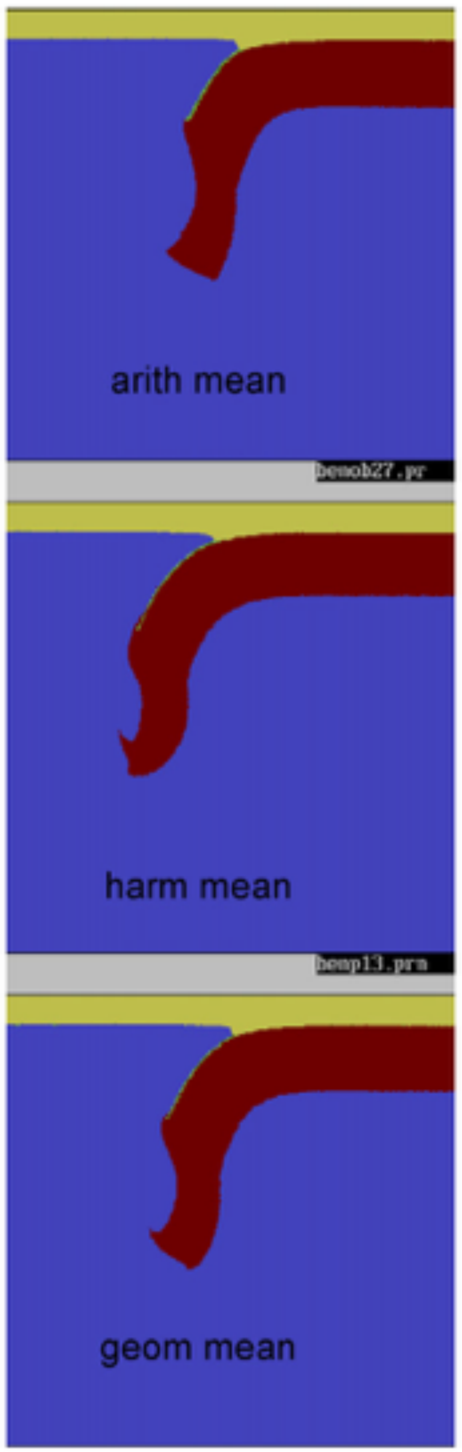


Fig. 5. Comparison of the shapes of the slabs for different viscosity averaging methods using I2VIS. Note that the snapshots are taken at different times (59.6, 24.4, 37.8 Myears from top to bottom), so that the slab tips have reached comparable levels.

Slab tip evolution

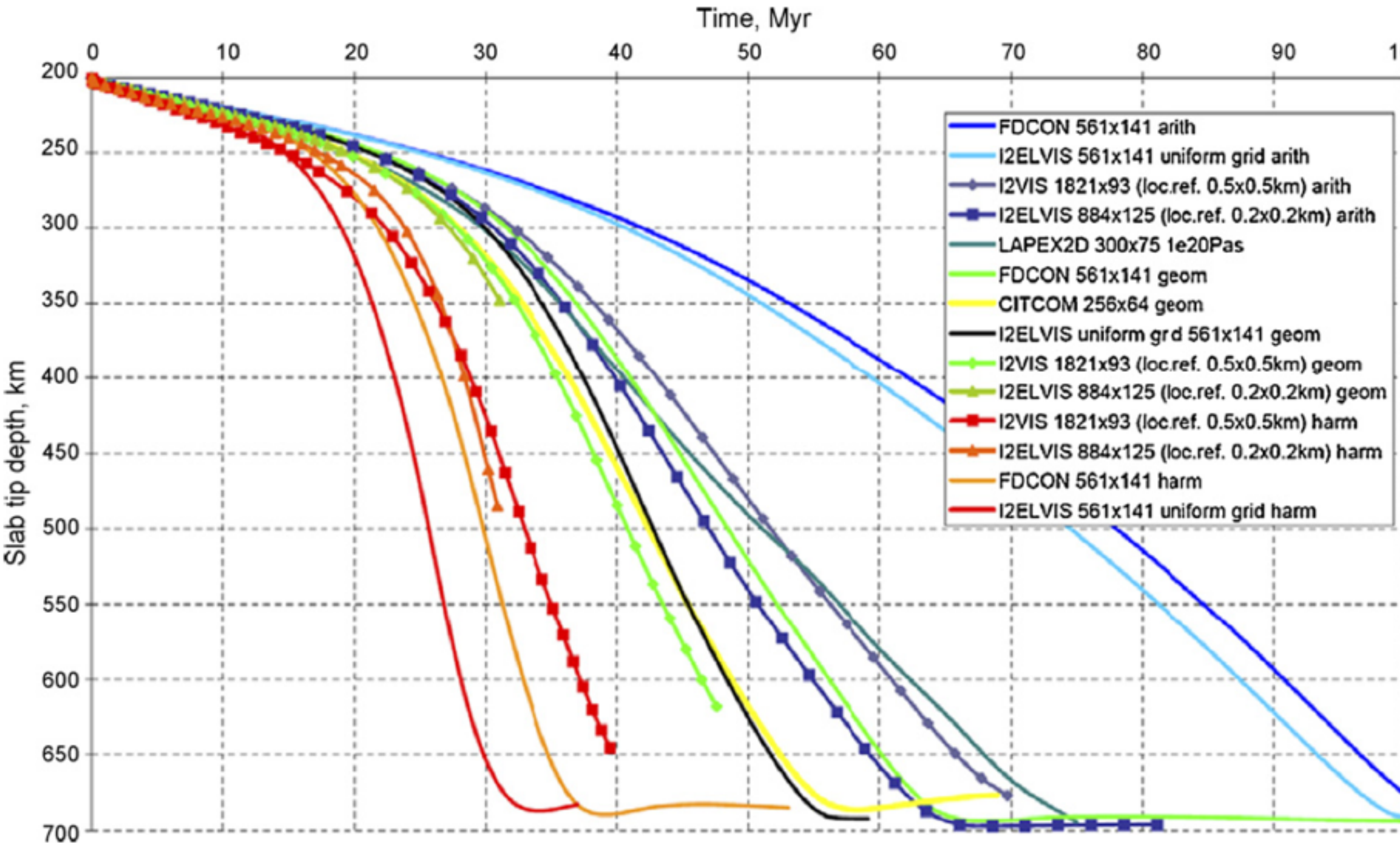


Fig. 6. Temporal behaviour of case 1 modelled by different codes with highest resolutions each. Each curve shows the position of the deepest part of the slab (slab tip) as a function of time below the initial surface of the lithosphere. See the legends for the used codes and grid resolution. Note that the codes I2VIS and I2ELVIS also use local refinement at the trench area (given in parentheses in the legend). Outside the trench area the resolution decreases to 10 × 46 km at model sides. At the lower boundary the vertical resolution was 1 km. The rheological means (cf. Section 3.2) are denoted as geom for geometric, harm for harmonic and arith for arithmetic, respectively. In contrast to the others, LAPEX2D was run with 10^{20} Pa s for the weak layer.

Viscous entrainment

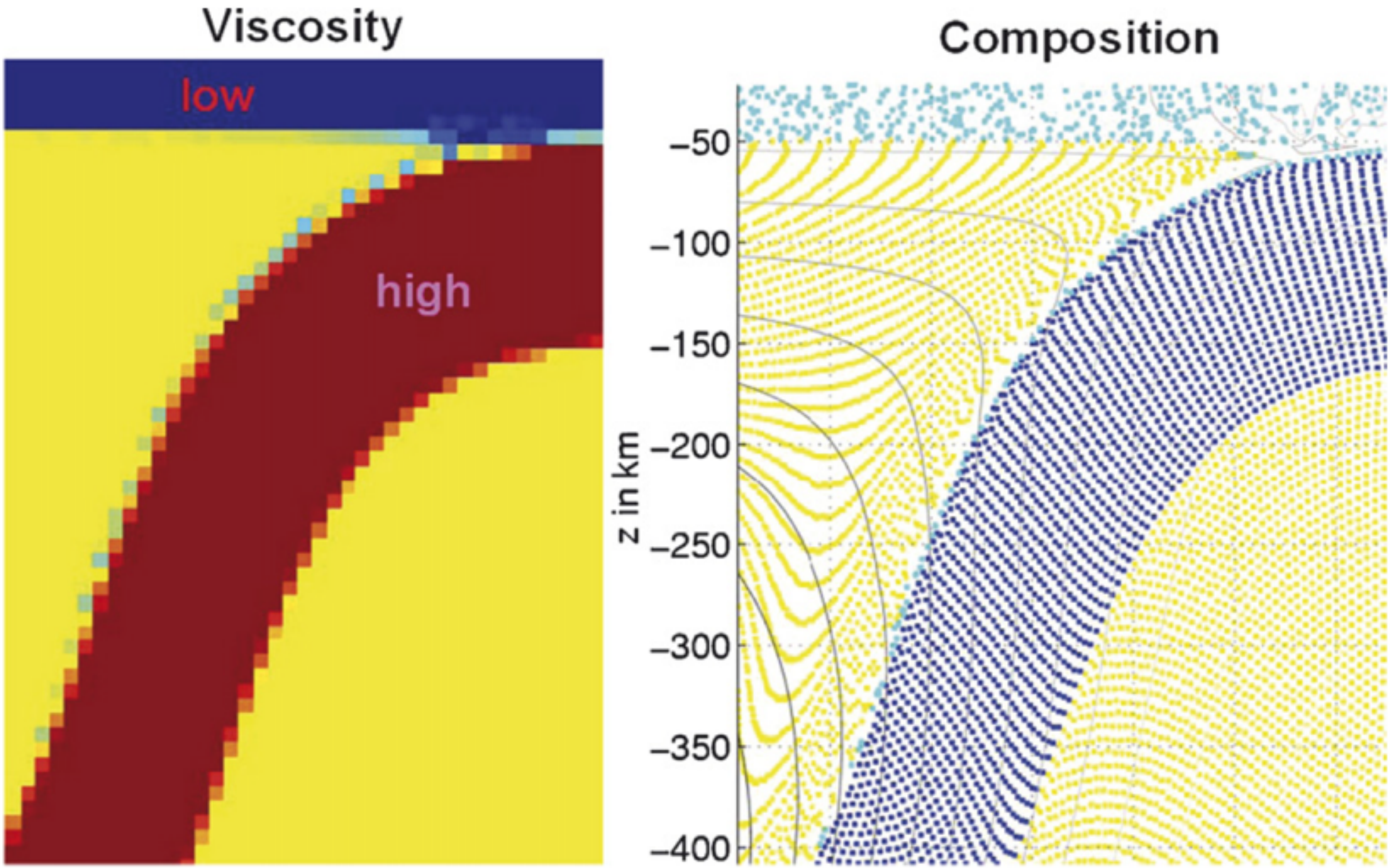


Fig. 9. Details of the entrainment and lubrication of the soft surface layer.

Slab tip evolution

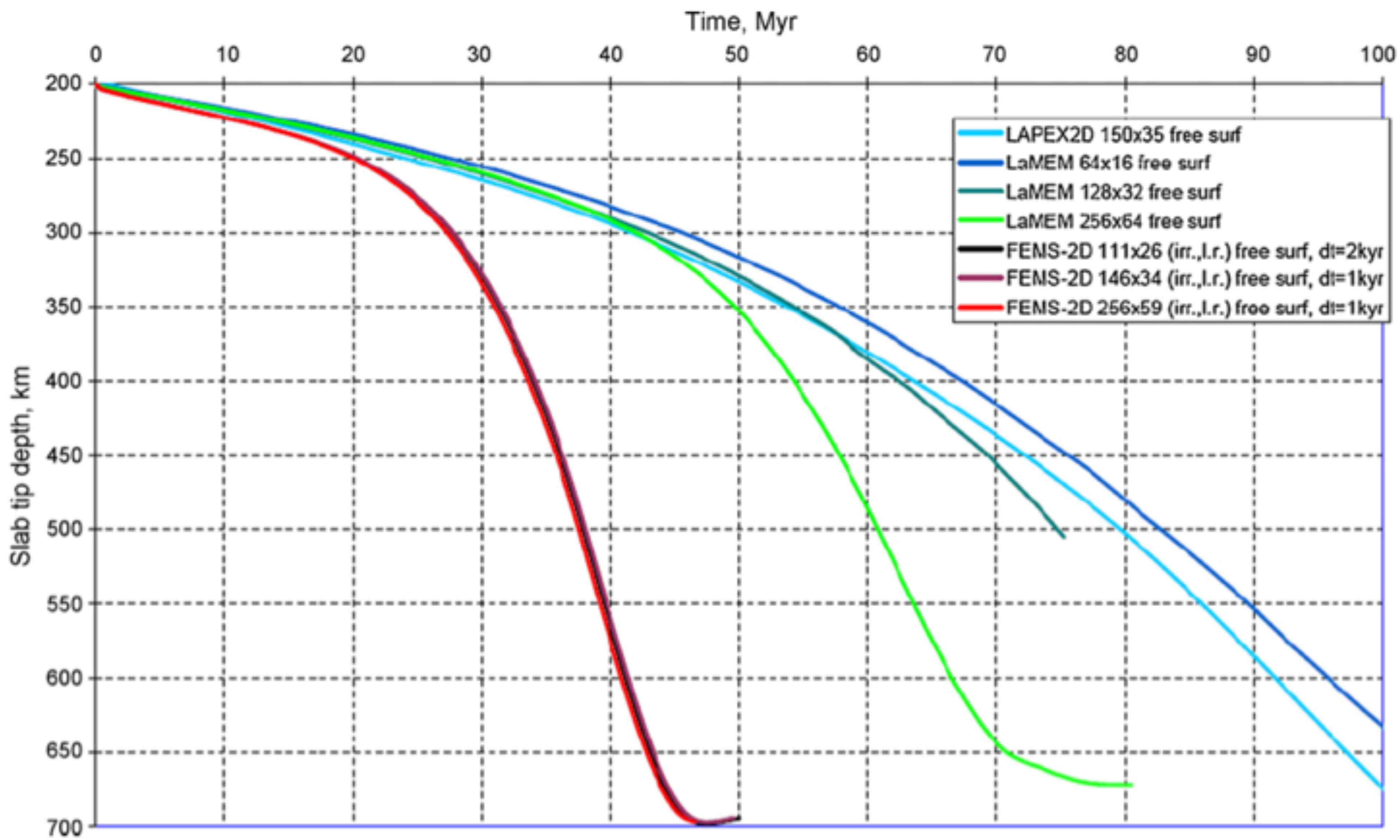
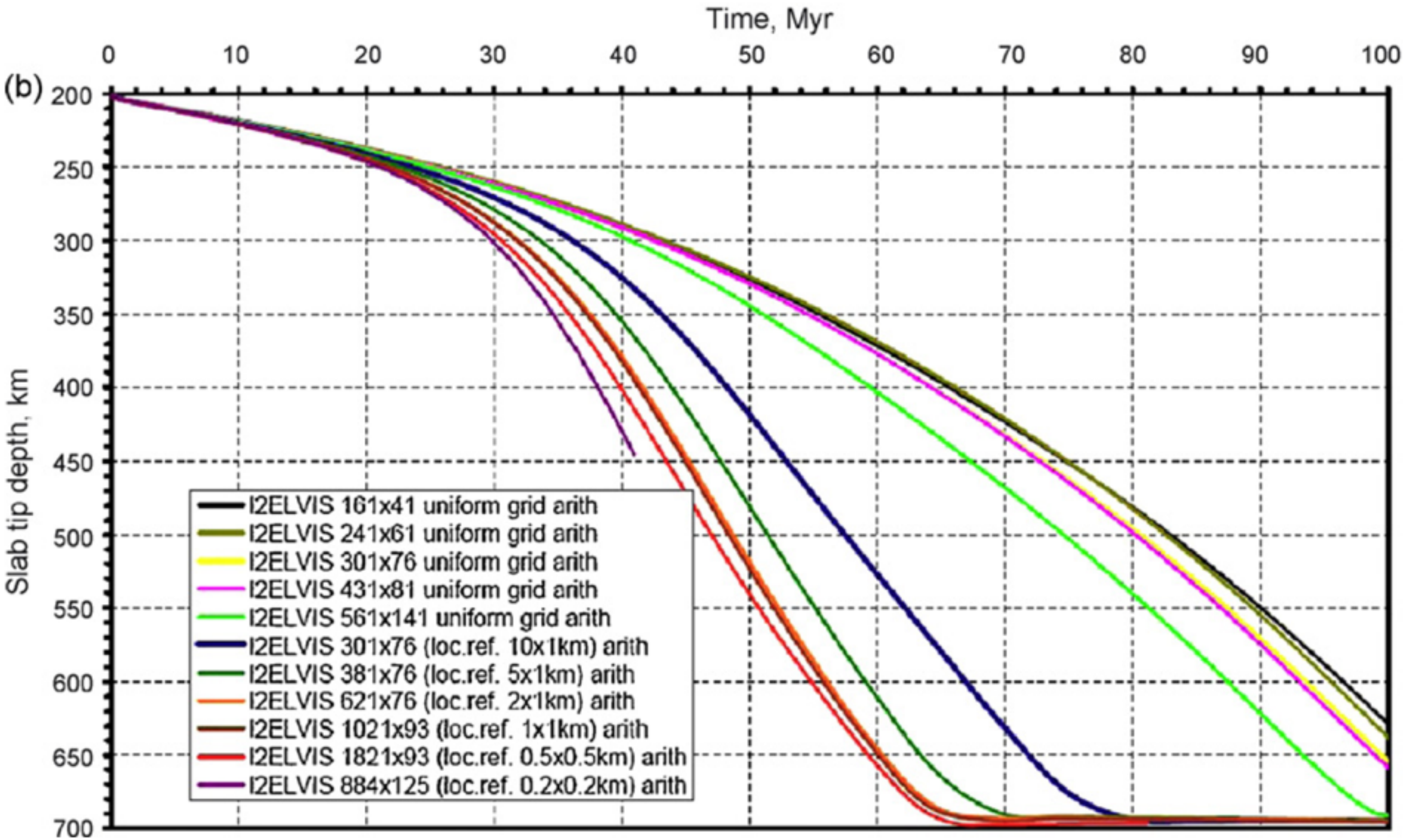
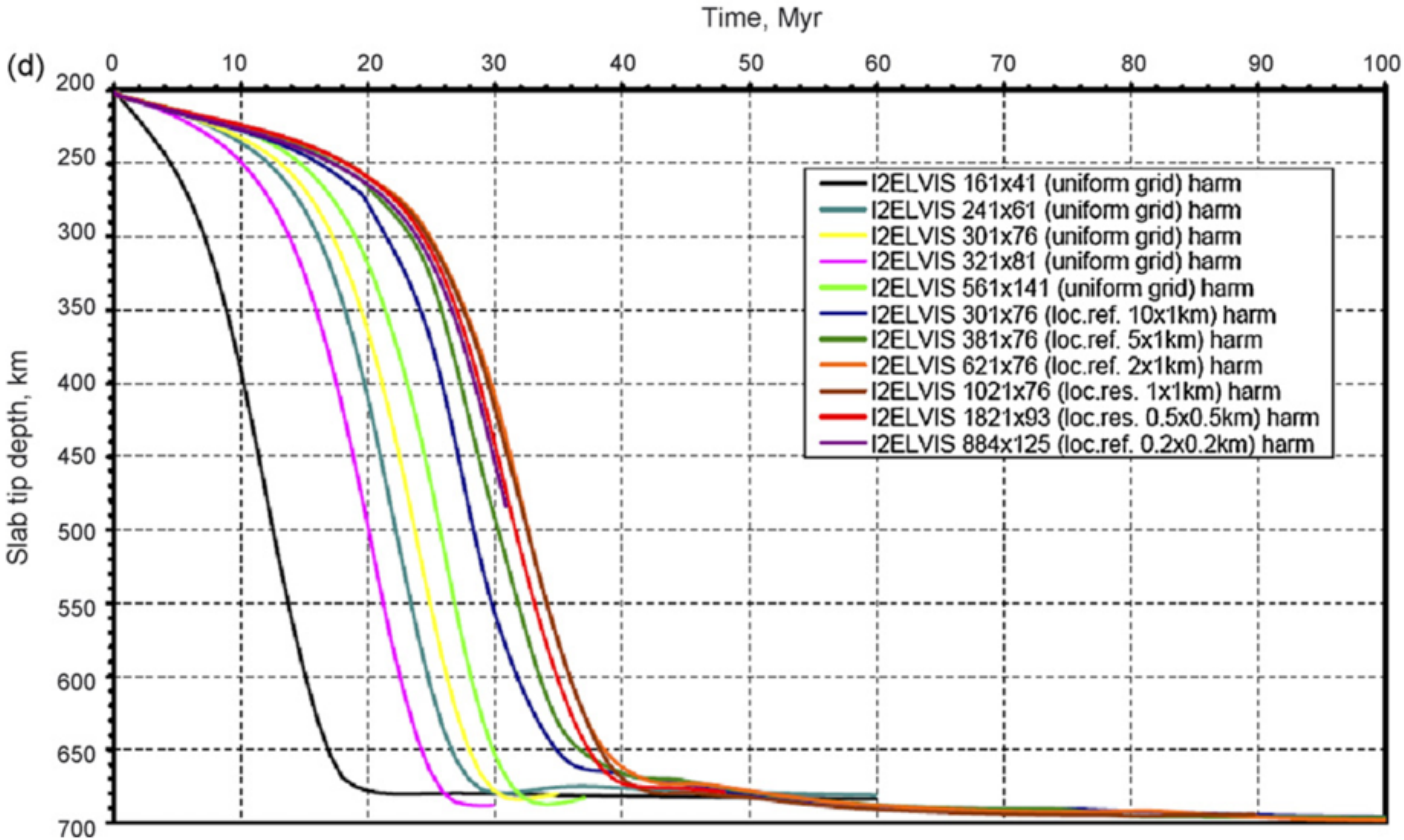


Fig. 10. Comparison of temporal behaviour of case 1 models assuming a free surface instead of a weak layer. The FEMS-models used an irregular (irr.) mesh with local refinment (l.r.) near the trench, the given x–y grid resolution are only approximate values for comparison, calculated from the total number of nodes. The three FEMS-2D models are very similar so that the curves partly overlap.

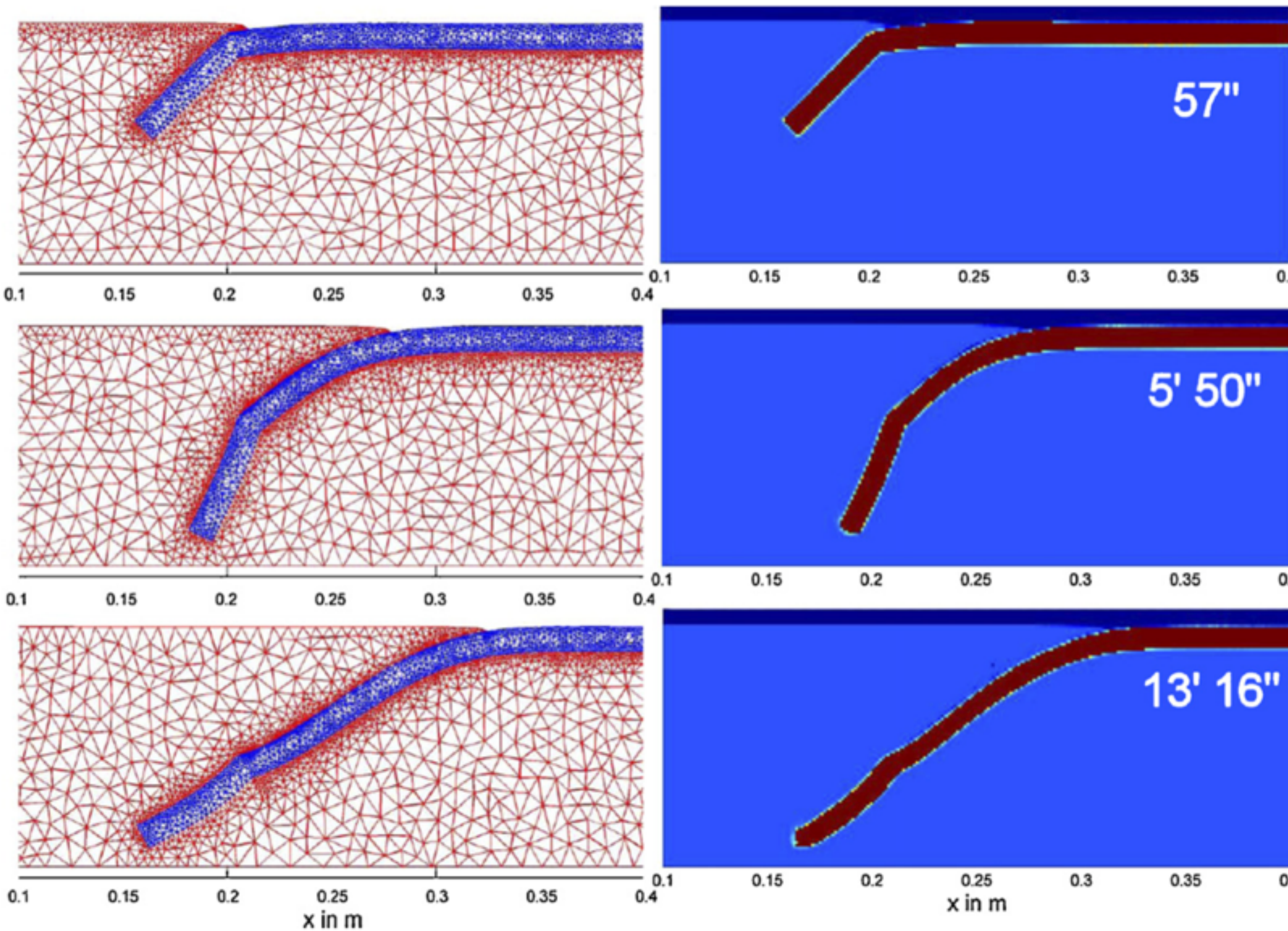
Resolution matters



Resolution matters

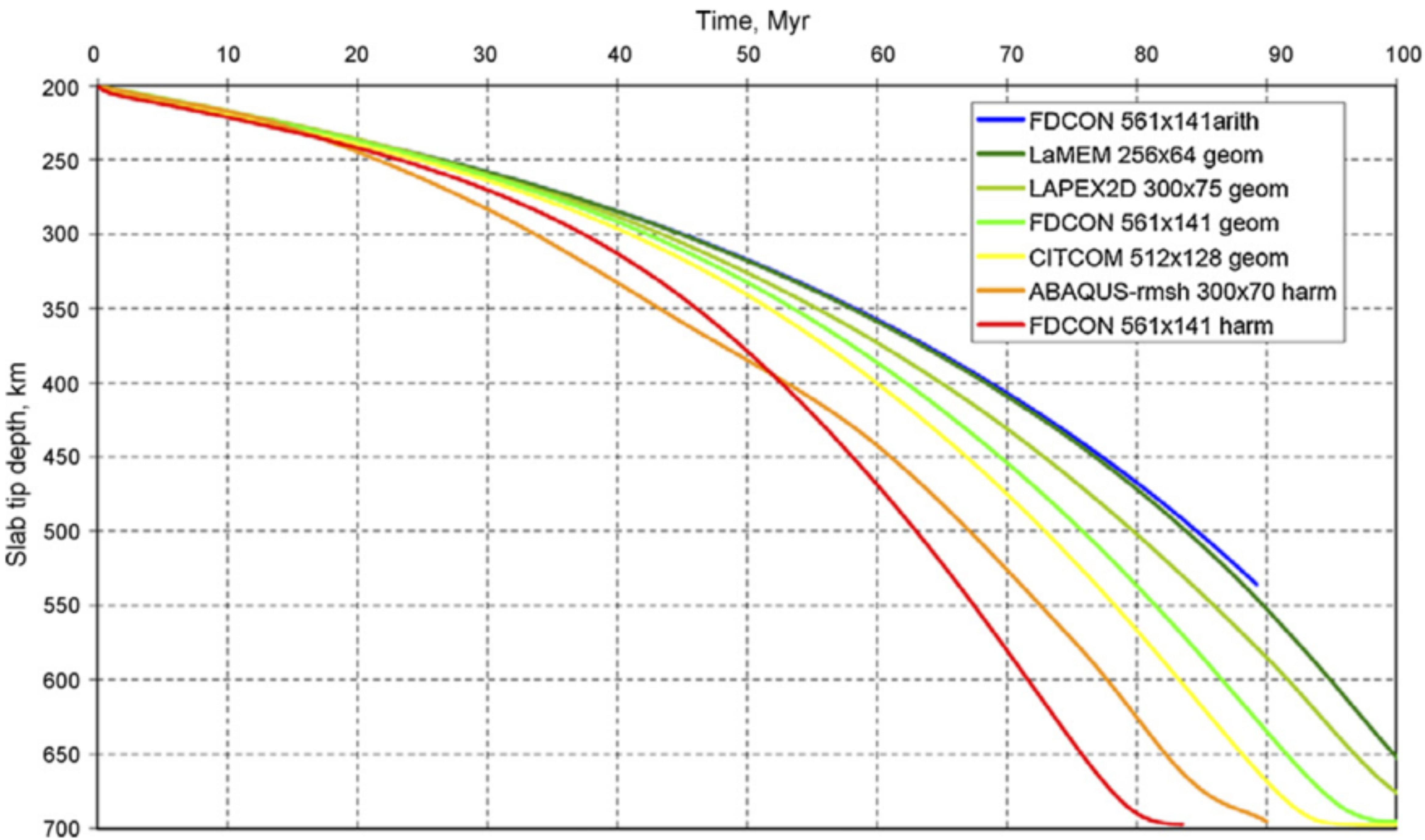


Averaging scheme

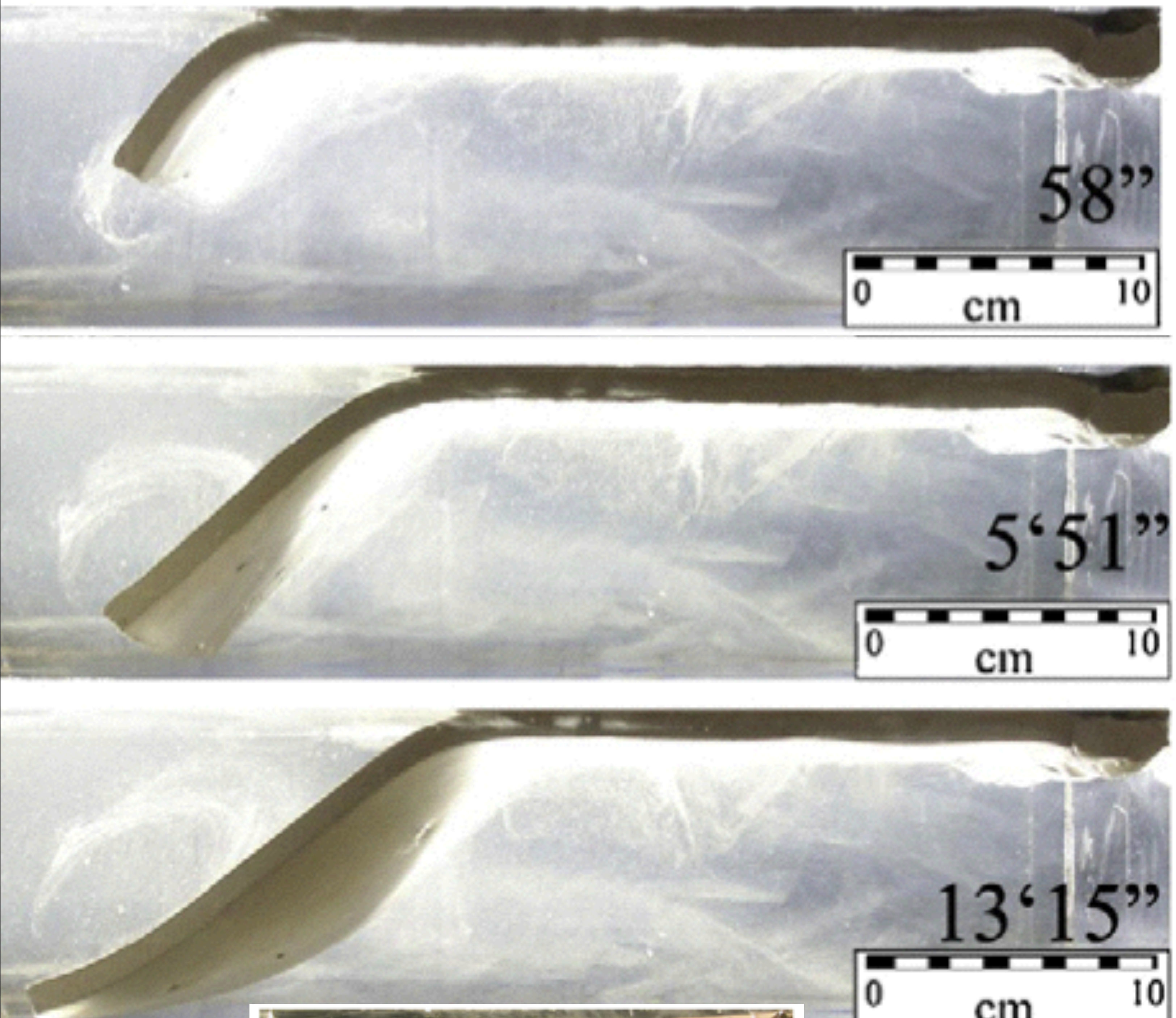


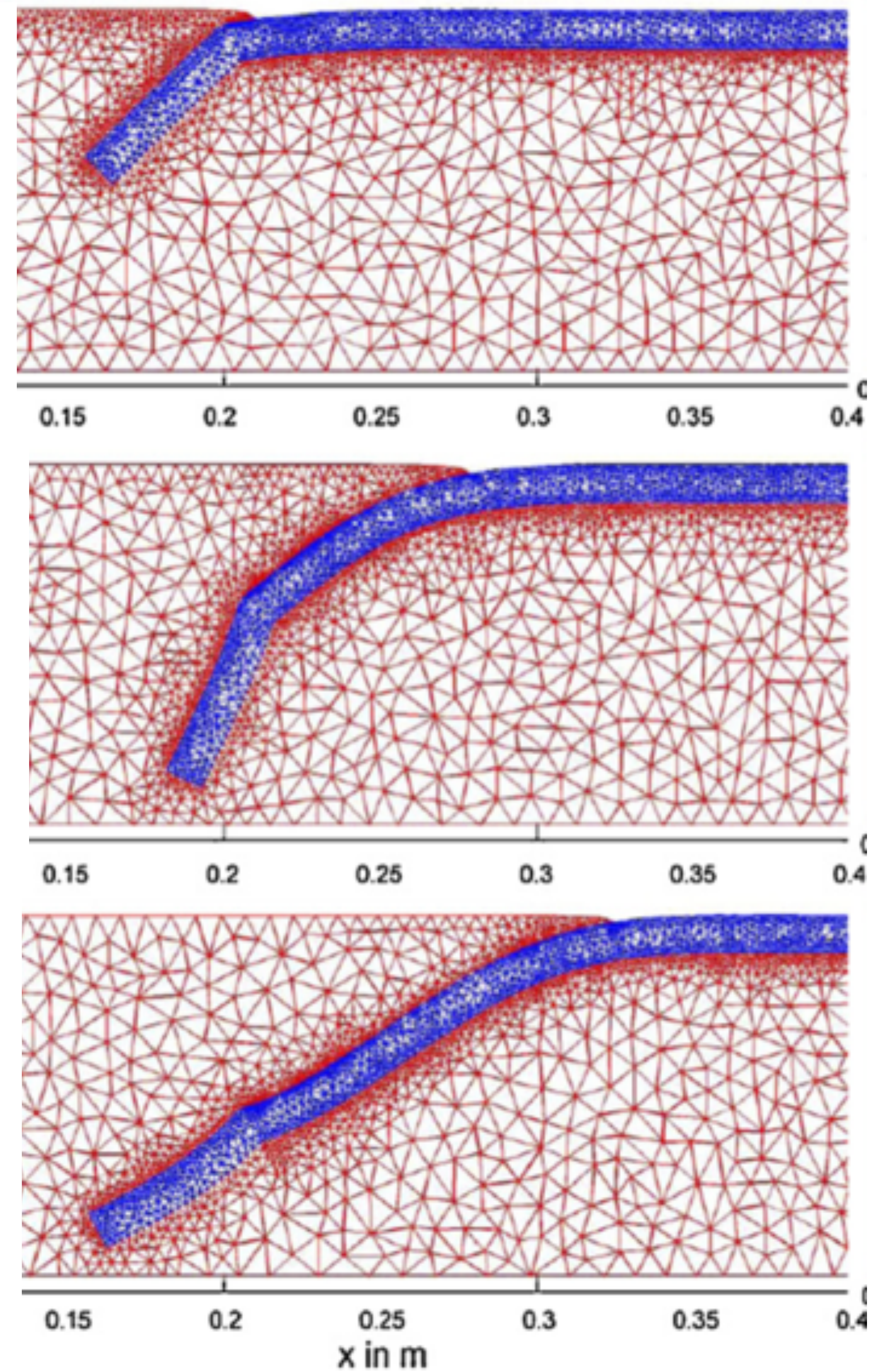
Is the approximate free surface the problem?

Free slip boundary (no free surface)



Comparison with lab models





top view at 19’15

Comparison with lab models

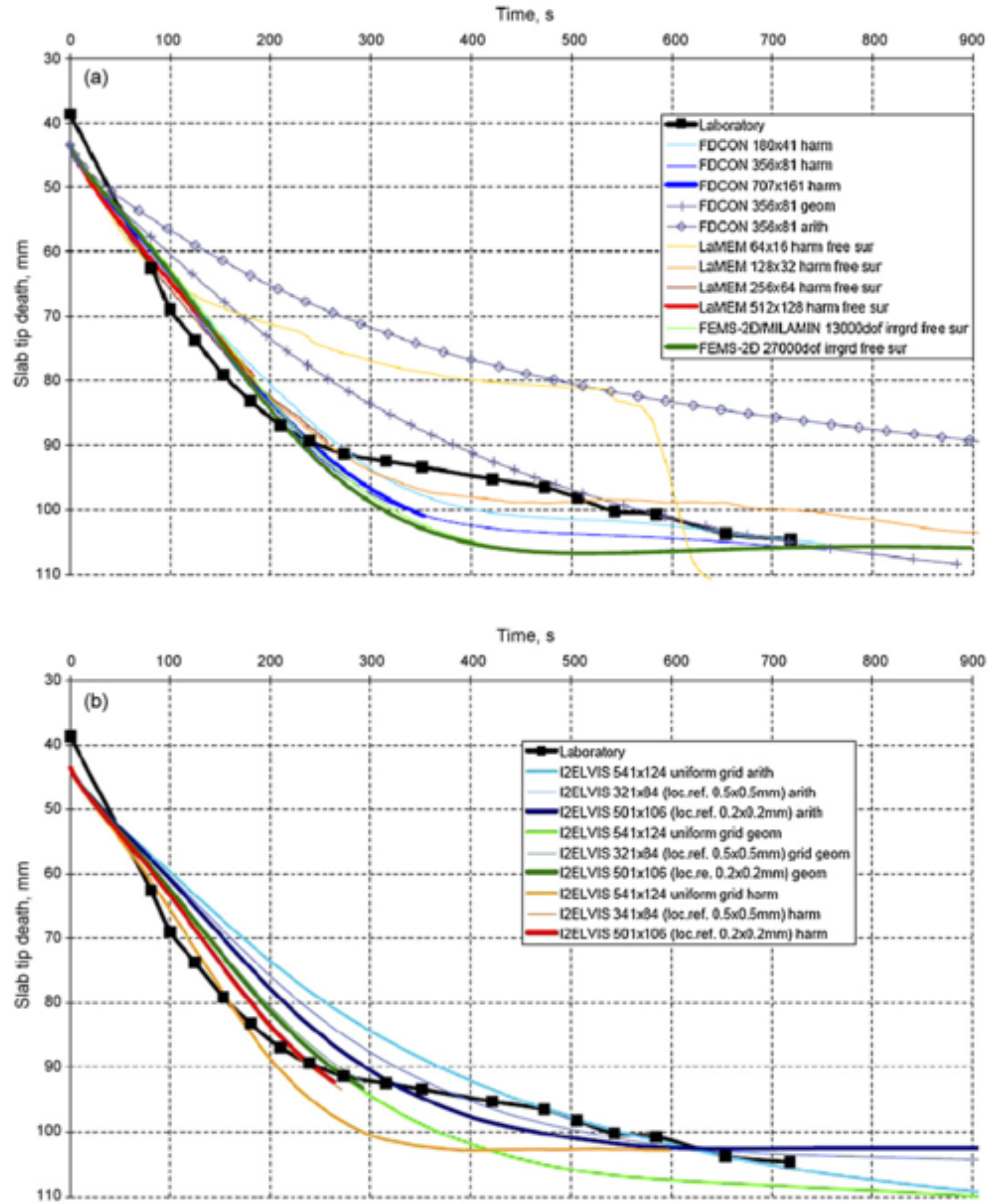


Fig. 17. (a) Comparison between laboratory and numerical slab tip depths obtained by the codes FDCON, LaMEM and FEMS-2D. FDCON used a viscosity of the soft surface layer equal to 1/10 of the mantle viscosity. (b) As (a) but for I2ELVIS, using a viscosity of the soft surface layer of 1/100 of the mantle viscosity.

Summary

- Models of buoyancy driven flows (subduction) are sensitive to the numerical methodology and numerical resolution
- Introducing free surface as “sticky air” can make the comparison between methods worse - although the subduction dynamics is better modelled
- Using harmonic or geometric averaging is advocated for subduction models
- Reasonable comparisons with lab models are obtained if a true free surface is adopted, or if low viscosity sticky air + harmonic averaging is used

Paper 3: Some open questions

- What is the explanation given to explain why the numerical scheme produce different answers?
- Different viscosity / density schemes were proposed in order to make the numerical methods agree. How applicable do you think these schemes are to general geodynamic contexts (e.g. when applied to models other than subduction)?
- FEMS-2D appears to produce sinking rates independent of resolution (Figure 10)? Why? Why don’t we simply use the method of FEMS-2D for all geodynamic calculations?
- Comment on why you think the laboratory models do not agree with the numerical results? (See figure 17)
- We rely on numerical models in geodynamics, however even very simple systems can produce a wide variety of answers - how could we resolve this issue? What do you think the consequences are if we include more complexity in the numerical models?

Further reading

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- OzBench, M., Regenauer-Lieb, K., Stegman, D. R., Morra, G., Farrington, R., Hale, A., & Moresi, L. (2008). A model comparison study of large-scale mantle–lithosphere dynamics driven by subduction. *Physics of the Earth and Planetary Interiors*, 171 (1), pp. 224-234.

- Quinquis, M. E., Buiter, S. J., & Ellis, S. (2011). The role of boundary conditions in numerical models of subduction zone dynamics. *Tectonophysics*, 497 (1), pp. 57-70.

Final remarks

- Assessment for the subject will be finalised within one week from Thursday, April 14
- I will send results via email to each one of you (marks will also be available via eDoz)

Final remarks

- Geophysical Fluid Dynamics project page (Bachelor, MSc)

www.gfd.ethz.ch/education/BSc_and_MSc_projects

- Semester block course

651-4144-00L Introduction to Finite Element Modelling in Geosciences

Semester	Spring Semester **2016**
Lecturers	**M. Frehner, D. A. May**
Periodicity	yearly course
Language of instruction	English
Comment	Block Course from **July 25–29, 2016 [9:15 - 17:00 every day]**

- GFD seminar series
  - Wed, 12:00 — 13:00, F39
  - **http://jupiter.ethz.ch/~ballmerm/seminar.html**