Dynamics of the mantle and lithosphere
Practical: Fluid mechanics

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Q1 The Rayleigh and Nusselt number

Recall that the Nusselt number is given by

$$Nu = \frac{R_m}{S_e\ (T_{\mathsf{CMB}} - T_0)} \int_S \left(\frac{\partial T}{\partial z}\right)_{z=0} dS,$$

where R_m represents the thickness of the mantle, S_e represent the surface area of the Earth, T_{CMB} is the temperature at the core-mantle boundary and T_0 is the surface temperature. For the following calculations, you can assume that the temperature at the CMB is 4500 K and that the mantle is entirely composed of dry Olivine with a conductivity given by $k = 1.67$ W m^{-1} K^{-1} and a thermal expansivity of $\alpha = 3 \times 10^{-5}$ K^{-1}.

(a) Derive an expression for the Rayleigh number (Ra) for a Boussinesq fluid convecting in a box of height h. Assume that the fluid is incompressible, constant viscosity, there are no internal sources of heating and that the thermal conductivity is constant. A fixed temperature boundary condition is applied on the top and the bottom of the domain. Use the following choices for the non-dimensional scaling:

$$\boldsymbol{x}' = \boldsymbol{x}/h, \quad t' = t/(h^2/\kappa), \quad T' = T/\Delta T, \quad \boldsymbol{v}' = \boldsymbol{v}/(\kappa/h), \quad p' = p/(\eta\kappa/h^2), \qquad (1)$$

where h and ΔT are the characteristic length (height of the box) and ΔT is the temperature difference between the lower and upper boundary. When deriving Ra, make sure you consider the *perturbed* form of the momentum equations.

(b) Compute the Nusselt number using typical heat flux estimates from (i) the oceanic lithosphere and (ii) the continental lithosphere.

(c) Under the assumption that the CMB is at a constant temperature, and there are no internal sources of heating, determine the Rayleigh number for each region using the appropriate $Nu - Ra$ scaling law. Given what you know about convective systems, what does the Ra you computed imply about the mantle - e.g. is the mantle actively convecting? Explain your answer.

(d) Deduce the average viscosity in the mantle under the oceanic and continental lithosphere. State all assumptions made regarding the characteristic values used. Do these viscosity estimates seem physically reasonable? Discuss the limitations or short-comings of the analysis used to determine the viscosity. How could the viscosity estimates be improved?

Q2 Mantle flow under the lithosphere

We will consider a plate-mantle flow system. Assume that a 100 km thick rigid lithosphere moves horizontally with uniform 10 cm/yr (in the negative x direction) with respect to the lower mantle. Within the upper mantle, between 100 km and 660 km depth a pressure-driven return flow is established such that the total horizontal flux in the total upper mantle (between 0 and 660 km depth) is zero. The situation is shown in Fig. 1. You can assume

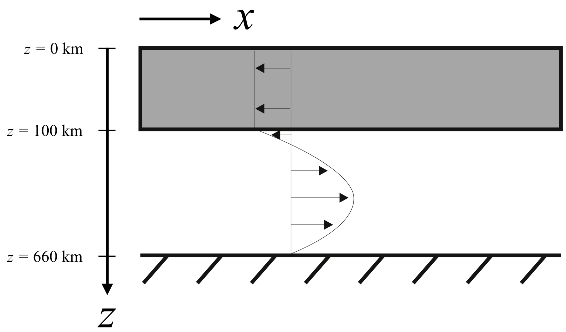

Figure 1: Simplified flow under the lithosphere

that the pressure gradient $\frac{\partial P}{\partial x}$ between 100 km $\leq z \leq 660$ km is a constant given by α, and the fluid viscosity within this region is a constant which we will denote by η.

(a) The horizontal velocity V_x as a function of depth z in the upper mantle below the plate is a parabolic function of the form $V_x(z) = Az^2 + Bz + C$. Derive this expression for the velocity field. Clearly state and justify all assumptions made.

(b) Solve for the coefficients A, B, and C. The coefficients of the parabolic flow can be found by assuming the following:

- $V_x(z = 100$ km$) = -10$ cm/yr
- $V_x(z = 660$ km$) = 0$ cm/yr
- The integrated value of V_x between the surface and 660 km depth should equal zero, e.g.:

$$\int_{z=0\text{ km}}^{z=660\text{ km}} V_x(z)\, dz = 0$$