Spring Semester 2014
651-4144-00L

Introduction to Finite Element Modelling in Geosciences

July 7 - 11, 2014

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Overview of this course

- OBJECTIVES

“Learn how to program the finite element method and apply it to equations relevant for geodynamics. The course is given in the form of MATLAB exercises, with an introduction of the relevant theory. The emphasis is on practical exercises, and students obtaining knowledge of how to write their own codes.”

- TEACHING STYLE

This is a “hands on” course. “Hands on” means “you code” whilst “we” assist you. In our experience, this is the most effective way to teach Earth scientists the finite element method. This is NOT a theoretical class. Some theory will be discussed in the material presented (although if interested please ask).

The emphasis is on learning the practical aspects of writing your own finite element code.

Overview of this course

- SCHEDULE
  - ⋆ 9:15 -- 1 hour presentation / overview / Q&A
  - ⋆ 14:00 -- 1 hour presentation / overview / Q&A
  - ⋆ The rest of the time, you will be reading the handouts provided, completing the exercises, writing MATLAB code, and asking the lecturers and assistants for help with your code.
- TEACHING MATERIAL

http://jupiter.ethz.ch/~gfdteaching/femblockcourse/2014/IntroToFEMForGeosciences.html

Assessment

- ***To obtain a mark of 5.0***

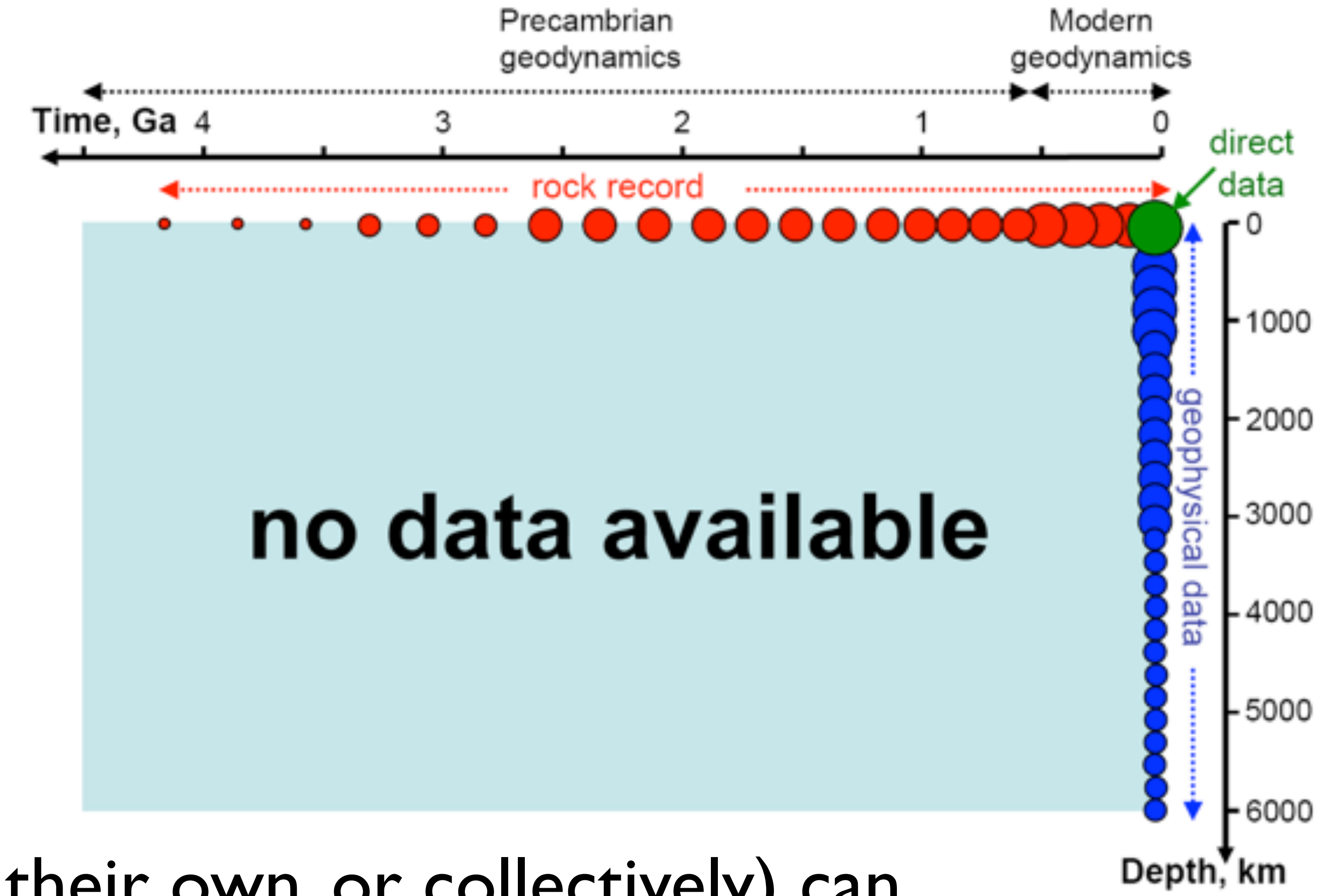
  + Program the 2D diffusion equation with linear and with quadratic elements.
  + Compute the order of accuracy of your code for both element for the steady state diffusion equation (look at the online lecture notes).
  + Send an email to dave.may@erdw.ethz.ch AND marcel.frehner@erdw.ethz.ch with
  (i) a 1-page report including a description of the code and figures which show the convergence plots,
  (ii) all the source code so we can run your code and reproduce the results.
  + ***It is required that you visit D. May or M. Frehner during the week of July 21 to discuss your code. This is simply to ensure that you wrote it yourself. All reports must be submitted by July 31.***
  + You can ask as many questions and visit as often as you require (appointment via email please!) to get your code working. This is encouraged and will not negatively influence your mark.

- ***To obtain a mark greater than 5.0, do any of the following***

  1) Write a 2D elasticity code (see online lecture notes) and demonstrate that it is working correctly with a suitable test problem.
  2) Write a 2D stokes code (this will definitely give you a 6.0 and free beer all night at FB)
  3) Perform a convergence test with the method of manufactured solutions for a time dependent 2D diffusion problem (using quadratic elements). What is the order of accuracy in space and time of your implementation?
  4) Find a cool geological application for any of the codes you have developed and show how your numerical model gives new insight into this problem.

Why do we need numerics?

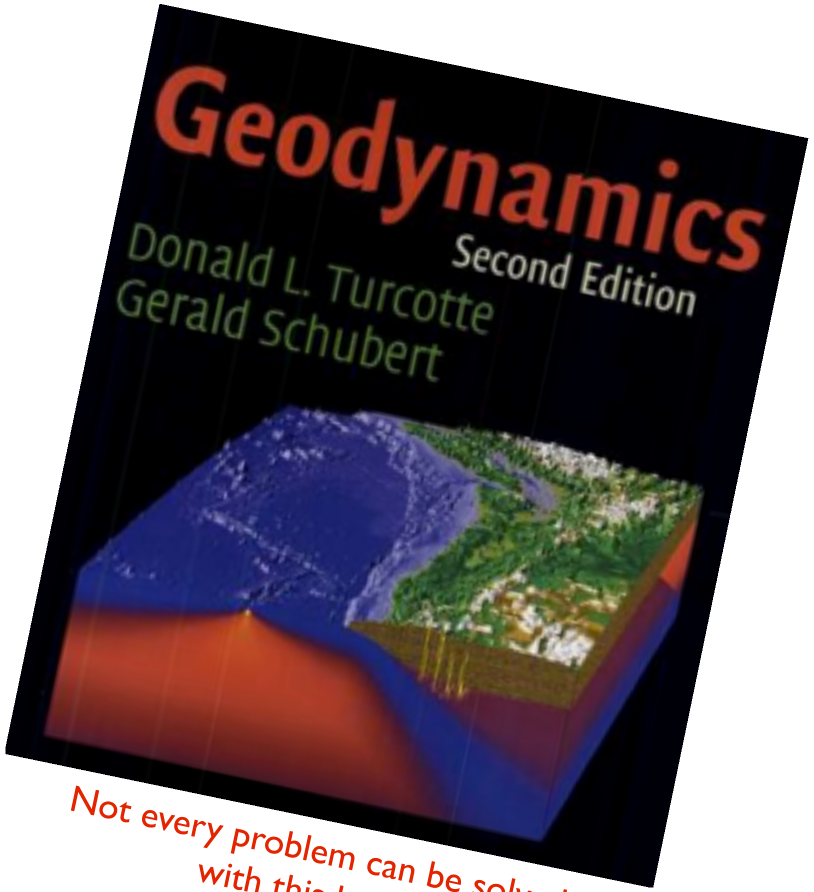
- Traditional methods to understand the Earth consist of geological, geophysical field based approaches - together analog experiments.



- None of these (one their own, or collectively) can satisfactorily provide a detailed understanding to develop deep insight into the governing physics and evolution of the system.

Why do we need numerics?

- Analytic methods only take us so far...



Not every problem can be solved with this book!

  - Overly simplified to the point where they don't resemble anything 'Earth like'.
  - Boundary conditions not appropriate.
  - Restricted dimensionality.
  - Mathematically too complex for most normal people...

Why FEM?

The Finite Element Method (FEM) possess a few very important characteristics which are relevant to studying geological processes in a reliable and accurate manner:

1. Simplicity in applying method to new equations.

2. Simplicity in meshing complicated geometries (internal and external). Meshes can be constructed using triangles or quadrilaterals. Meshes can be deformed and evolve with time.

3. Wide range of boundary conditions are easily able to be used. e.g. free surface boundary conditions are trivial to implement.

5. Jumps in material properties do not introduce any additional complexity.

6. Minimal programming complexity between 2D and 3D is low (much code can be re-used).

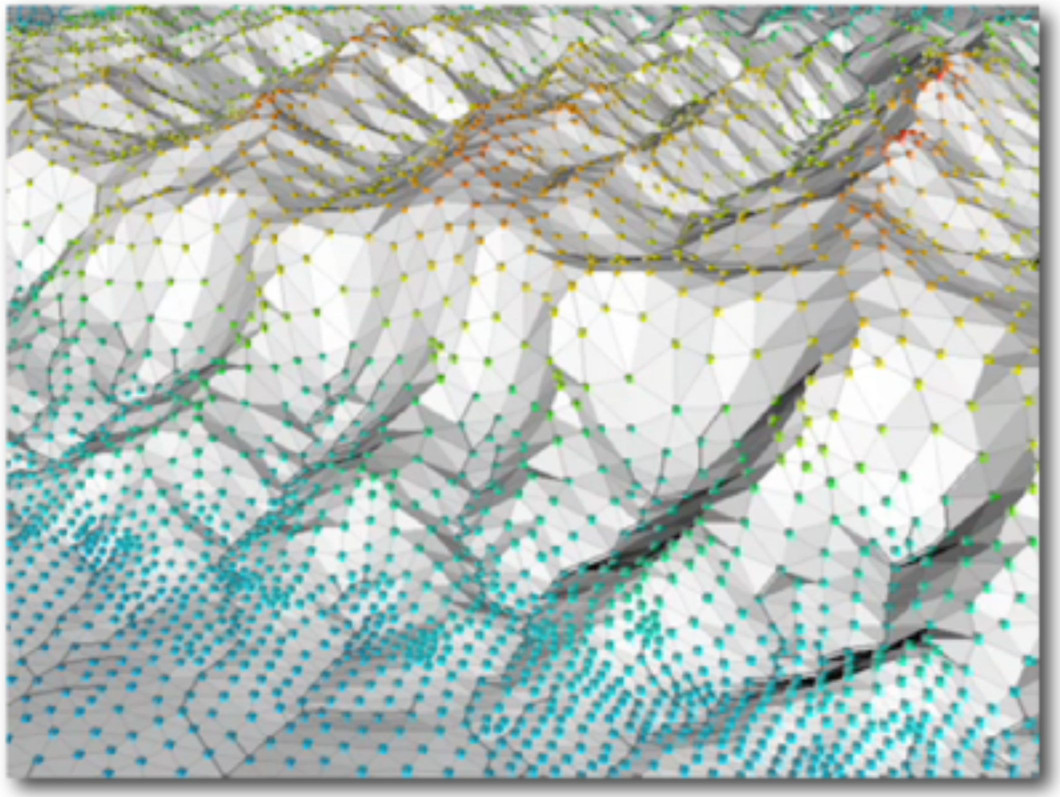
7. Rich mathematical foundations $\longrightarrow$ *reliable and accurate results.*

FEM examples

Thermochronology
(Braun, Comput. & Geosci., 2003)

[Energy transport + surface processes]

+ Deformed upper boundary.
+ Non-regular mesh on interior.
+ Flux boundary conditions.



"Pecube"

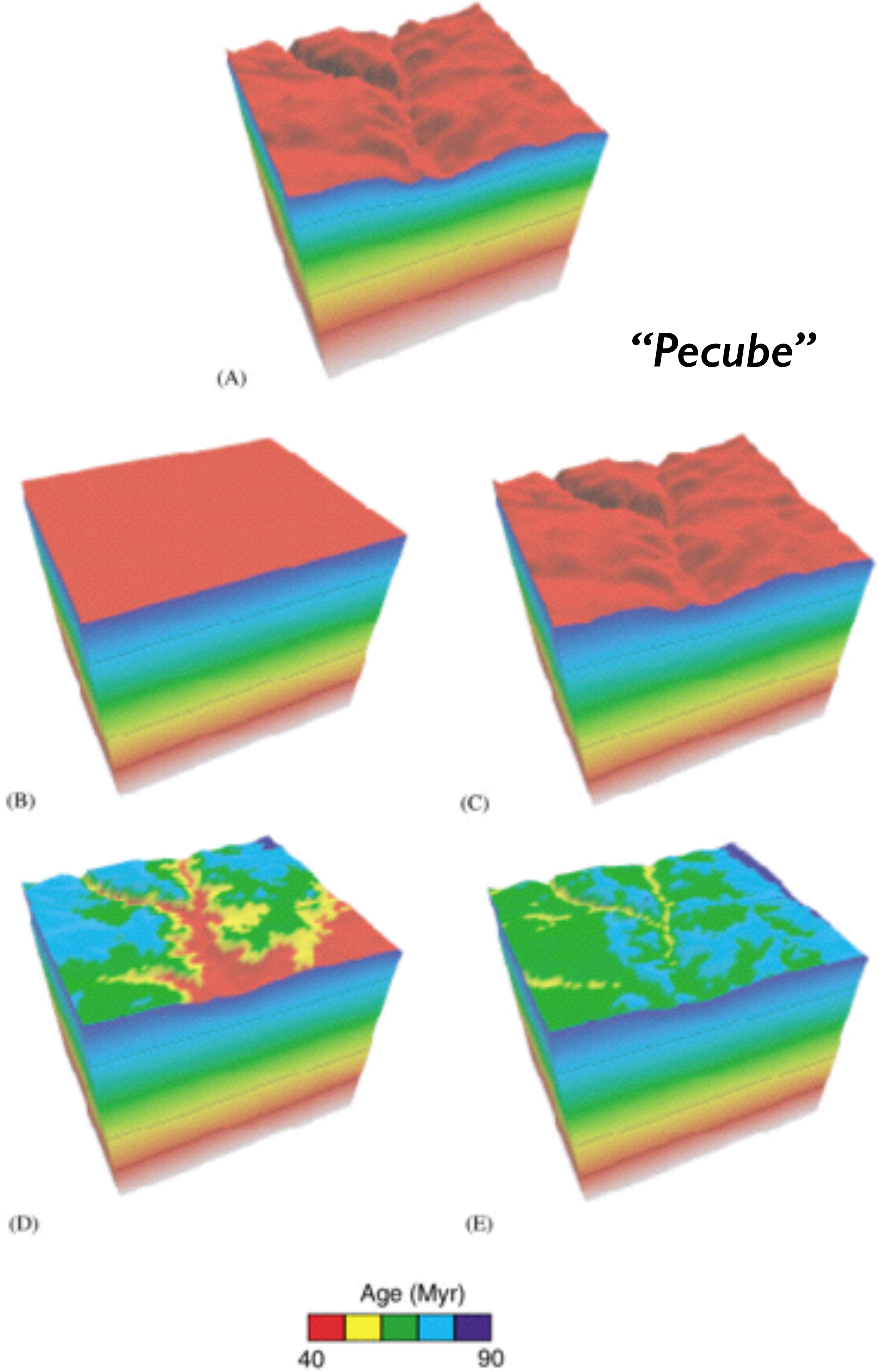


Fig. 4. Perspective view of finite-element domain with temperature field contoured on sides. (A) Solution at end of Laramide Orogeny; (B) 20 Myr later following scenario 1 and (C) scenario 2; (D) solution at end of computations (corresponding to present day) following scenario 1 and (E) scenario 2. Age contours superimposed on surface in panels (D) and (E).

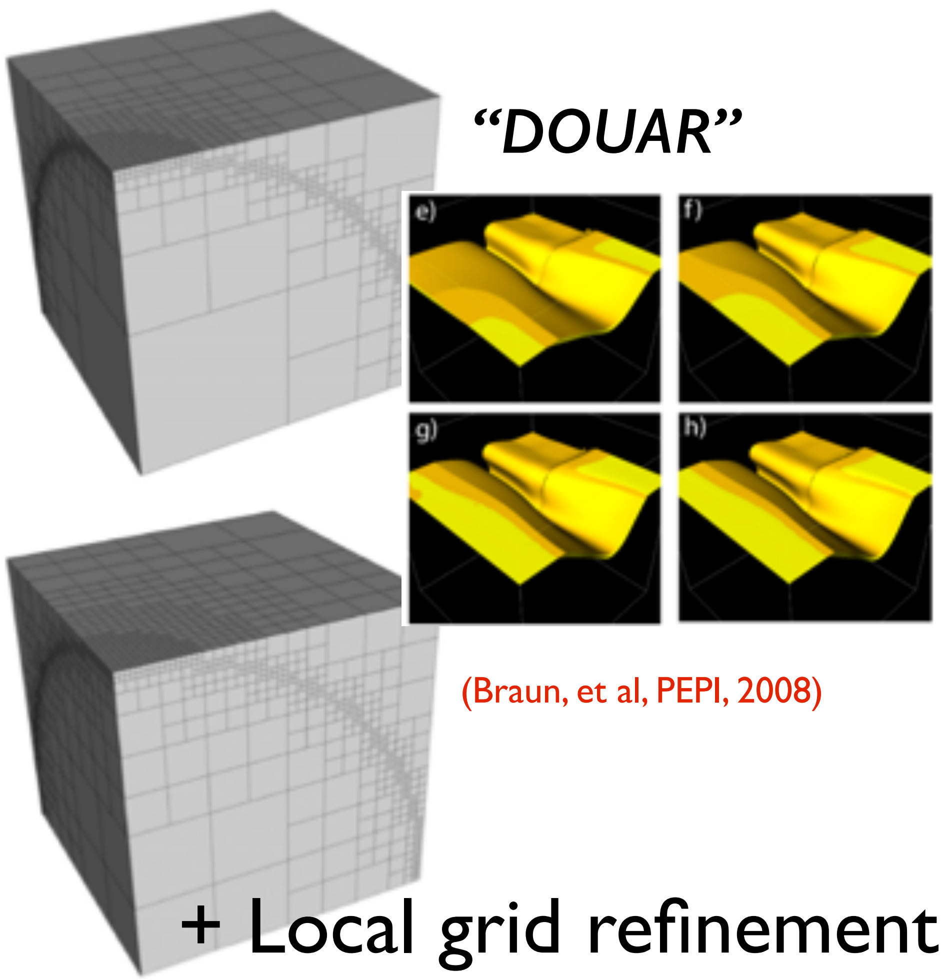
FEM examples

Lithospheric deformation

[Stokes flow + Energy transport + Surface processes]

+ Deformed upper boundary, coupled to surface process models.
+ Non-regular mesh on interior.
+ Flux boundary conditions.

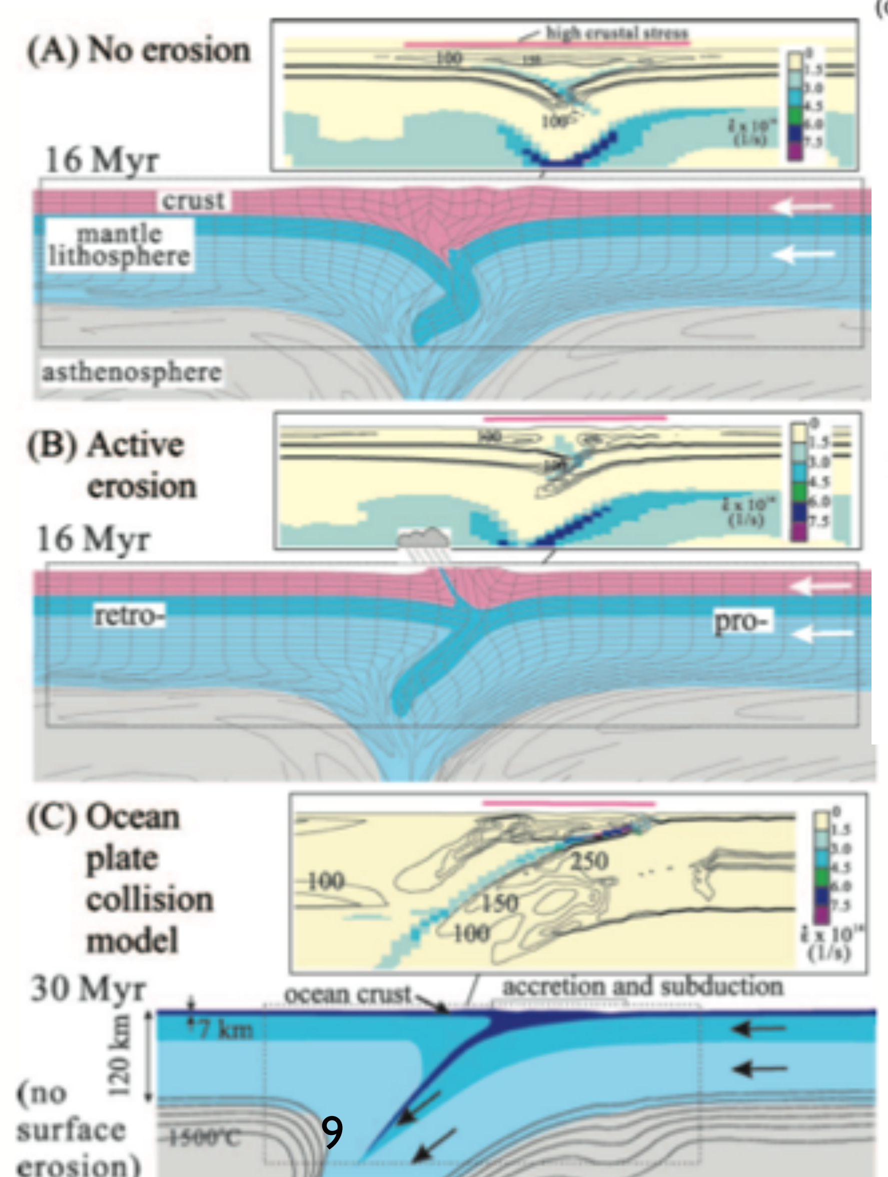
"DOUAR"



(Braun, et al, PEPI, 2008)

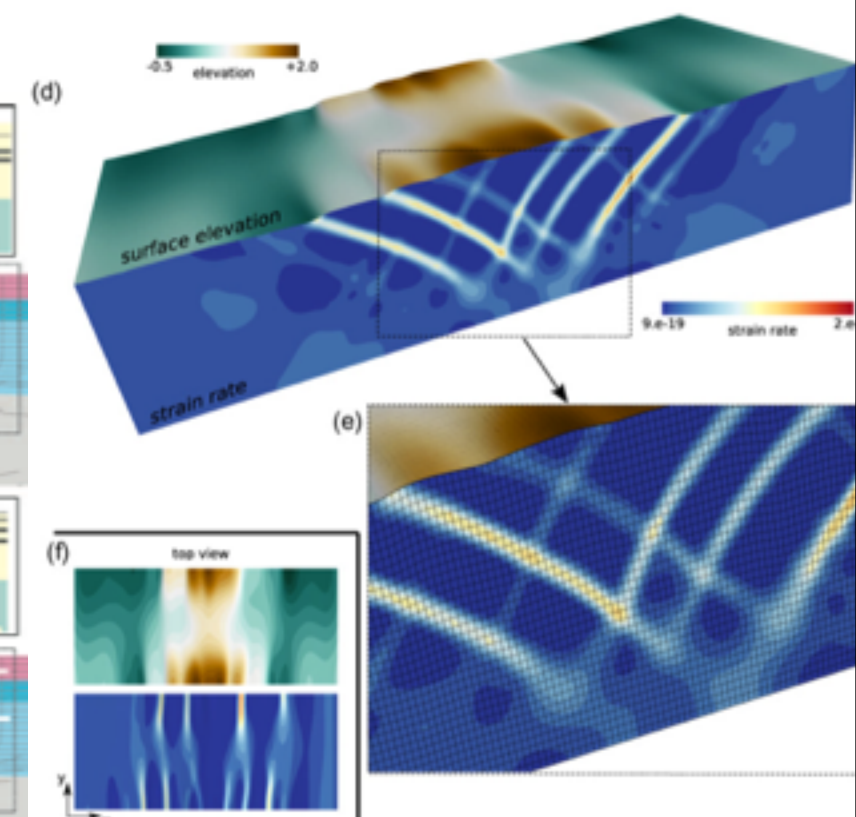
+ Local grid refinement

"SOPALE"



(Pysklywec, Geology, 2006)

"FANTOM"



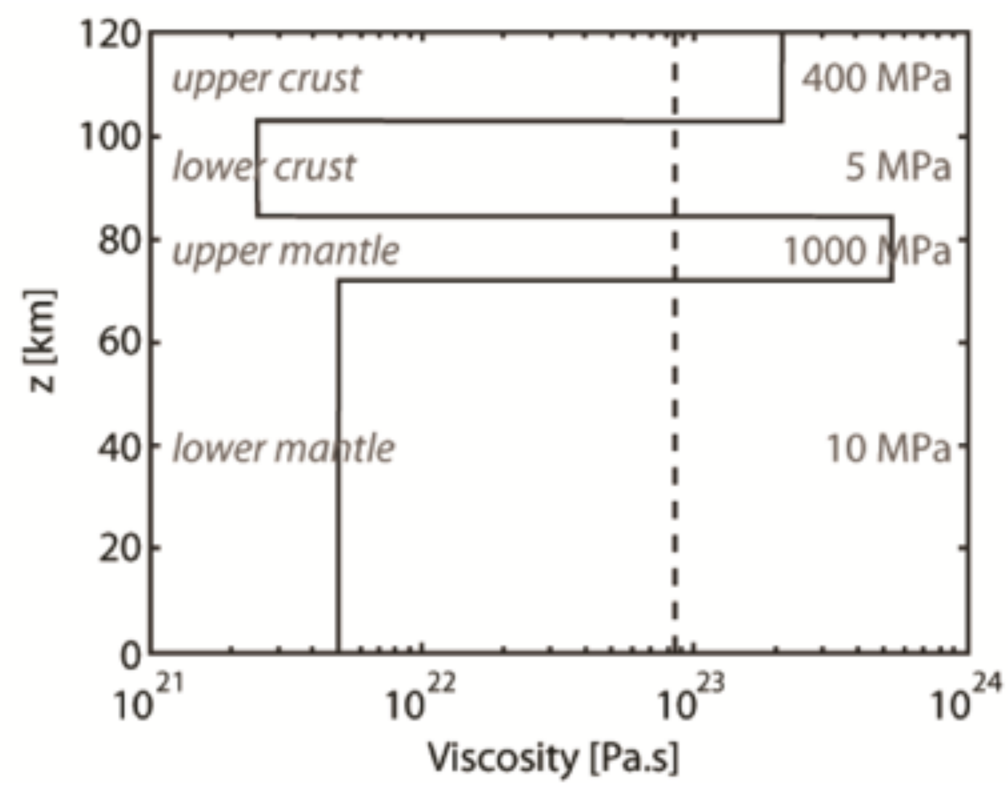
(Thieulot, PEPI, 2011)

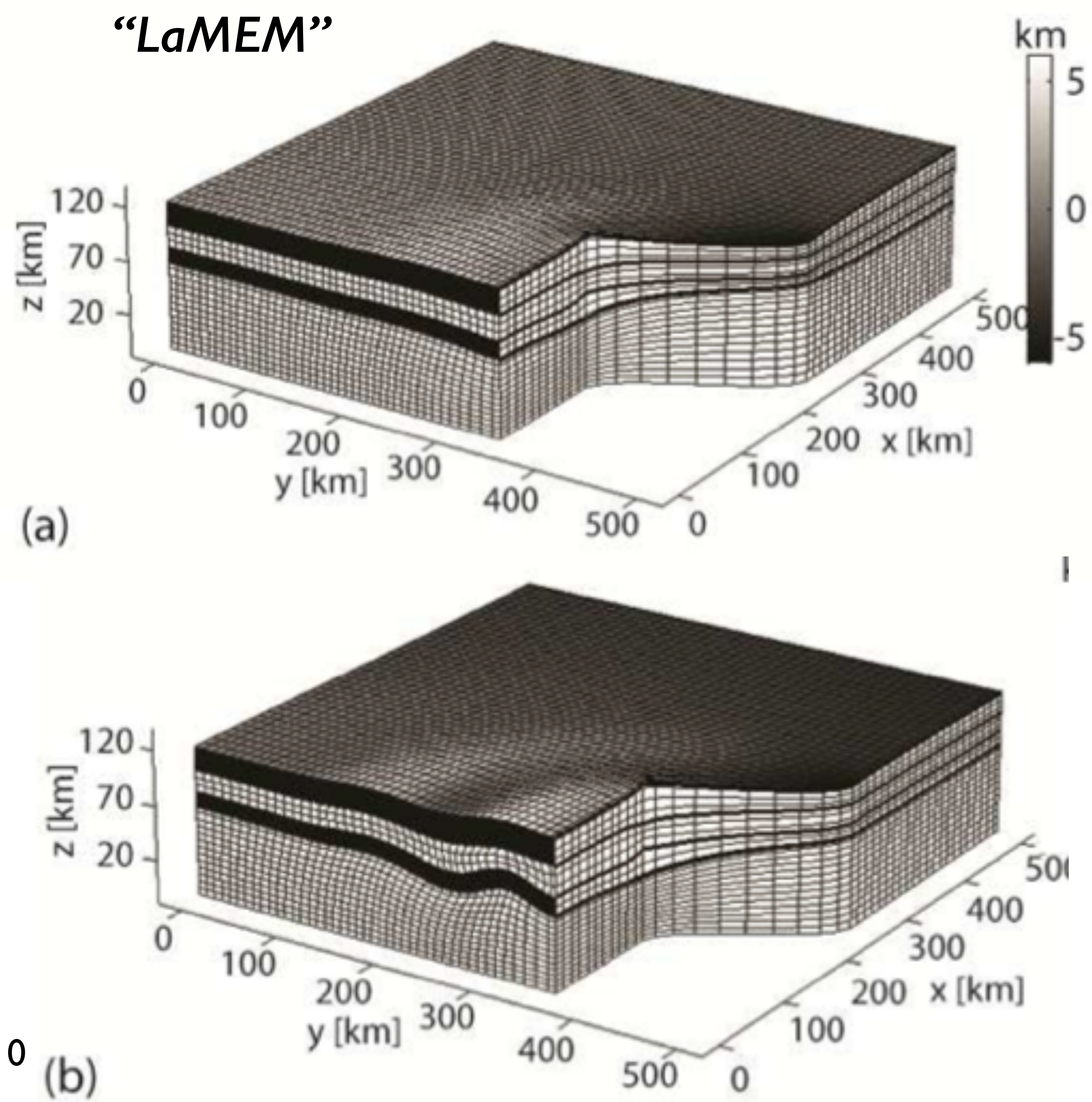
FEM examples

Viscous folding
(Lechmann, et al, GJI, 2011)

[Stokes flow]

+ Free surface boundary condition.
+ Deformed upper boundary.
+ Non-regular mesh on interior.
+ Viscosity layering resolved via mesh.





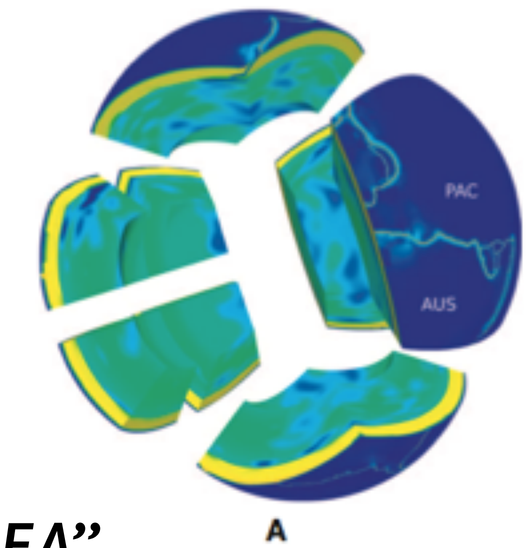
FEM examples

Global scale mantle convection
(Burstedde. et al, SC, 2009)
+ (Stadler, et al, Science, 2010)

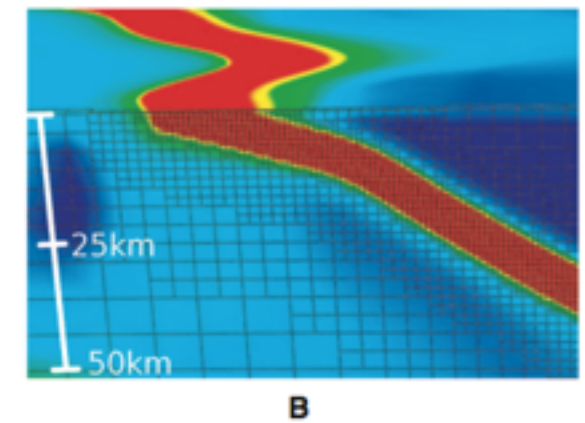
[Stokes flow + Energy transport]

+ Locally adaptive mesh.
+ Adaptation follows flow features.
+ 1 km resolution at the plate boundaries.

"RHEA"



A



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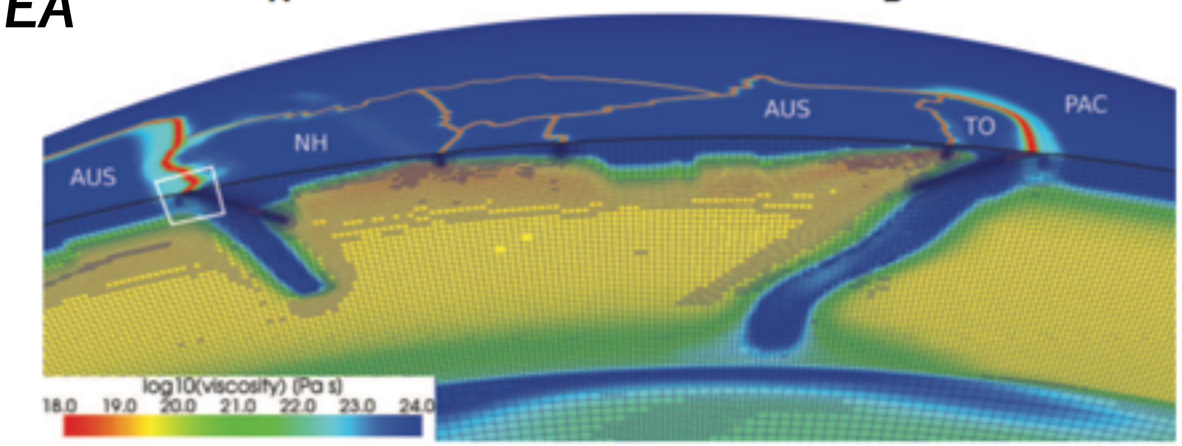
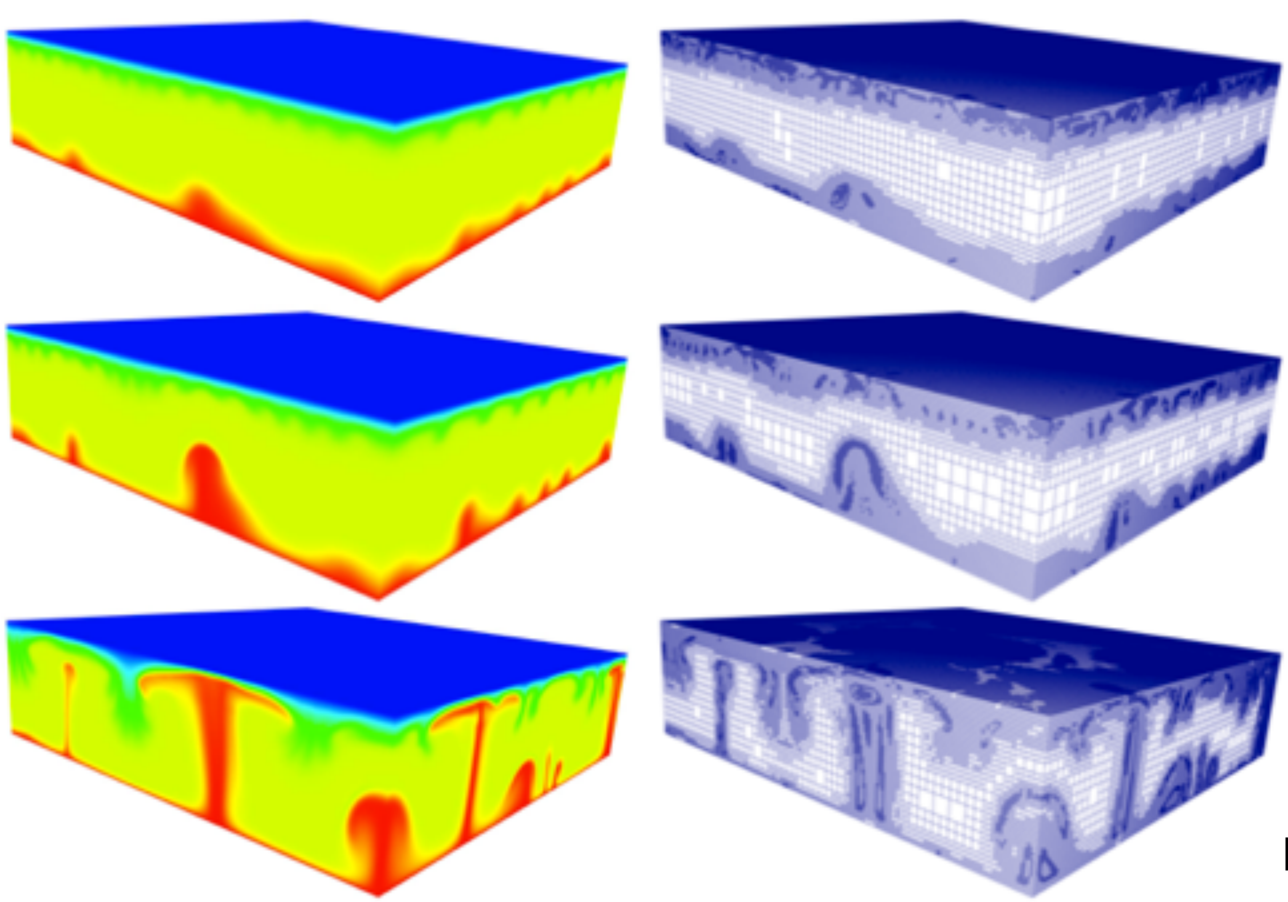
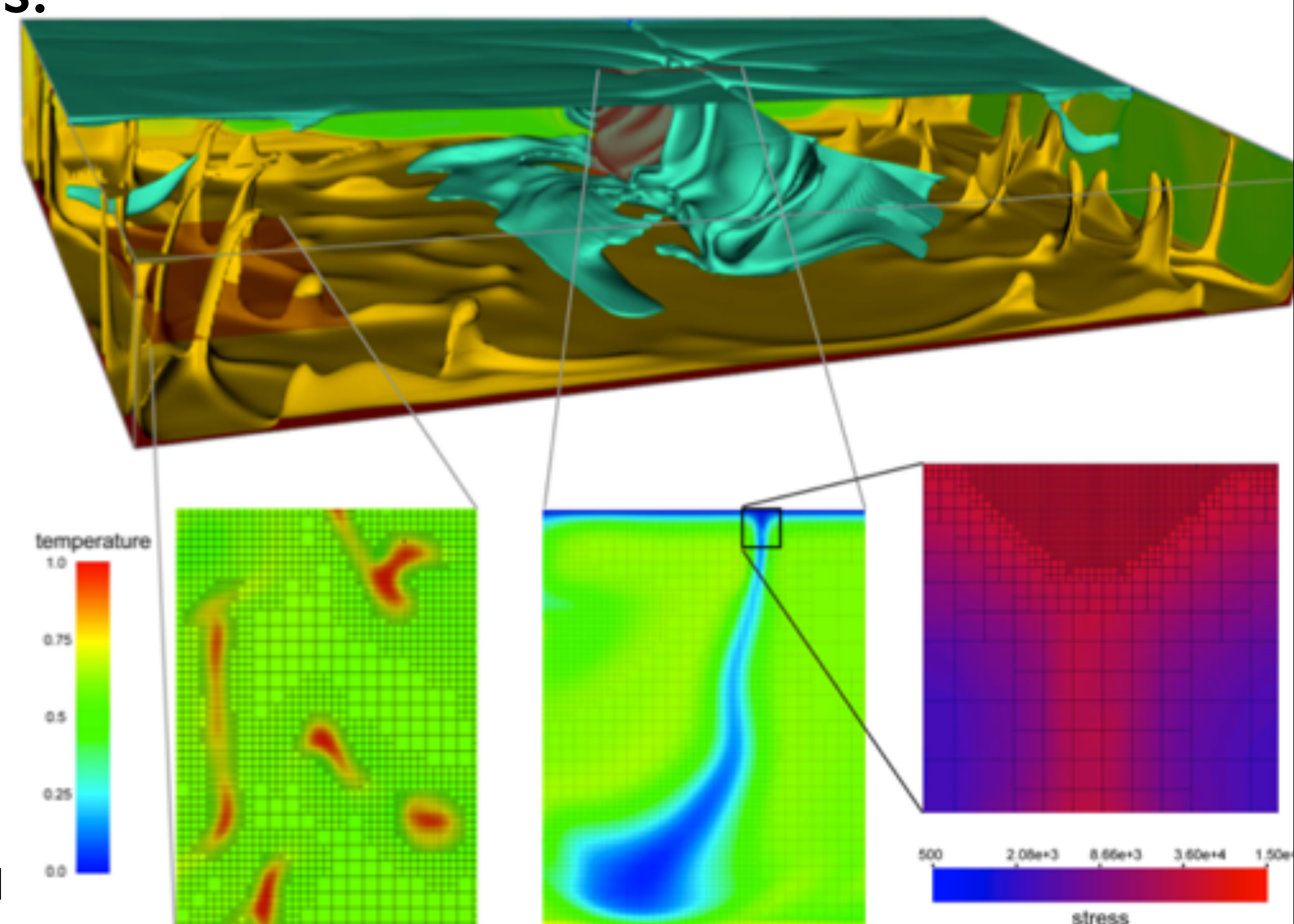


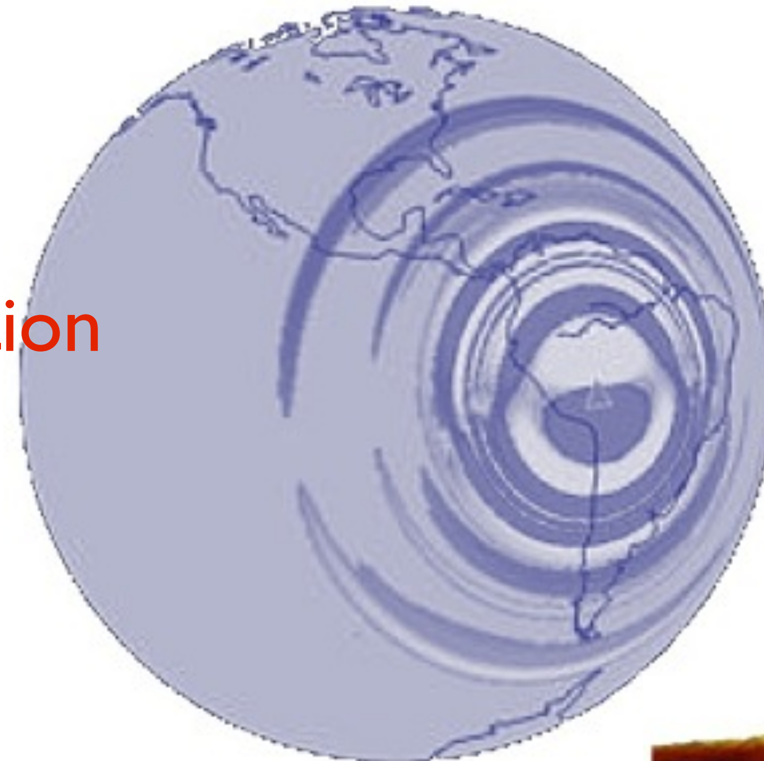


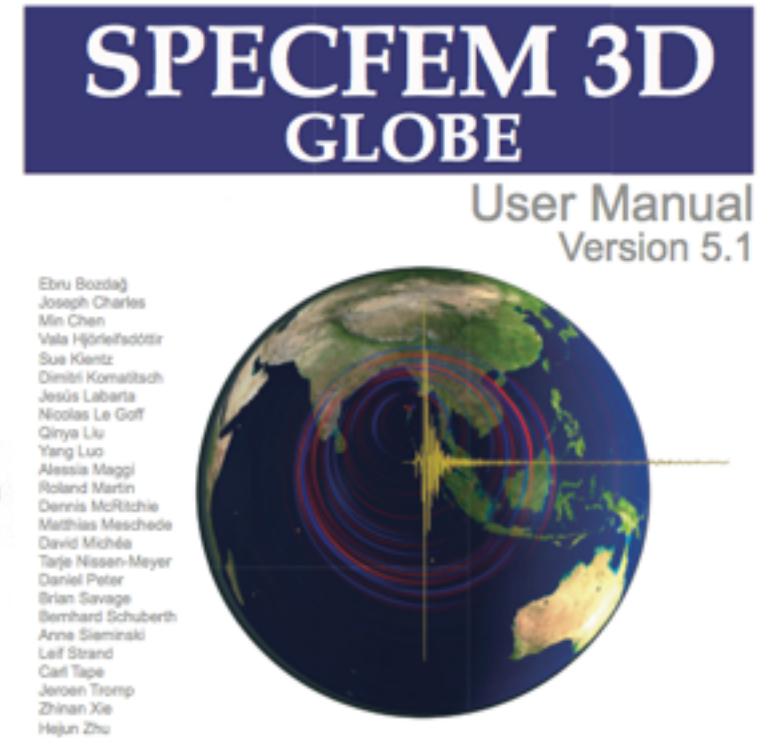


FEM examples

Global / local scale wave propagation
(CIG : www.geodynamics.org)

[Elasto-dynamics]





+ High order polynomials.
+ Locally adaptive mesh.
+ Topography tracked (deformed upper boundary).
+ Material jumps captured by the mesh.

"SPECFEM"