Introduction to Finite Element Modelling in Geosciences

651-4144-00L

July 25 - 29, 2016
NO Bld., Room D39
ETH Zurich

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Who are we?

- Oberassistant — Geophysical Fluid Dynamics group (P. Tackley)
- Regional scale geodynamic processes, HPC algorithms and software for Earth sciences
- Australian

- Oberassistant — Structural Geology & Tectonics group (J.-P. Burg)
- Computational structural geology and digital rock physics
- Swiss

Who are we?

- Postdoc — Advanced Computing Laboratory (Olaf Schenk, USI Lugano)
- Software and algorithms for accelerator technology (e.g. GPU, MIC)
- Californian

Objectives

To learn:

1. the basics of the finite element method;
2. how to program the finite element method;
3. how to apply it to equations relevant for Earth science applications.

The course is given in the form of MATLAB exercises, with an introduction of the relevant theory.

Format

This is a “*hands on*” course. “Hands on” means **you code** whilst **we assist** you.

In our experience, this is the most effective way to teach Earth scientists the finite element method. This is not a theoretical class. Only the essential components of the theory will be discussed, however, if you require further details, please ask.

Learning the practical aspects of finite elements is only achieved by writing your own code

Schedule

- 9:15 -- 1 hour presentation / overview / Q&A
- 14:00 -- 1 hour presentation / overview / Q&A
- The rest of the time you will be reading the course notes, hand-outs, completing the exercises, writing MATLAB code, or asking us for assistance.

Teaching Material

http://jupiter.ethz.ch/~gfdteaching/femblockcourse/2016/

Assessment

- ***To obtain a mark of 5.0***

1. Program the 2D diffusion equation with bilinear and with biquadratic elements.
2. Compute the order of accuracy of your code for both element types for the steady state diffusion equation (look at the online lecture notes).

- ***To obtain a mark greater than 5.0, do one (or more) of:***

1. Write a 2D elasticity code and demonstrate that it is working correctly with a suitable test problem.
2. Write a 2D stokes code (this will definitely earn a mark of 6.0).
3. Perform an order of accuracy test with the method of manufactured solutions for a *time dependent* 2D diffusion problem (using biquadratic elements).
4. Find a cool geological application for any of the codes you have developed and show how your numerical model gives new insight into this problem.

Assessment

- Prepare a short report (<10 pages) including a description of the model, the code implementation and any figures/graphs. All figures/graphs must have labelled axis, etc.

- The source code used to generate your results must be submitted with your report. It is a requirement that I can reproduce your results.

- **All reports and code must be submitted by August 31.** Please email your submission to dave.may@erdw.ethz.ch

- You can ask as many questions and visit as often as you require (appointment via email please!) to get your code working. *This is encouraged and will not negatively influence your final mark.*

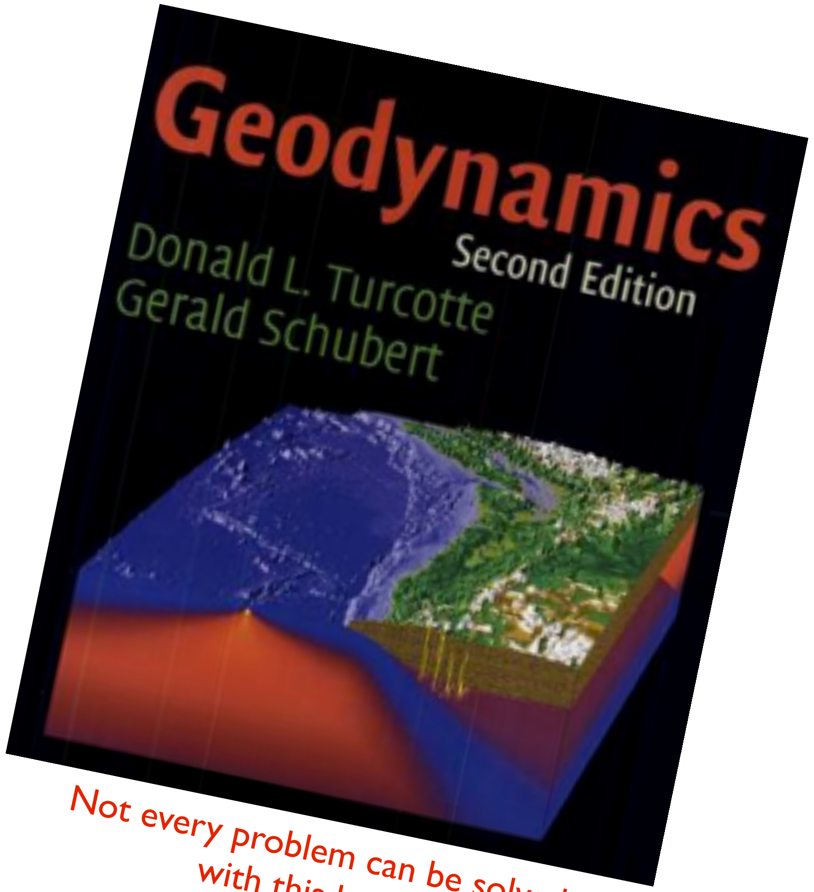
Why do we need numerics?

- Traditional methods to understand the Earth consist of geological, geophysical field based approaches - together with analogue experiments.

- None of these, on their own, or collectively, can satisfactorily provide a complete understanding of the dynamics in *z*-*t* space.

Why do we need numerics?

- Analytic methods only take us so far...
  - Overly simplified to the point where they don't resemble anything "Earth like".
  - Boundary conditions are often not appropriate.
  - Restricted dimensionality.
  - Mathematically too complex for most normal people...



Not every problem can be solved with this book!

Why use Finite Elements?

The Finite Element Method (FEM) possesses several important characteristics which are relevant to studying geological processes in a reliable and accurate manner:

1. Simplicity in applying the method to new equations.

2. Simplicity in meshing complicated geometries (internal and external). Meshes can be constructed using triangles or quadrilaterals. Meshes can be deformed and evolve with time.

3. Wide range of boundary conditions are easily able to be used; e.g. free surface boundary conditions are trivial to implement.

5. Jumps in material properties do not introduce any additional complexity.

6. Minimal programming complexity when changing from 2D to 3D (much code can be re-used).

7. Rich mathematical foundations —> reliable and accurate results.

Finite Element Examples

- Mantle convection
- Lithospheric deformation
    - crustal scale
    - regional scale
- Heat flow, surface processes
- Two-phase flow
- Wave propagation

Convection

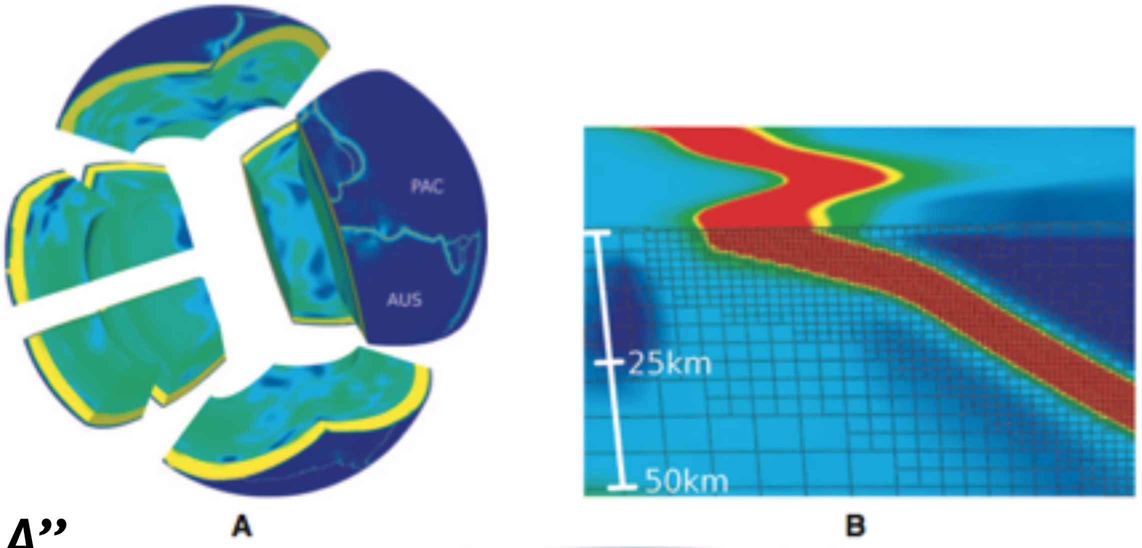
Global scale mantle convection

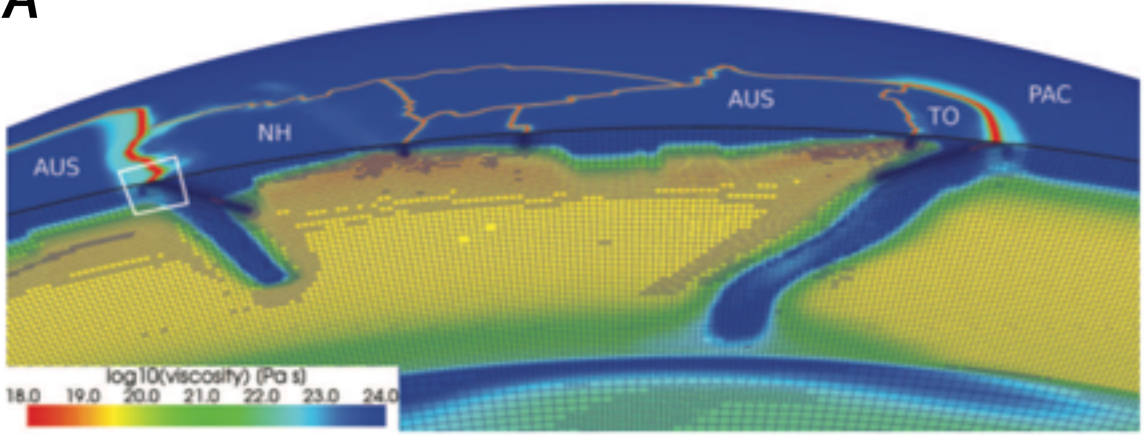
(Burstedde. et al, SC, 2009;
Stadler, et al, Science, 2010)

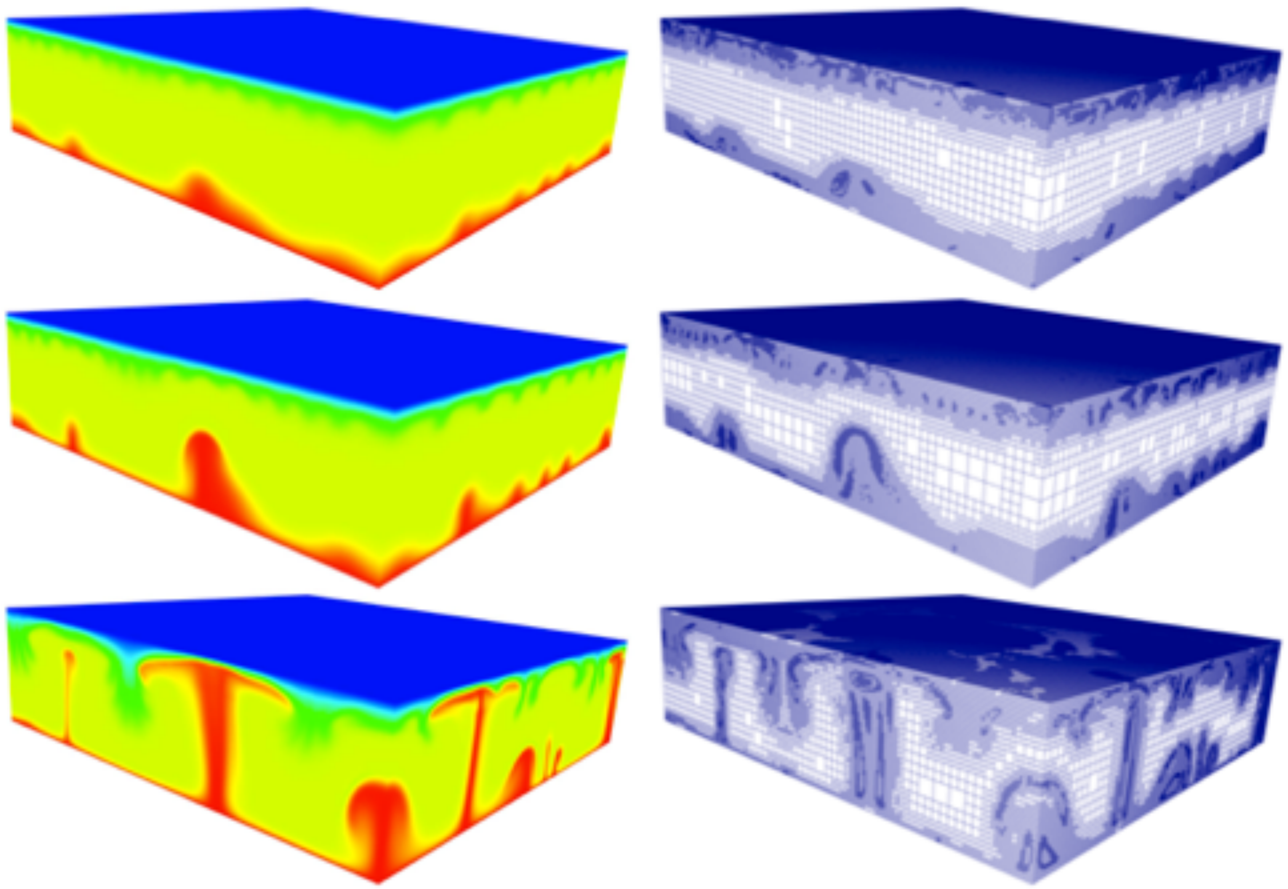
[Stokes flow + Energy transport]

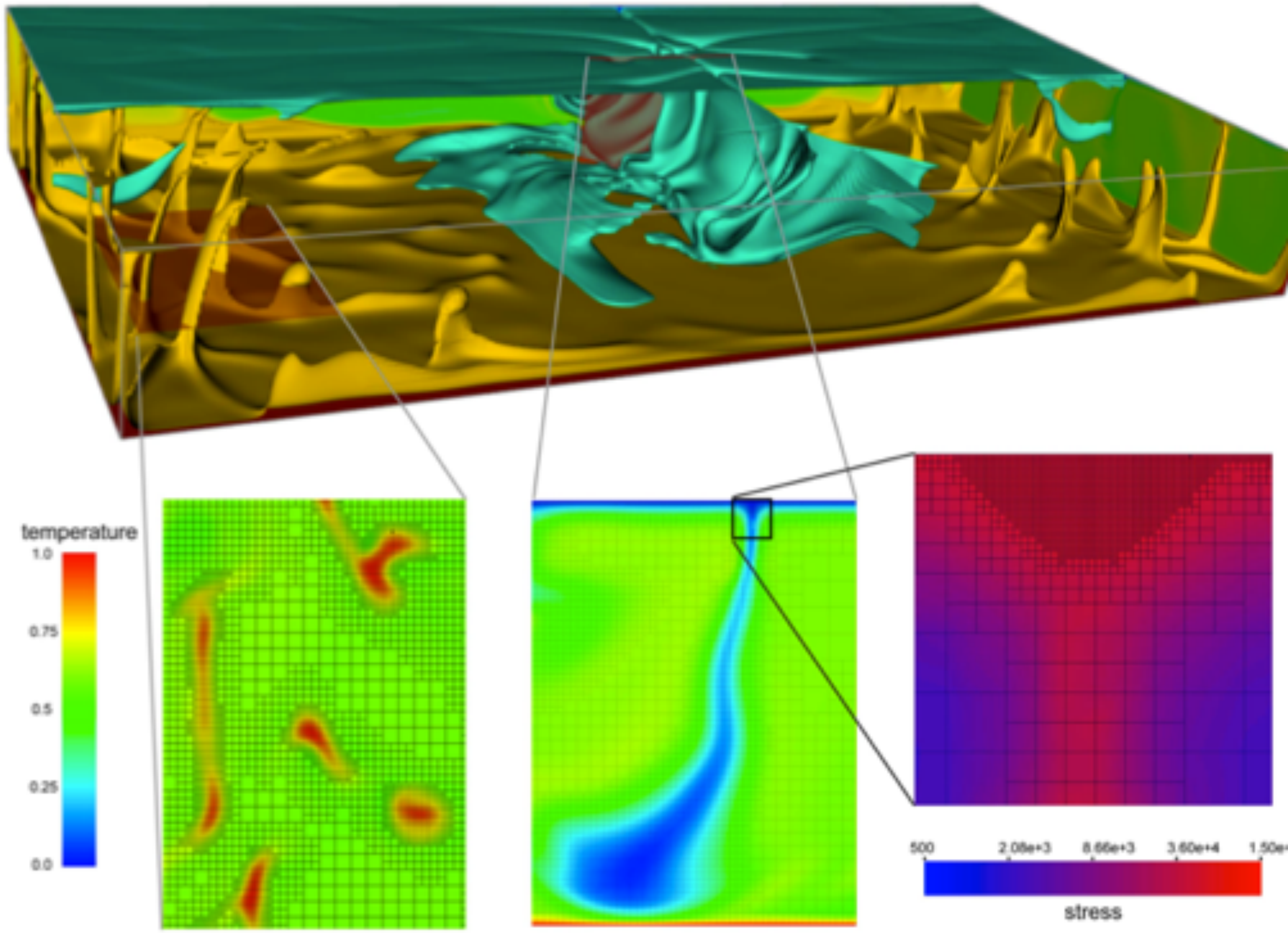
+ Locally adaptive mesh.
+ Adaptation follows flow features.
+ 1 km resolution at the plate boundaries.

“RHEA”









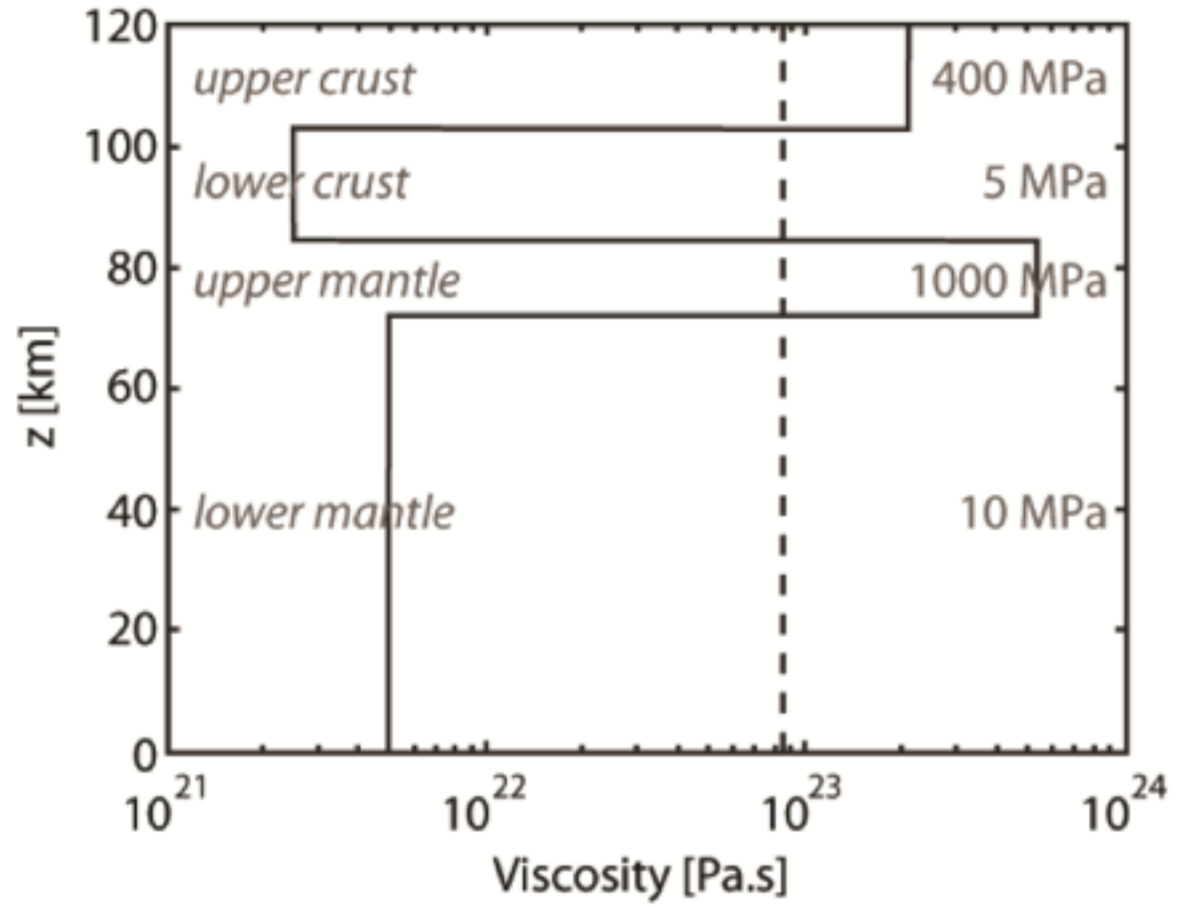
Lithospheric deformation

Viscous folding

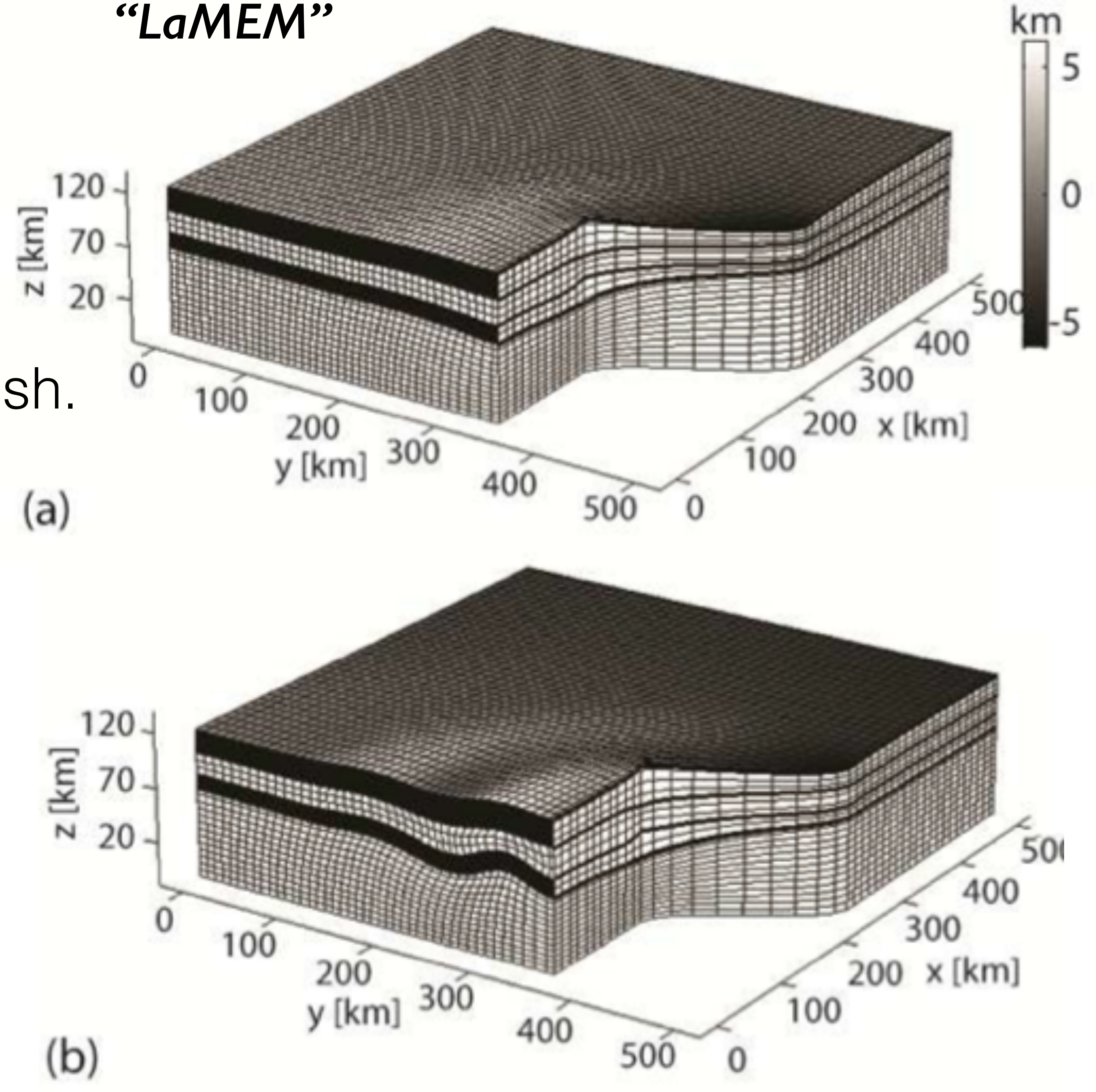
(Lechmann, et al, GJI, 2011)

[Linear 3D Stokes flow]

+ Free surface boundary condition.
+ Deformed upper boundary.
+ Non-regular mesh on interior.
+ Viscosity layering resolved via mesh.



“LaMEM”



urich, 2016

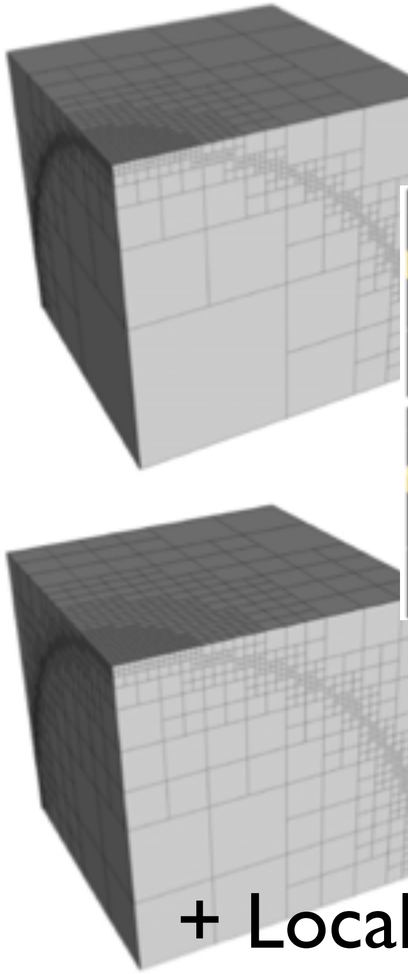
Lithospheric deformation

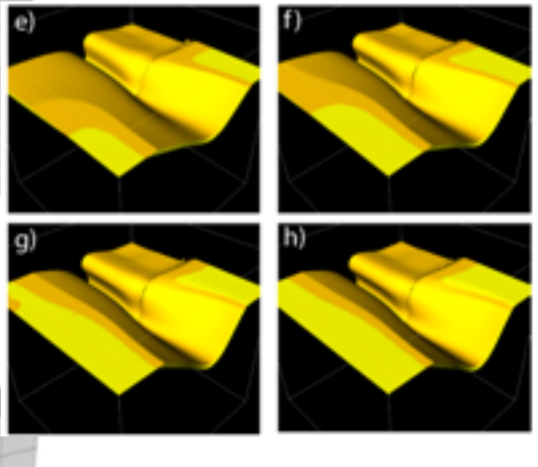
Coupled thermo-mechanical models

[Non-linear 3D Stokes flow + Energy transport + Surface processes]

+ Deformed upper boundary, coupled to surface process models.
+ Non-regular mesh on interior.
+ Flux boundary conditions.

“DOUAR”

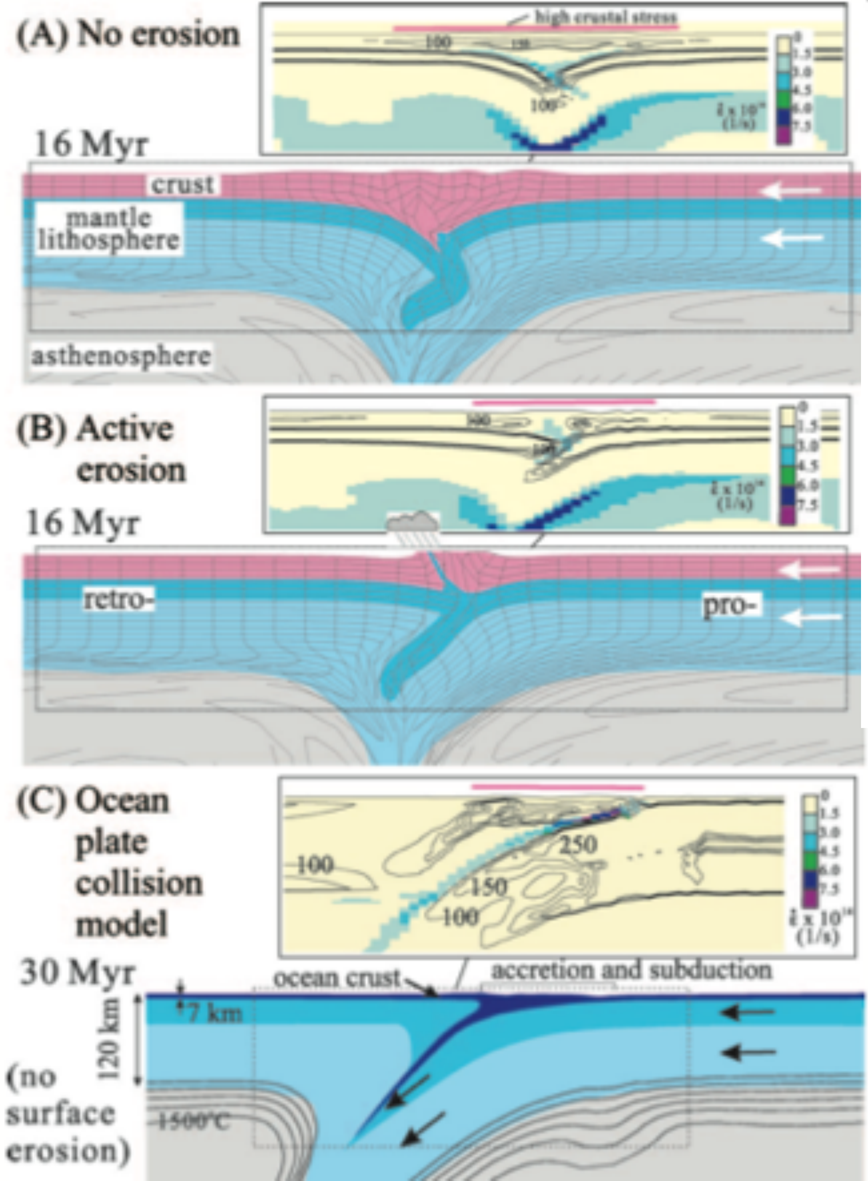




(Braun, et al, PEPI, 2008)

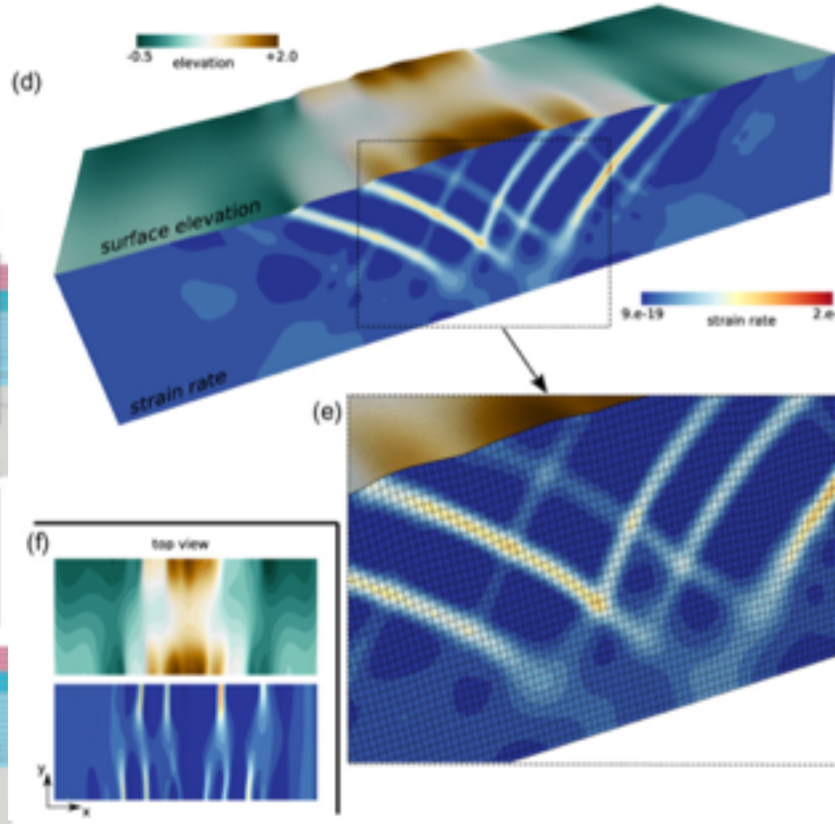
+ Local grid refinement

“SOPALE”



(Pysklywec, Geology, 2006)

“FANTOM”



(Thieulot, PEPI, 2011)

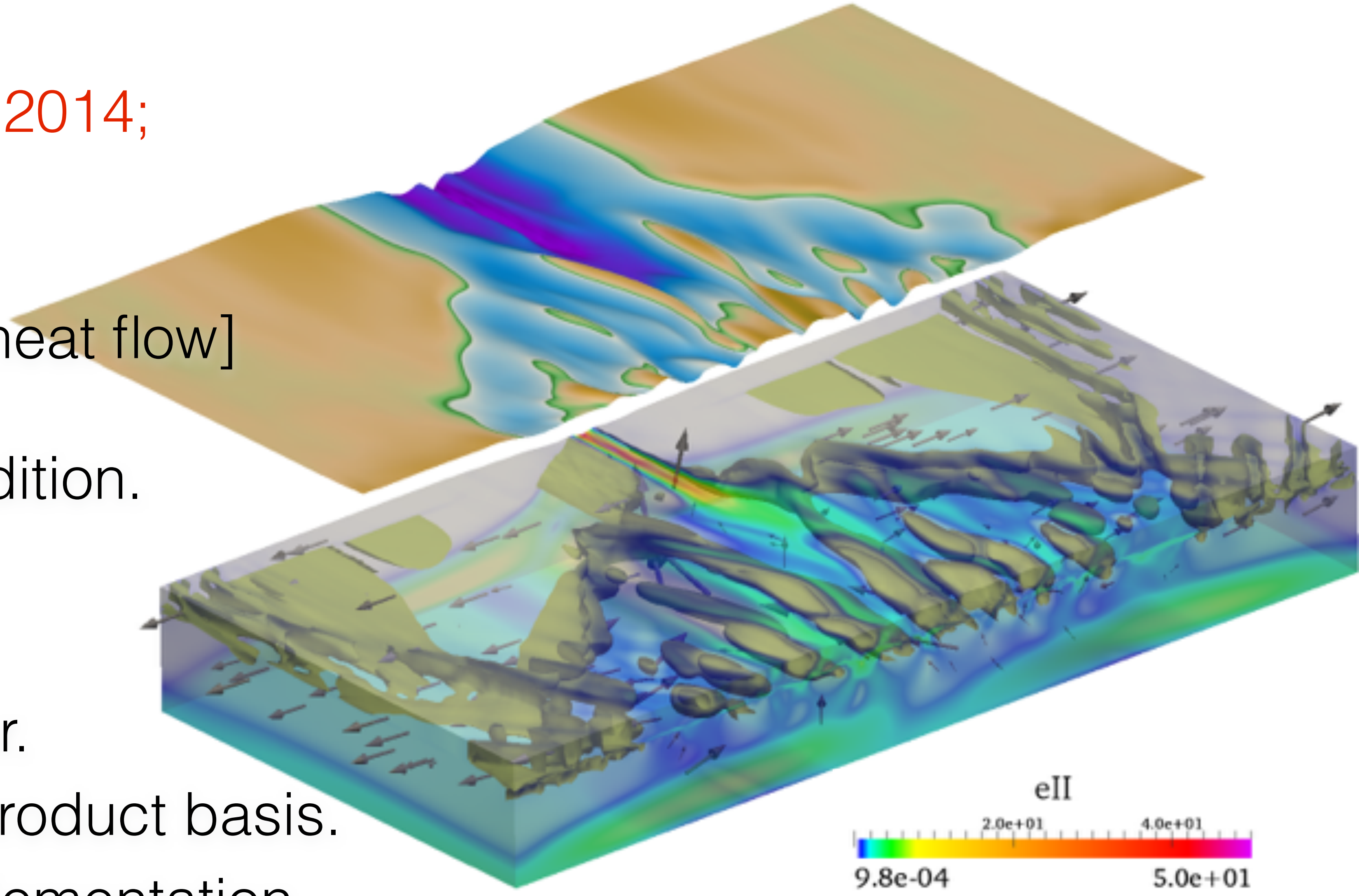
Lithospheric deformation

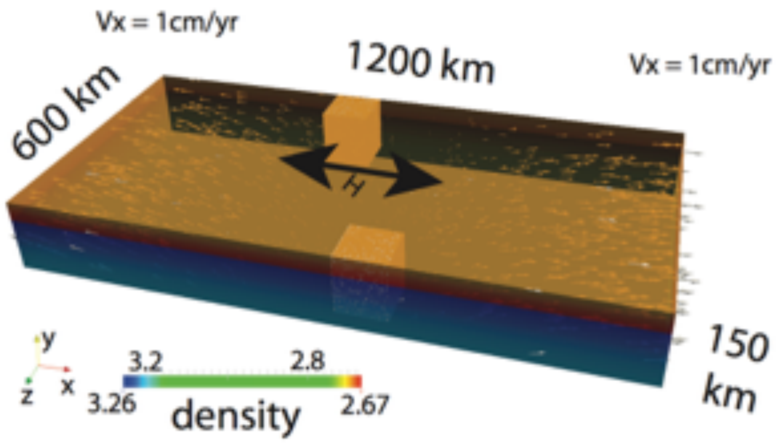
Continental break-up

(May et al, Super Computing, 2014;
May et al, CMAME, 2014)

[Non-linear 3D Stokes flow + heat flow]

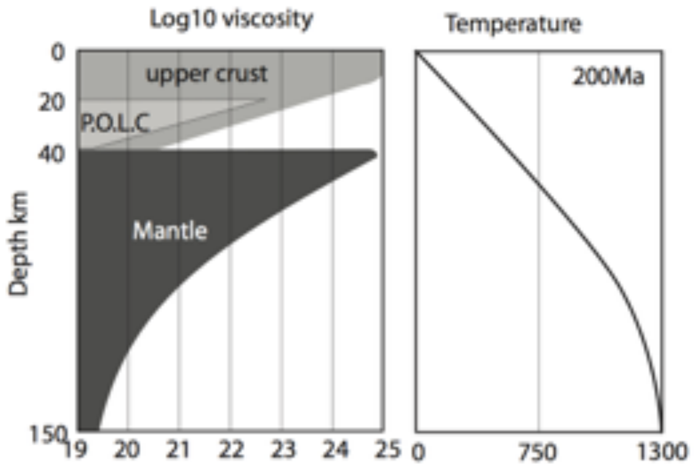
+ Free surface boundary condition.
+ Visco-plastic rheology.
+ Deformed upper boundary.
+ Non-regular mesh on interior.
+ Fast —> exploiting tensor product basis.
+ Massively parallel HPC implementation.

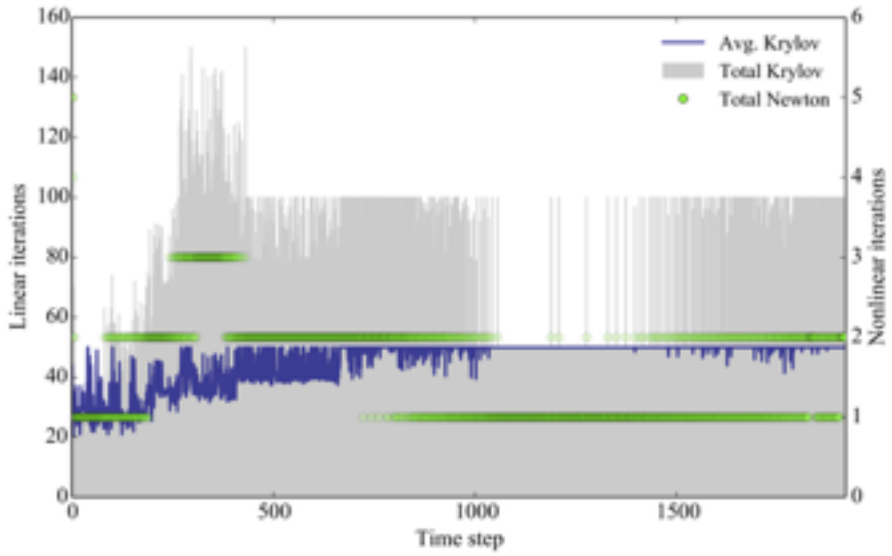




Material inflow at the base of the box to compensate outflow

Localisation is triggered by two cubes of elevated plastic strain





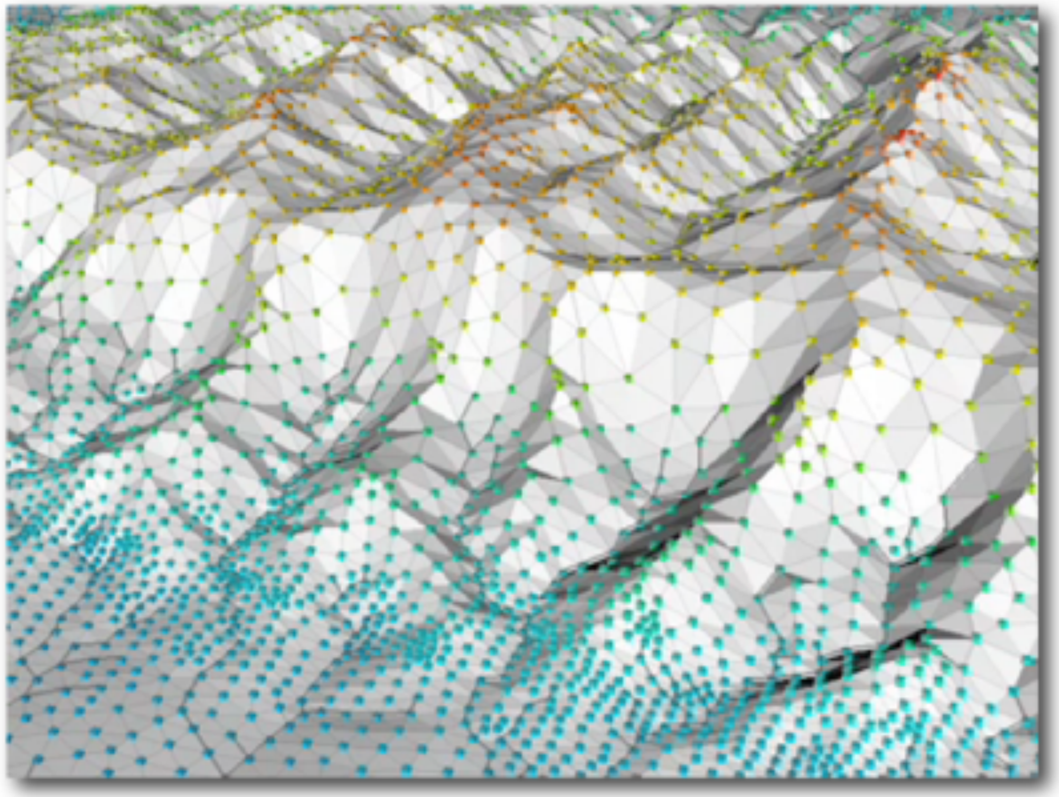
Heat Flow and Surface Processes

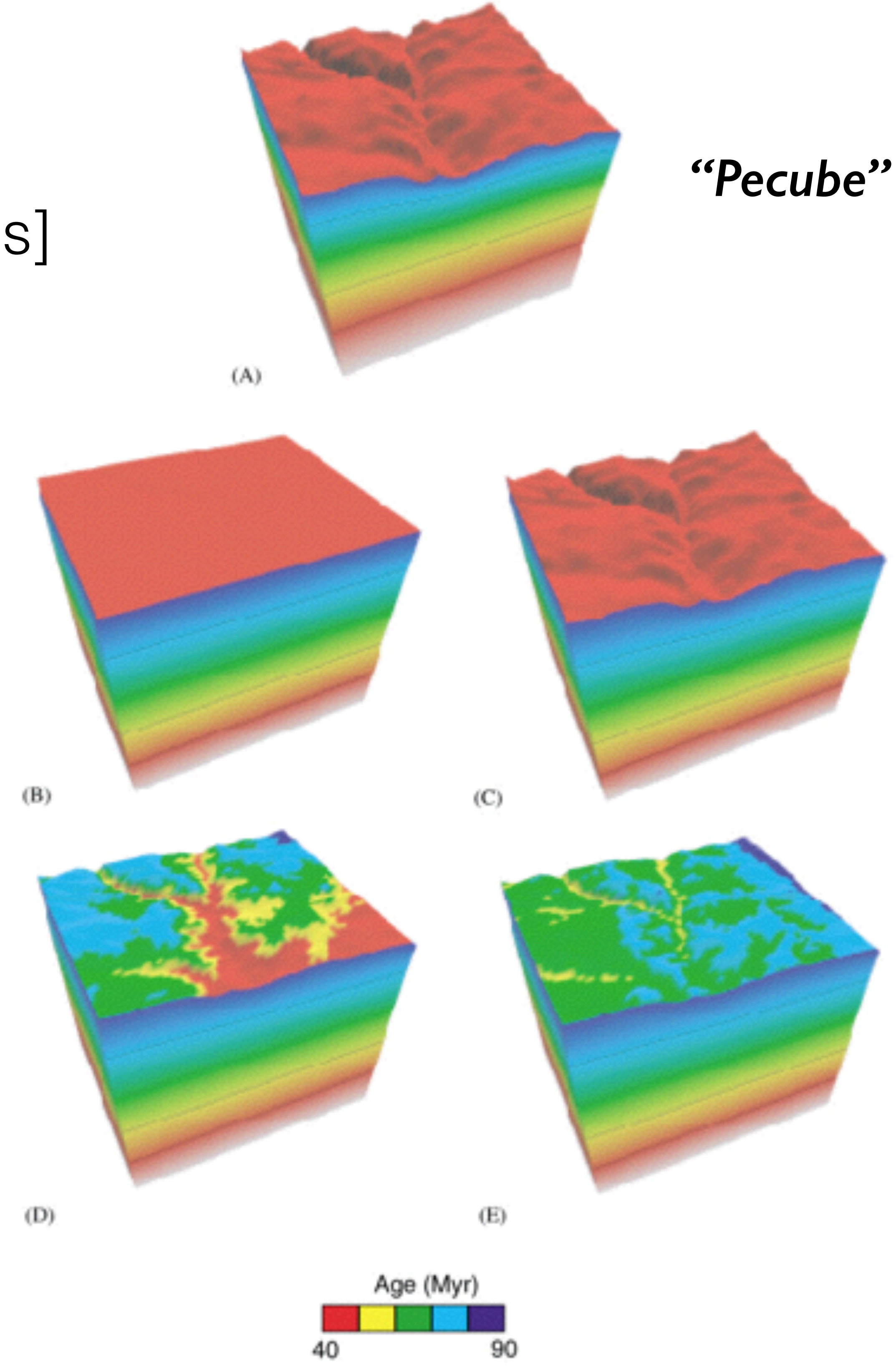
Thermochronology

(Braun, Comput. & Geosci., 2003)

[Energy transport + surface processes]

+ Deformed upper boundary.
+ Non-regular mesh on interior.
+ Flux boundary conditions.



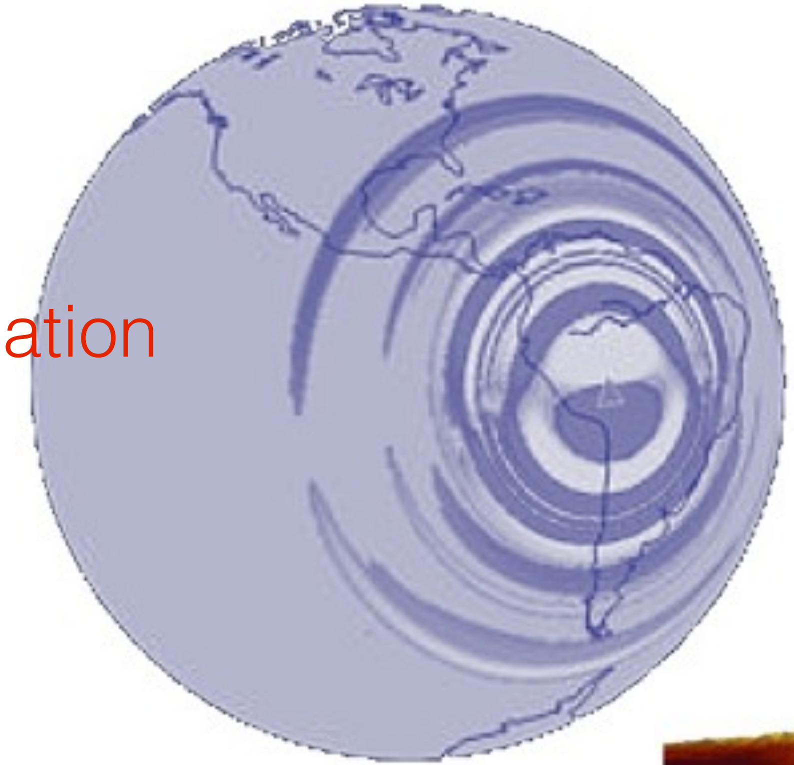


"Pecube"

Wave Prop.

Global / local scale wave propagation
(CIG : www.geodynamics.org)

[Elasto-dynamics]



SPECFEM 3D
GLOBE
User Manual
Version 5.1

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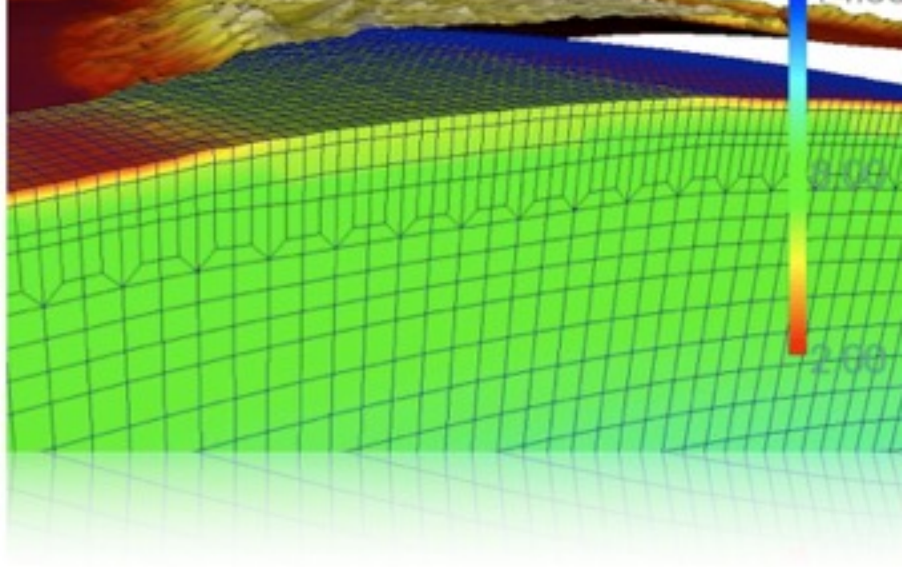
Bernhard Schuberth
Anne Sieminski
Leif Strand
Carl Tape
Jeroen Tromp
Zhinan Xie
Hejun Zhu

+ High order polynomials.
+ Locally adaptive mesh.
+ Topography tracked (deformed upper boundary).
+ Material jumps captured by the mesh.

"SPECFEM"



SPECFEM 3D
User Manual
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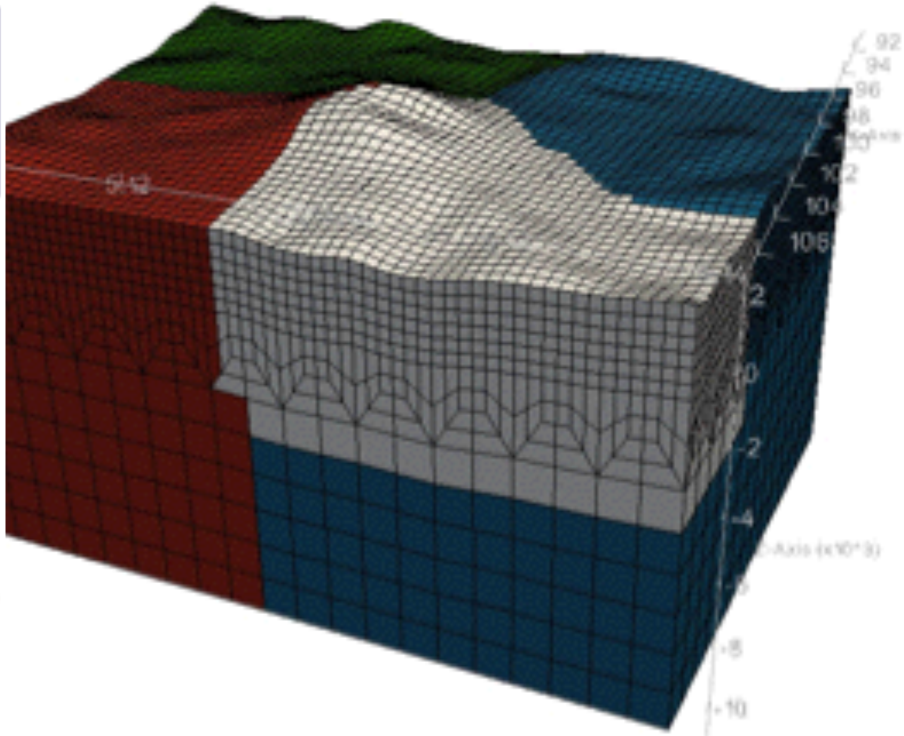
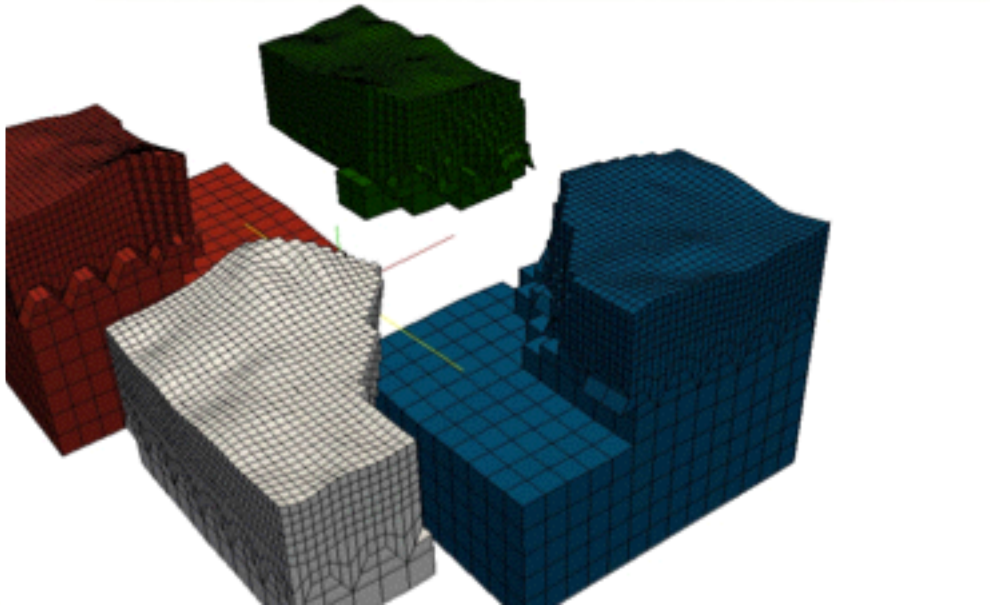
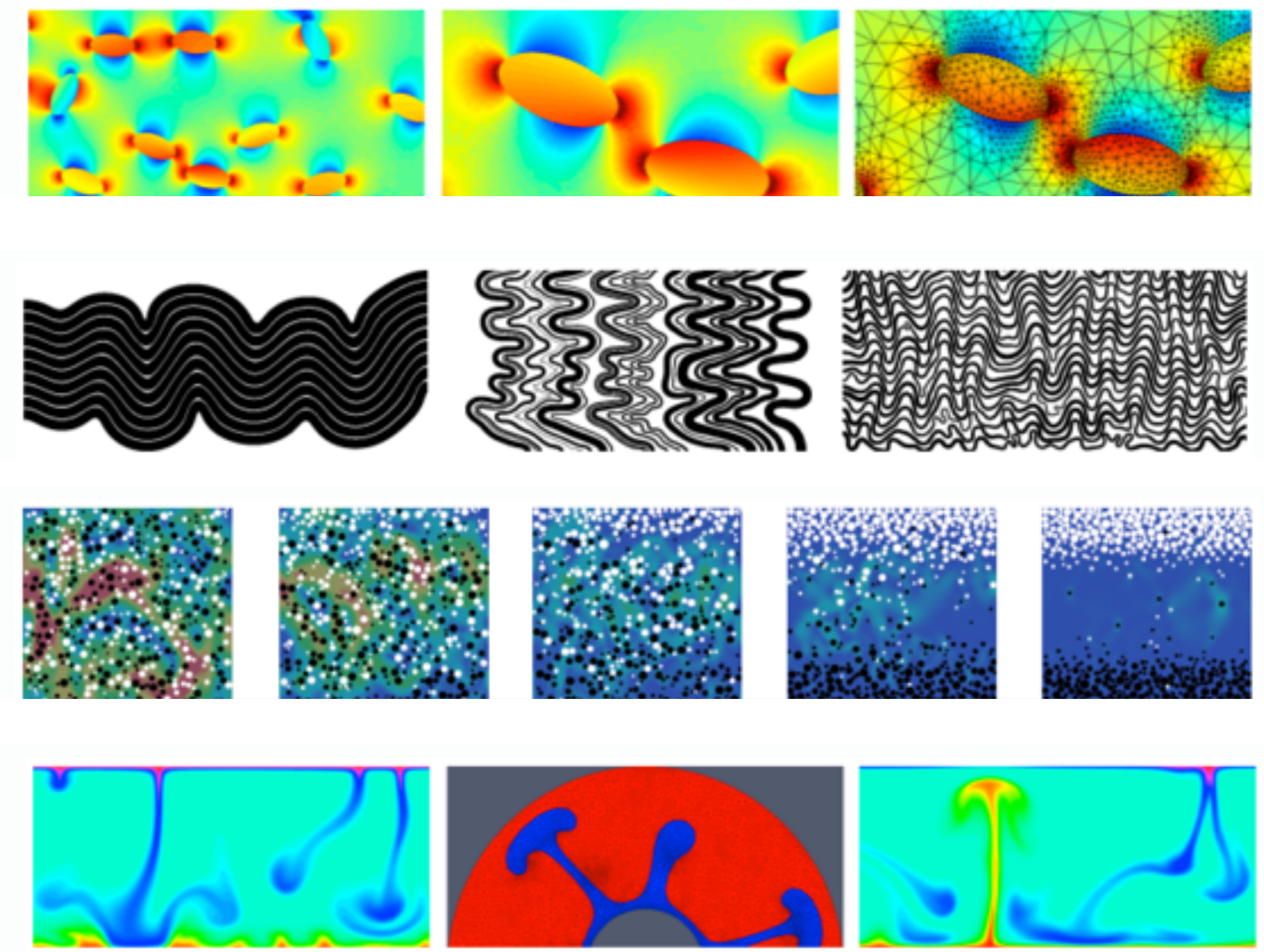


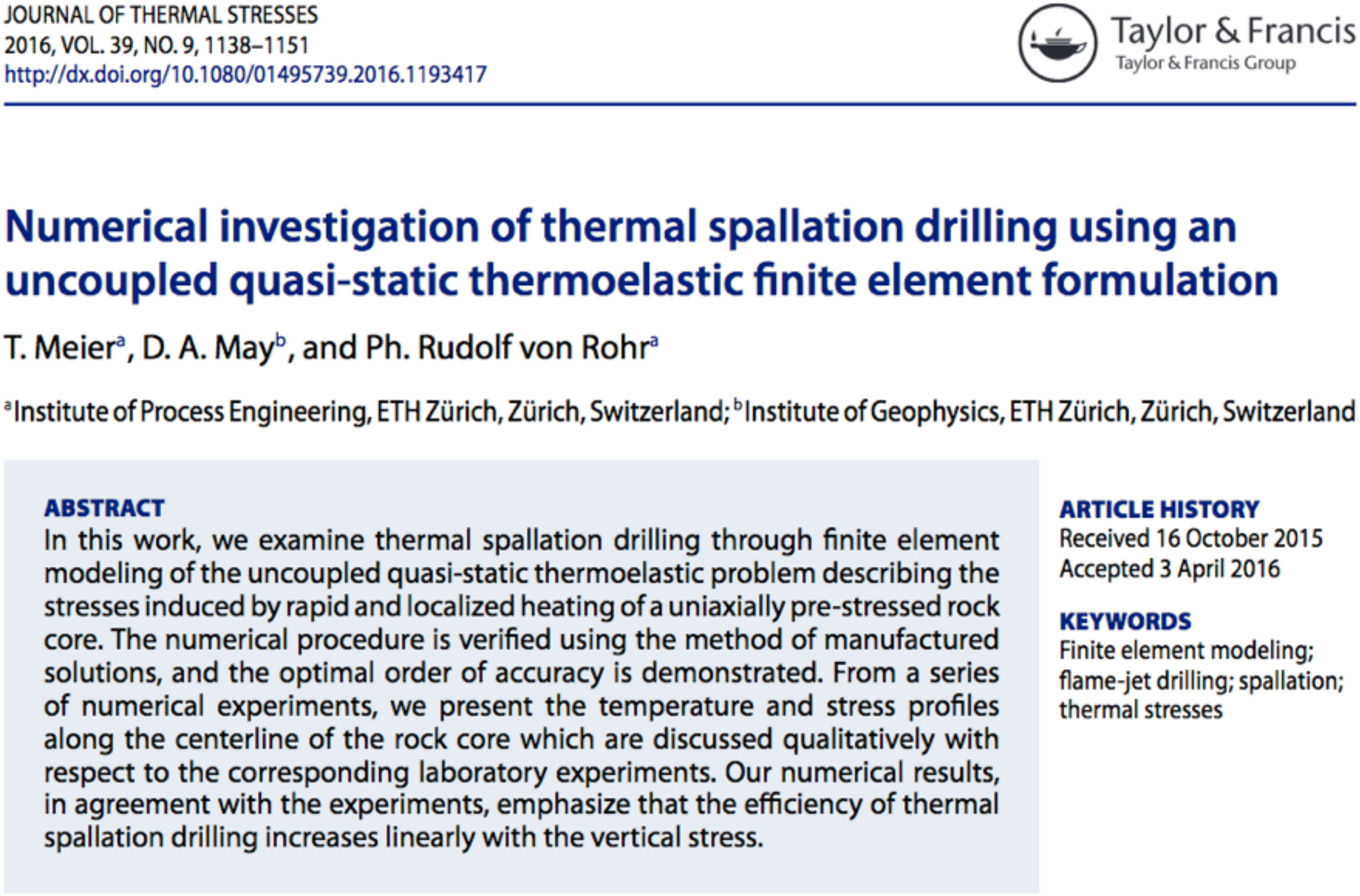


MATLAB FE codes for research?

http://milamin.sourceforge.net/

- Unstructured triangular meshes
- **Fast** 1M DOFs per minute
- Lots of applications
    - mixing
    - folding
    - sedimentation
    - thermo-chemical convection





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Numerical investigation of thermal spallation drilling using an uncoupled quasi-static thermoelastic finite element formulation

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ABSTRACT
In this work, we examine thermal spallation drilling through finite element modeling of the uncoupled quasi-static thermoelastic problem describing the stresses induced by rapid and localized heating of a uniaxially pre-stressed rock core. The numerical procedure is verified using the method of manufactured solutions, and the optimal order of accuracy is demonstrated. From a series of numerical experiments, we present the temperature and stress profiles along the centerline of the rock core which are discussed qualitatively with respect to the corresponding laboratory experiments. Our numerical results, in agreement with the experiments, emphasize that the efficiency of thermal spallation drilling increases linearly with the vertical stress.

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KEYWORDS
Finite element modeling; flame-jet drilling; spallation; thermal stresses

now it is your turn…