Some other popular quadrilateral isoparametric elements are eight-node and nine-node elements as shown in Fig. 6.2.6. Their shape functions are given below.

Eight-node element:

$$H_1 = \frac{1}{4}(1-\xi)(1-\eta)(-1-\xi-\eta) \tag{6.2.45}$$

$$H_2 = \frac{1}{4}(1+\xi)(1-\eta)(-1+\xi-\eta) \tag{6.2.46}$$

$$H_3 = \frac{1}{4}(1+\xi)(1+\eta)(-1+\xi+\eta) \tag{6.2.47}$$

$$H_4 = \frac{1}{4}(1-\xi)(1+\eta)(-1-\xi+\eta) \tag{6.2.48}$$

$$H_5 = \frac{1}{2}(1-\xi^2)(1-\eta) \tag{6.2.49}$$

$$H_6 = \frac{1}{2}(1+\xi)(1-\eta^2) \tag{6.2.50}$$

$$H_7 = \frac{1}{2}(1-\xi^2)(1+\eta) \tag{6.2.51}$$

$$H_8 = \frac{1}{2}(1-\xi)(1-\eta^2) \tag{6.2.52}$$

Nine-node element:

$$H_1 = \frac{1}{4}(\xi^2-\xi)(\eta^2-\eta) \tag{6.2.53}$$

$$H_2 = \frac{1}{4}(\xi^2+\xi)(\eta^2-\eta) \tag{6.2.54}$$

$$H_3 = \frac{1}{4}(\xi^2+\xi)(\eta^2+\eta) \tag{6.2.55}$$

$$H_4 = \frac{1}{4}(\xi^2-\xi)(\eta^2+\eta) \tag{6.2.56}$$

$$H_5 = \frac{1}{2}(1-\xi^2)(\eta^2-\eta) \tag{6.2.57}$$

$$H_6 = \frac{1}{2}(\xi^2+\xi)(1-\eta^2) \tag{6.2.58}$$

$$H_7 = \frac{1}{2}(1-\xi^2)(\eta^2+\eta) \tag{6.2.59}$$

$$H_8 = \frac{1}{2}(\xi^2-\xi)(1-\eta^2) \tag{6.2.60}$$

$$H_9 = (1-\xi^2)(1-\eta^2) \tag{6.2.61}$$

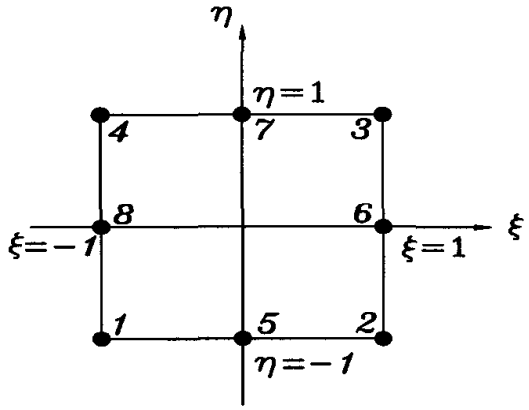

Figure 6.2.6 Eight-Node Isoparametric Element

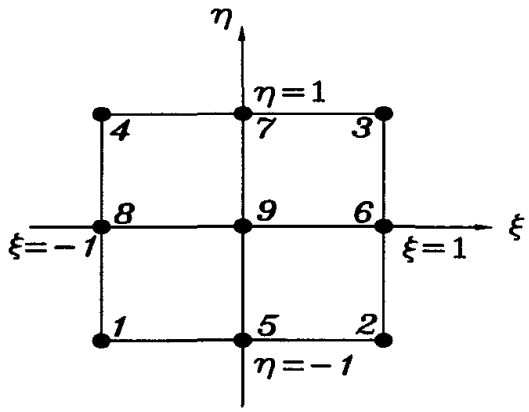

Figure 6.2.7 Nine-Node Isoparametric Element

6.3 Triangular Elements

Like quadrilateral isoparametric elements, triagular isoparametric elements can be defined. Shape functions of the linear triangular element are in terms of the *natural* coordinate system

$$H_1 = 1 - \xi - \eta \tag{6.3.1}$$

$$H_2 = \xi \tag{6.3.2}$$

$$H_3 = \eta \tag{6.3.3}$$

for the nodes shown in Fig. 6.3.1. The quadratic triangular element has the following shape functions with reference to Fig. 6.3.2.

$$H_1 = (1 - \xi - \eta)(1 - 2\xi - 2\eta) \tag{6.3.4}$$

$$H_2 = \xi(2\xi - 1) \tag{6.3.5}$$

$$H_3 = \eta(2\eta - 1) \tag{6.3.6}$$

$$H_4 = 4\xi(1 - \xi - \eta) \tag{6.3.7}$$