A global model of mantle conductivity derived from 5 years of CHAMP, Ørsted, and SAC-C magnetic data

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We present a global 1-D conductivity model which is obtained by analysis of five years (2001–2005) of simultaneous magnetic data from the three satellites Ørsted, CHAMP and SAC-C. After removal of core and crustal fields as predicted by a recent field model we used non-polar scalar and vector observations from the night-side sector, and interpret the field residuals in terms of a large-scale contribution from the magnetospheric ring current and its induced counterpart. We then derive transfer functions between internal (induced) and external expansion coefficients of the magnetic potential and provide globally-averaged C-responses in the period range between 14 hours and 4 months. Since the satellite responses are probably influenced by induction in the oceans for periods shorter than a few days, we correct the data for this effect. Interpreting the corrected responses yields a 1-D conductivity model which is rather similar to models derived from ground-based data.


1. Introduction

Temporal variations of the Earth’s magnetic field have long been used to infer global conductivity-depth (1-D) profiles of the Earth’s mantle, mostly from continental geomagnetic observatories [cf. Schmucker, 1985; Schultz and Larsen, 1987; Olsen, 1998]. In contrast to the data from observatories, which are sparse and irregularly distributed (with only few in oceanic regions), data collected by satellites (such as Magsat, Ørsted, CHAMP and SAC-C) provide full coverage of the Earth’s surface. This enables one to derive a globally-averaged conductivity profile which is not biased toward continental regions (as is probably the case for most observatory-based models). However, satellite data analysis is more difficult compared to observatory data for two reasons: First, low-orbit satellites move typically with a speed of 7–8 km/s and thus measure a mixture of temporal and spatial changes of the magnetic field. Second, satellites pass over both continents and oceans, and therefore the magnetic satellite data are affected by induction in the oceans [cf. Tarits and Grammatica, 2000; Everett et al., 2003; Kuvshinov and Olsen, 2005b] in a complicated way. In spite of these difficulties, a number of successful attempts have been made to probe mantle conductivity from space [Didwali, 1984; Oraevsky et al., 1993; Olsen, 1999b; Olsen et al., 2002; Constable and Constable, 2004; Velimsky et al., 2006]. However, the results of previous satellite data analyses are generally more scattered than those obtained from observatory data, due to the shorter time-series (1 year or less in most previous analyses) of satellite data, which yields noisier results compared to an analysis of the several decades of data that are available from ground observatories.

In this paper we estimate C-responses (and a 1-D model of mantle conductivity) from a considerably extended satellite data set which includes five years of simultaneous magnetic field recordings from Ørsted, CHAMP and SAC-C satellites.

2. Data Processing

We use five years (2001–2005) of scalar and vector magnetic field measurements from the Ørsted and CHAMP satellites, and scalar magnetic field measurements from the SAC-C satellite. These data have been processed as follow. First, the core and crustal contributions as predicted by the CHAOS model [Olsen et al., 2006] have been removed from the observations. As a part of the CHAOS model, an improved alignment of the CHAMP vector data (i.e., the transformation from the instrument frame to the geocentric frame) was performed; for our study we used these improved CHAMP vector data. In order to reduce the influence of ionospheric currents we only use night-side (magnetic local time between 18:00 and 06:00) and non-polar (geomagnetic latitude equatorward of 50°) data. The resulting magnetic field residuals, dB, are considered to contain the large-scale contribution from the magnetospheric ring-current and its induced counterpart. Outside the conducting Earth (r > a), and assuming a laterally uniform conductivity in the Earth, these residuals, dB = −∇V, are approximated by a magnetic scalar potential

\[ V(t, r, \theta) = a \left( \epsilon_1^0(t) \left( \frac{e^2}{\alpha^2} \right) + \epsilon_1^0(t) \left( \frac{e^2}{\rho^2} \right)^2 \right) \Phi_1^0(\cos \theta), \]

where \( a = 6371.2 \text{ km} \) is the mean radius of the Earth, \( \theta \) is geomagnetic colatitude, \( \Phi_1^0 = \cos \theta \) is the Legendre function of degree 1 and order 0, and \( \epsilon_1^0 \) and \( \epsilon_1^0 \) are the Gauss coefficients describing sources internal and external to the Earth, respectively. In this equation we have only used the spherical harmonic \( \Phi_1^0 \); inclusion of higher harmonics did not improve the result.

Estimation of the coefficients on an orbit-by-orbit basis is the usual approach when analyzing single satellite data. In this case the sampling interval of the recovered time series of \( \epsilon_1^0 \) and \( \epsilon_1^0 \) is given by the orbit period (approxi-
Table 1. C-Responses, $\delta C$, and Squared Coherency $coh^2$ for 17 Periods $T$, Derived From 5 Years of CHAMP, Ørsted, and SAC-C Magnetic Data

<table>
<thead>
<tr>
<th>$T$, s</th>
<th>Uncorrected Responses</th>
<th>Corrected Responses*</th>
</tr>
</thead>
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<tr>
<td></td>
<td>$Re,C$, km</td>
<td>$Im,C$, km</td>
</tr>
<tr>
<td>50800</td>
<td>236</td>
<td>$-368$</td>
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<tr>
<td>71600</td>
<td>367</td>
<td>$-267$</td>
</tr>
<tr>
<td>100900</td>
<td>555</td>
<td>$-207$</td>
</tr>
<tr>
<td>142300</td>
<td>606</td>
<td>$-153$</td>
</tr>
<tr>
<td>200600</td>
<td>693</td>
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</tr>
<tr>
<td>12388800</td>
<td>1520</td>
<td>$-323$</td>
</tr>
</tbody>
</table>

*Responses derived from data that were corrected for induction in the oceans.

4. 1-D Conductivity Model Estimation

Using the quasi-Newton (QN) algorithm of Byrd et al. [1995] we derived spherical 1-D conductivity models from the various C-response estimates. The spherical layer thicknesses increase with depth as a geometric series of step size 1.1, starting from a top layer thickness of 0.5 km. Each model terminates with a highly conducting core at a depth of 2900 km. The solution is stabilized by requiring minimum first derivative of log(conductivity) with respect to log(depth).

For the considered 1-D problem this QN algorithm produces results that are very similar to those obtained with the Occam algorithm of Constable et al. [1987]. However, we prefer to use the QN approach since it allows for imposing constraints on the model parameters (in our case log(conductivity)). In addition, we believe that the limited memory QN scheme is more suitable for an efficient solution of the three-dimensional (3-D) inverse problems [cf. Newman and Boggs, 2004] that we plan to perform in the future.

The regularization parameter $\alpha$ of our inversion scheme describes the trade-off between model misfit and model smoothness. $\alpha$ is found by means of a cooling approach; starting from a large value of $\alpha$, several inversions are performed with decreasing $\alpha$ until the misfit reaches its target value (which depends on the assumed data errors). However, due to different ways of calculating
data errors when deriving the various data sets we used a heuristic approach and selected $\alpha$ such that similar model smoothness is obtained.

[14] Figure 2 shows the result of the inversion. At depths greater than 400 km the conductivity obtained from our satellite C-responses is rather similar to that derived from ground-based data. All models show a monotonic increase of conductivity from 0.03–0.08 S/m at $z = 400$ km depth to 1–2 S/m at $z = 900$ km depth. However, conductivity based on our new satellite responses is slightly (but systematically) higher at all depths below 400 km.

[15] At depths greater than 900 km the conductivities recovered from ground-based data and our satellite responses are almost constant, as opposed to the large conductivity increase (up to 200 S/m or higher) reported by Constable and Constable [2004]. However, we believe that the latter result is an artefact due to the multi-taper approach in combination with the short Magsat time-series of only 6 months used by Constable and Constable [2004], as can be demonstrated from analyzing synthetic time series of similar length.

[16] At depths shallower than 400 km our conductivity model is more conducting compared to the other profiles. In the next section we show that correction of the data for induction in the oceans removes this difference. Note, however, that results at shallow depths have to be taken with caution because the responses in the considered period range are not sensing the uppermost part of the model very well.

5. Correction for Induction in the Oceans

[17] In order to correct for the effect of induction in the non-uniform oceans, we used the following scheme:

[18] 1. The external (inducing) field (i.e., time series of the expansion coefficient $c_i^j(t)$) is determined, as described in section 2.

[19] 2. Using this time-space structure of the external field, we predict synthetic satellite magnetic signals for 1-D and 3-D conductivity models, respectively. Here the 3-D model consists of the 1-D mantle conductivity, overlaid by nonuniform oceans. Details of this magnetic field prediction, which is based upon a Fourier transformation of the inducing field and a frequency domain forward modeling, is given by Kuvshinov et al. [2006].

[20] 3. A correction of the magnetic field residuals, $\delta B$, for induction in the oceans is performed as

$$\delta B_{corr} = \delta B + \delta B^{1D} - \delta B^{3D}. \tag{5}$$

[21] 4. New C-responses and 1-D conductivity profiles are estimated from the corrected residuals $\delta B_{corr}$ using the approach described in sections 2–4.
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Olsen, N. (1998), Estimation of C-responses (3 h to 720 h) and the electrical conductivity of the mantle beneath Europe, Geophys. J. Int., 133, 298–308.


6. Conclusions

We estimated C-responses for periods ranging from 14 hours to four months using years (2001–2005) of simultaneous magnetic data collected by the three satellites Ørsted, CHAMP and SAC-C.

Interpreting these responses in terms of mantle conductivity leads to values that are similar to those derived from ground-based data, at least for depths greater than 400 km. All models show a monotonic increase of conductivity from 0.03–0.08 S/m at z = 400 km to 1–2 S/m at z = 900 km. However, our new 1-D model is slightly (but systematically) more conducting at all depths greater than 400 km.

At periods shorter than 7 days, satellite responses are probably influenced by induction in the oceans. We therefore applied a scheme to correct the data for this effect, which resulted in significant changes of the estimated C-responses. Inversion of the corrected C-responses makes the conductivity at depths shallower than 400 km more consistent with values derived from ground-based data.

Good consistency of our satellite and ground-based results support the idea that magnetic satellite data can also be applied to probe the internal structures of other planets. This is a major advantage of the EM method compared to seismological methods, which require installation of seismographs on the surface.

The results reported here are based on the assumption that mantle conductivity only varies with depth. Work is ongoing to detect lateral variations of mantle conductivity. The results of this effort will be the subject of a forthcoming publication.

[24] Applying this correction leads to significant changes in the estimated C-responses at periods shorter than 7 days (see the right parts of Table 1 and Figure 1). As expected, Re{C} increases, whereas Im{C} gets smaller (less negative). Inverting the corrected C-responses leads to more resistive conductivity at depths shallower than 400 km, and thus to values that are closer to those determined from observatory data (see Figure 2). The results are almost independent of the 1-D conductivity model used for calculating the correction of equation (5).

[25] An attempt has been made to improve the correction by iterative use of equation (5). However, such an iteration did not cause significant changes in the responses. Moreover, our correction scheme neither results in increasing coherency nor reduced misfit. The normalized misfit of the optimal 1-D conductivity model (known as D’ model [cf. Parker and Whaler, 1981]), $\chi^2/2K = 1.7$, still exceeds its expectation value of 1. Whether this is due to contamination of the residuals by non-induction signals, shortcomings of the data analysis, inconsistency of the external field model, an inaccuracy in the correction prediction, or the effect of hypothetic mantle inhomogeneities is presently unknown. One may speculate if magnetic field variations due to motional induction in the oceans contaminate the response functions at shorter periods. However, estimates of tidal magnetic fields at satellite altitudes [cf. Kuvshinov and Olsen, 2005a] show that these signals are much smaller than those of magnetospheric origin and hence negligible in this context.


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